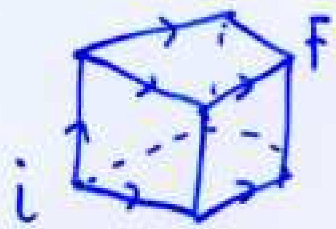
Higher $\vec{\pi}_n$



$\vec{\pi}_1(X, i, f) = \{*\}$

Surface of a cube

$\vec{\pi}_2$ should be nontrivial

constant maps from S^{n-1}

$$\vec{\pi}_n(X, x_0, x_1) = \vec{\pi}_1(X^{S^{n-1}}, x_0, x_1)$$

$f, g : S^{n-1} \to X$

$f \leq g \iff \forall x \in S^{n-1}\ f(x) \leq g(x)$

Two compositions

$$\vec{\pi}_n(X, x_0, x_1) \times \vec{\pi}_n(X, x_1, x_2) \to \vec{\pi}_n(X, x_0, x_2)$$

$* \in S^{n-1}$, basepoint, $\alpha : \vec{I} \to X$, $\alpha(0) = x_0$, $\alpha(1) = x_1$

$$\vec{\pi}_n(X, \alpha, x_0, x_1) = \vec{\pi}_1\left((X, \alpha)^{(S^{n-1}, *)}, x_0, x_1\right)$$

$f_t(x) = \alpha(t)$

$$\vec{\pi}_n(X, \alpha, x_0, x_1) \times \vec{\pi}_n(X, \alpha, x_0, x_1) \to \vec{\pi}_n(X, \alpha, x_0, x_1)$$

$$[f_t], \quad [g_t] \longrightarrow [\underbrace{f_t * g_t}_{\text{as in } \pi_{n-1}}]$$

GROUPSTRUCTURE!