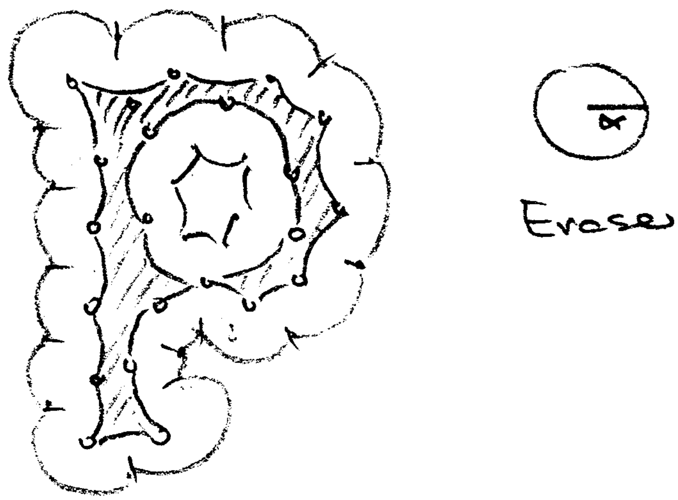
ALPHA SHAPES

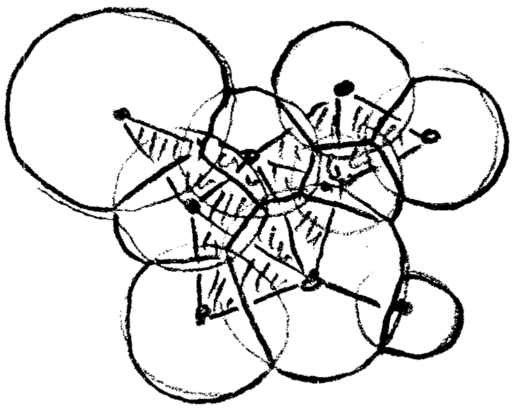
Alpha hull.



α-hull = all pts. that cannot be reached by (open) empty erasers of rad. α.

α-shape = obtained by making all edges straight.

Space-filling diagram.



... are popular models for molecules in chemistry and structural biology

$$\mathcal{B} = \{ b_i = (p_i, r_i) \mid 1 \le i \le n \}$$

Voronoi diagram decomposes the union of balls into convex regions of the form

$$\bigcup \mathcal{B} \cap V_i = b_i \cap V_i .$$

$\boxed{\text{Dual complex.}}$

<u>dual complex</u> K is nerve of $\{ b_i \cap V_i \}$.

$$\begin{array}{c} K \\ \cap \\ \mathrm{Del}(\mathcal{B}) \end{array}$$

homotopy equivalent (from Nerve Lemma; explicit

$\boxed{\text{Alpha complex.}}$

$$b_{i,\alpha} = (p_i, \sqrt{r_i^2 + \alpha^2}) \qquad \alpha^2 \in \mathbb{R}$$

bisector $\pi_{i,\alpha}(x) = \pi_{j,\alpha}(x)$

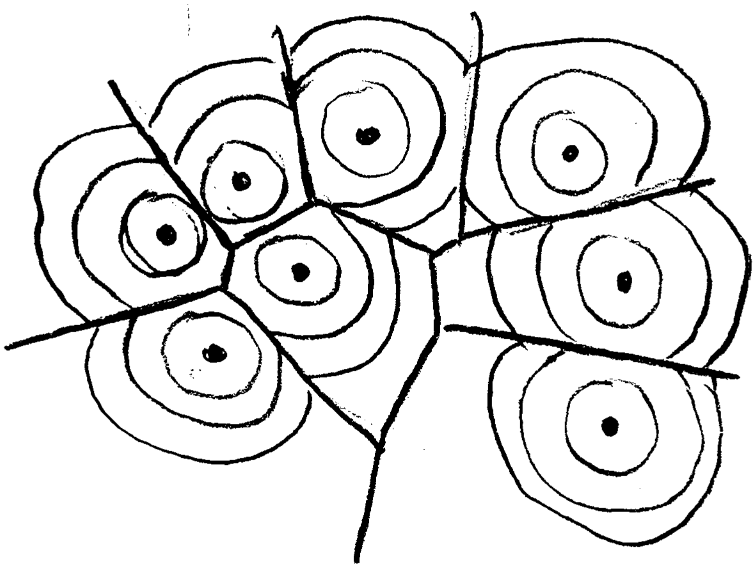
$$\|x - p_i\|^2 - r_i^2 - \cancel{\alpha^2} = \|x - p_j\|^2 - r_j^2 - \cancel{\alpha^2}$$

is the same for all α.

<u>alpha complex</u> K_α is nerve of $\{ b_{i,\alpha} \cap V_i \}$

$$\begin{array}{c} K_\alpha \\ \cap \\ \mathrm{Del}(\mathcal{B}) \end{array}$$

<u>alpha shape</u> = | alpha complex |. (underlying space)



Filtration.

$$\emptyset = K^0 \subset K^1 \subset \ldots \subset K^n = \mathrm{Del}(B)$$

we usually use finest nested sequence in which consecutive complexes differ by few simplices only.

History.

1983: α-shapes in $\mathbb{R}^2$ by E., Kirkpatrick, Seidel

1990: implementation and publically available software by E. Mücke.

1994: α-shapes in $\mathbb{R}^3$ by E. Mücke

α-shapes in biology

α-shapes for surface reconstruction

others