Ambiguity in Codes

Fernando Guzmán

Binghamton University

fer@math.binghamton.edu
The code $C = \{a, b, c, d\}$ where:

\[
\begin{align*}
    a &= 1 \ 1 \ 0 \\
    b &= 1 \ 1 \ 0 \ 1 \ 1 \\
    c &= 1 \ 0 \ 1 \\
    d &= 0 \ 1 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1
\end{align*}
\]

is an MSD code but it is not UD. It satisfies the relation $abcd = bdac$ as we can see in:

\[
\begin{array}{cccccc}
    a & b & c & d \\
    \hline
    1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\
    b & d & a & c
\end{array}
\]

and any other relation it satisfies is a consequence of this one. This code was introduced by Lempel in 1986, as the first known example of a proper MSD code.
The code $C = \{a, b, c, d\}$ where:

\[
\begin{align*}
    a &= 0 \ 1 \\
    b &= 1 \ 0 \\
    c &= 0 \ 0 \ 1 \ 0 \ 0 \\
    d &= 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1
\end{align*}
\]

is an SD code but it is not MSD.

\[
\begin{array}{cccc}
    a & a & c & b \\
    \hline
    0 & 1 & 0 & 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \\
    \hline
    d & c
\end{array}
\]

0:

\[
\begin{array}{cccc}
    a & c & b & d \\
    \hline
    0 & 1 & 0 & 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \\
    b & a & c & b
\end{array}
\]