Definition 1: A generator matrix is called a generator matrix whose \( r \times k \) minors are coprime if it is a non-polynomial code.

Definition 2: A generator matrix is called a convolutional code.

We denote \( G(a) \) is a generator matrix for the code.

\[
\{(a)_m : a \in \mathbb{F}_q^n \mid (a)_m G(a)_m = e\}
\]

by \( G(a) \) is defined as the set of \( (a)_m \) for a convolutional code \( G(a) \).

Let \( \mathbb{F}_q \) be a finite field and \( G(a) \) be a \( k \times r \) matrix over \( \mathbb{F}_q \).
choose the basic generator matrix.

Note that the definition is independent of the particular coin matrix of $G$.

We define the degree of a convolutional code $C$ to be the maximum degree of the code $C(G)$. The convolutional code $C(G)$ is said to be the convolutional code generated by the matrix $G$ if and only if there exists a matrix and $G$ which generate the same convolutional code.

Let $W$ be any two basic matrices $G_1$ and $G_2$.
2. The Generalized Singleton Bound

Remark 12. An upper bound on $d_a$ is given by

$$d_a \leq 2 \sum_i f_i$$

If the encoder is non-catastrophic, then

$$d_{\infty} = d_a \leq d_{\infty}$$

Also,

$$\frac{d_a}{d_{\infty}} = \frac{d_{\infty}}{d_{a}}$$

The limit exists if $\lim_{i \to \infty} f_i = 0$ and

$$\lim_{i \to \infty} f_i = 0$$

Then the upper bound on distance is defined as:

$$d_a \leq \sum_i f_i$$

Definition 10. Let $G_{f}$ denote the $k \times (1 + f)u$ matrix with indices $(a, b)$.

Definition 11. Let $G_{f}$ be the $k \times (1 + f)u$ matrix with indices $(a, b)$. Let $G_{f}$ be the $k \times (1 + f)u$ matrix with indices $(a, b)$. Let $G_{f}$ be the $k \times (1 + f)u$ matrix with indices $(a, b)$.
Theorem 15: For any positive integer $k > n$, and each field $\mathbb{F}$, we have:

\[ 1 + (1 + n - u)^{-1} \leq \sum_{i=0}^{n-k} \begin{pmatrix} n-k \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \]

\[ \Rightarrow \sum_{i=k}^{n-k} \begin{pmatrix} n-k \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \leq 1 + (1 + n - u)^{-1} \]

The main result of [RS95] states:

\[ 1 + g + (1 + \lceil \sqrt{g} \rceil) (x - u) \leq \sum_{i=0}^{n-k} \begin{pmatrix} n-k \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \]

bounded by conditional code $C$ of degree $d$, the free distance is $\text{dist}(C)$.

For these indices we have:

\[ 1 + n - u \leq \sum_{i=0}^{n-k} \begin{pmatrix} n-k \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \]

Remark 15: For given $n$, $k$, and $d$, the upper bound (3) reduces to the Singleton block code case.

Proof: We may assume that

\[ (a), C \]

with

\[ [(a), C]_{u+n-k} \subseteq [(a), C]_{u+k} \]

and

\[ (a), C \]

\[ \begin{pmatrix} n \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \]

\[ \Rightarrow \sum_{i=k}^{n-k} \begin{pmatrix} n-k \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \leq 1 + (1 + n - u)^{-1} \]

Remark 14: If $a = 0$ and $k = 1$, i.e., block code case.

Proof: We may assume that

\[ (a), C \]

\[ \begin{pmatrix} n \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \]

and

\[ \Rightarrow \sum_{i=k}^{n-k} \begin{pmatrix} n-k \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \leq 1 + (1 + n - u)^{-1} \]

Theorem 11: For any positive integer $k > n$, and each field $\mathbb{F}$, we have:

\[ 1 + g + (1 + \lceil \sqrt{g} \rceil) (x - u) \leq \sum_{i=0}^{n-k} \begin{pmatrix} n-k \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \]

bounded by conditional code $C$ of degree $d$, the free distance is $\text{dist}(C)$.

For these indices we have:

\[ 1 + n - u \leq \sum_{i=0}^{n-k} \begin{pmatrix} n-k \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \]

Remark 15: For given $n$, $k$, and $d$, the upper bound (3) reduces to the Singleton block code case.

Proof: We may assume that

\[ (a), C \]

with

\[ [(a), C]_{u+n-k} \subseteq [(a), C]_{u+k} \]

and

\[ (a), C \]

\[ \begin{pmatrix} n \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \]

\[ \Rightarrow \sum_{i=k}^{n-k} \begin{pmatrix} n-k \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \leq 1 + (1 + n - u)^{-1} \]

Remark 14: If $a = 0$ and $k = 1$, i.e., block code case.

Proof: We may assume that

\[ (a), C \]

\[ \begin{pmatrix} n \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \]

and

\[ \Rightarrow \sum_{i=k}^{n-k} \begin{pmatrix} n-k \end{pmatrix} \binom{i}{k} R_{1}^{(i)} \leq 1 + (1 + n - u)^{-1} \]
Define the generator matrix:

\[
\begin{pmatrix}
(a)^{e-v} & \cdots & (a)^{e-v}a \\
\vdots & \ddots & \vdots \\
(a)^{e-v} & \cdots & (a)^{v}a \\
(a)^{e-v} & \cdots & (a)^{v}a
\end{pmatrix}
= (a)^G
\]

We want an MDS convolutional code.

Consider convolutional codes. When this reduction was able to construct MDS convolutional codes, we saw that it is possible to construct an MDS convolutional code.
Example 22. We want a triple $$(2/3)$$-NDS-code with $\gamma = 2$.

Example 23. We want a triple $$(2/3)$$-NDS-code with $\gamma = 2$.

Remark 21. A red $g$ which satisfies

$$\frac{(\gamma - u)}{g + 1} + \frac{\gamma}{g} \geq \frac{\gamma}{g}$$

where

$$|1 - (1 + a)u| - N = \gamma$$

and $u = 1 - \gamma < b$

Conditions. The construction works under the

Remark 20. The construction works under the

Remark. This gives the representation of a cyclic block code.

Theorem 19. Let $p \in \mathbb{N}$. Then $q = p^n$.

Corollary. If the elements $a, b, c$ are called $R^4$-equivalent then

$$a = b.$$
Remark 23. Let $c$ be generated by $(a)^6$. Then $a$ is a generator of a cyclic block code (N, F). Let $H$ be the dual polynomial associated to $(a)^6$. The construction of the cyclic block code $(N, F)$ is given by the matrix:

\[
\begin{bmatrix}
(a)^{1+6} & (a)^{2+6} & \cdots & (a)^{6+6} \\
\vdots & \vdots & \ddots & \vdots \\
(a)^{1+6} & (a)^{2+6} & \cdots & (a)^{6+6} \\
(a)^{6} & (a)^{1+6} & (a)^{2+6} & \cdots & (a)^{6+6}
\end{bmatrix}
\]

where

\[-a(a)(a)^{1+6} + \cdots + a(a)(a)^{6+6} = (a)^6H.
\]

Example 23. Another example of an MDS code is the extended RS code for $B = 1 - b = N^2 = 222$. The construction of the smallest primitive power $b = 2$, i.e., $b = 1 - b = N = 2$. The extended binary primitive field is $F = \mathbb{F}_2$. The construction of an MDS code over $F_2$ and degree $6 = 12$.

\[
\begin{bmatrix}
0 & a^{1+6} & a^{2+6} & \cdots & a^{6+6} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & a^{1+6} & a^{2+6} & \cdots & a^{6+6} \\
0 & a^6 & a^{1+6} & \cdots & a^{6+6}
\end{bmatrix}
\]

Example 23. The extended RS code of degree 6 is given by $a$ with $a$ a primitive element of $F_2$. If $a$ is an encoder for a convolutional code of length $l$, then $a = a^{1+6}$, and the convolutional encoder becomes: $I_{1, n} = a^{1+6}$, and $I_{l} = a^{6}$, if $n = l$ then $I_{1} = a$. The convolutional encoder.

\[
\begin{bmatrix}
0 & a^{1+6} & a^{2+6} & \cdots & a^{6+6} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & a^{1+6} & a^{2+6} & \cdots & a^{6+6} \\
0 & a^6 & a^{1+6} & \cdots & a^{6+6}
\end{bmatrix}
\]

Example 23. The extended RS code of degree 6 is given by $a$ with $a$ a primitive element of $F_2$. If $a$ is an encoder for a convolutional code of length $l$, then $a = a^{1+6}$, and the convolutional encoder becomes: $I_{1, n} = a^{1+6}$, and $I_{l} = a^{6}$, if $n = l$ then $I_{1} = a$. The convolutional encoder.

\[
\begin{bmatrix}
0 & a^{1+6} & a^{2+6} & \cdots & a^{6+6} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & a^{1+6} & a^{2+6} & \cdots & a^{6+6} \\
0 & a^6 & a^{1+6} & \cdots & a^{6+6}
\end{bmatrix}
\]
Come up with algebraic decoders.

Construct near-MDS codes over small fields.

Find other constructions over possibly smaller fields.

Research Problems

\[ \begin{bmatrix}
1 & a + a + 1 & a + a + 1 \\
1 & a + 1 & a \\
a + 1 & a & a + 1
\end{bmatrix} = (a) H \]

A generator matrix is then:

\[ \begin{bmatrix}
1 & a + 1 & a + 1 \\
a + 1 & a & a + 1
\end{bmatrix} = (a) H \]

Matrix:

Let \( C \) be the convolutional code with the parity check \( BCH \)-code. Let \( h(x) = x^3 + x^2 + x + 1 \).

Example 26. Let \( C_{12^d} \) be the \( (12, 6) \) convolutional code. Describe what a generator polynomial of \( (a) H \) of a BCH code is. It was developed.

Following the same approach we can obtain.

\[ g = (a) H \cdot (a) H \]

Respectively, they are dual, i.e.,

\[ (a) H \]

Then the convolutional code defined by \( (a) H \)

\[ \left( X_{0} \right) \cdots \left( X_{N-1} - a \right) \left( X_{N} - a \right) \cdots \left( X_{N-1} - a \right) \left( X_{0} - a \right) = (a) H \]

\[ \left( X_{N} - a \right) \cdots \left( X_{N} - a \right) \left( X_{0} - a \right) \left( X_{0} - a \right) = (a) H \]

\[ (X_{N} - a) \cdots (X_{N} - a) = 1 - a \]

\[ \left( X_{N}^d - a \right) \cdots (X_{N}^d - a) = 1 - a \]

\[ \left( X_{N}^d - a \right) \cdots (X_{N}^d - a) = 1 - a \]
References