Pressure Waves and Premixed Flames

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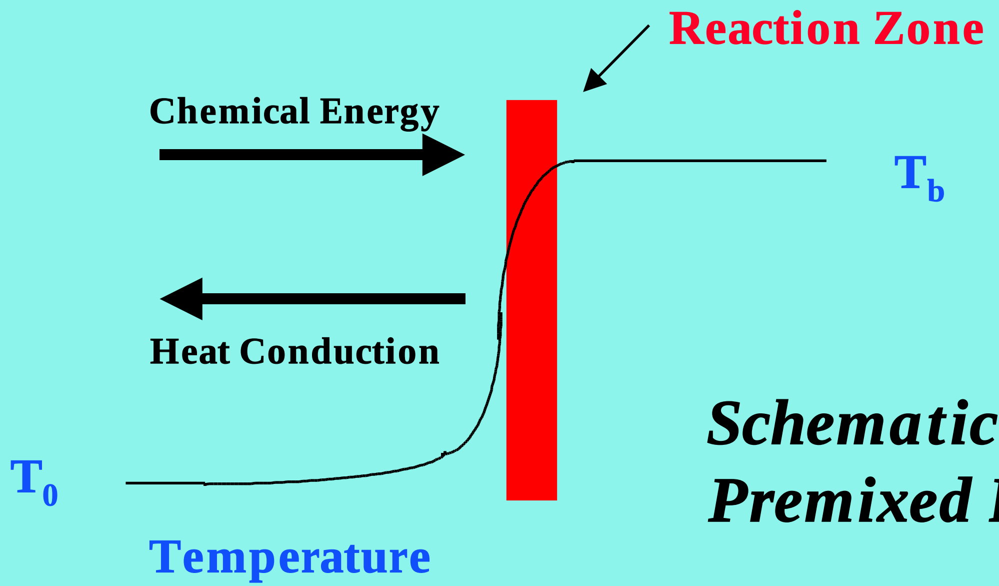
"Without theory,
practice is but the routine of habit....."

Louis Pasteur

Pressure Waves and Premixed Flames

- Flames, burning velocity and surface area
- 2-D Mechanisms
  - Rayleigh Taylor Effect
  - Baroclinic Effect
- 1-D acoustics
- 1-D extinction effects
  - Normal flow
  - Strained flow
- 1-D Weakly compressible flames

Pressure Waves and Premixed Flames



Schematic of Premixed Flame

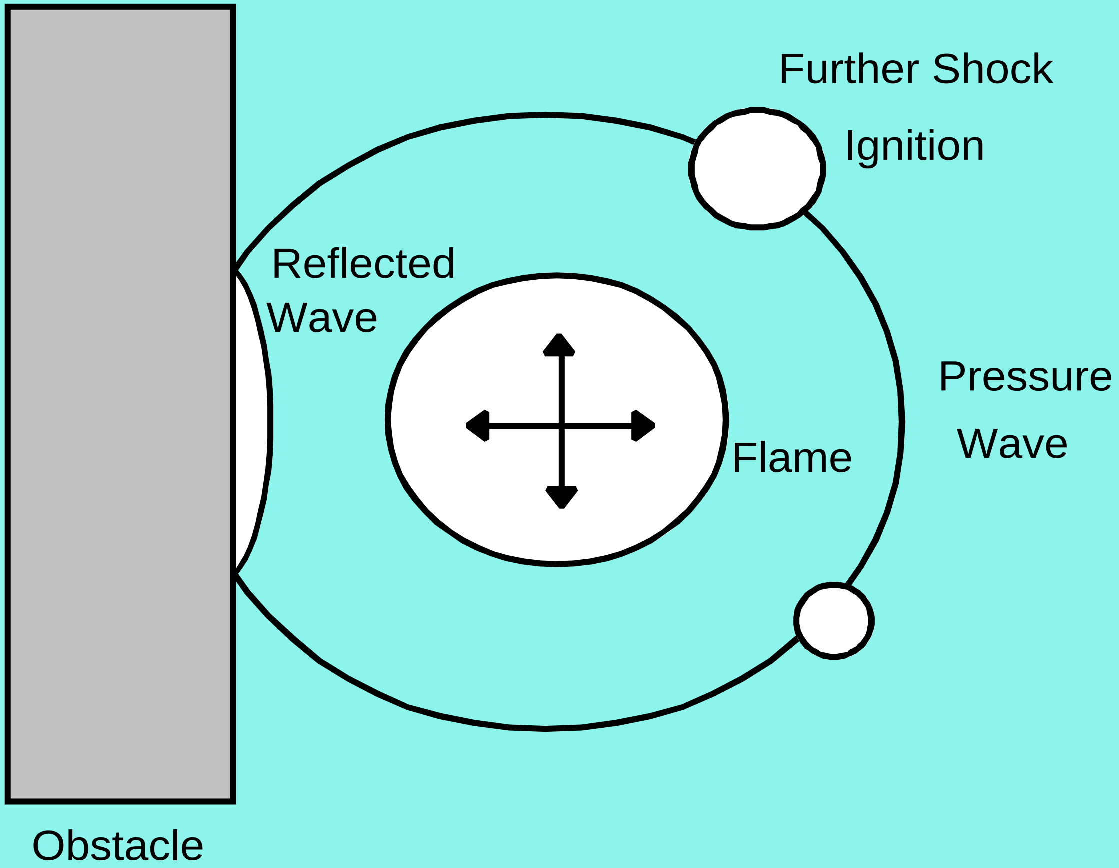
Flames, burning velocity and surface area

Pressure Waves and Premixed Flames

- **2-D Interactions**

Rayleigh-Taylor Effect

Pressure Waves and Premixed Flames



Pressure Interaction Effects during the early stages of Ignition

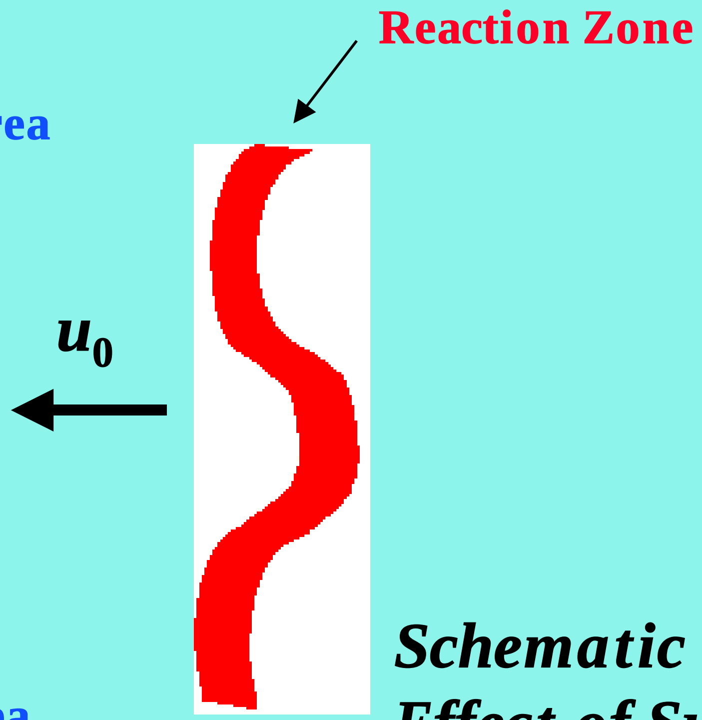
Pressure Waves and Premixed Flames

Mass burning rate — Mass Flux — Surface Area

$$M = (\rho u_0)S$$

$$\bar{u}_0 = \frac{M}{\rho A}$$

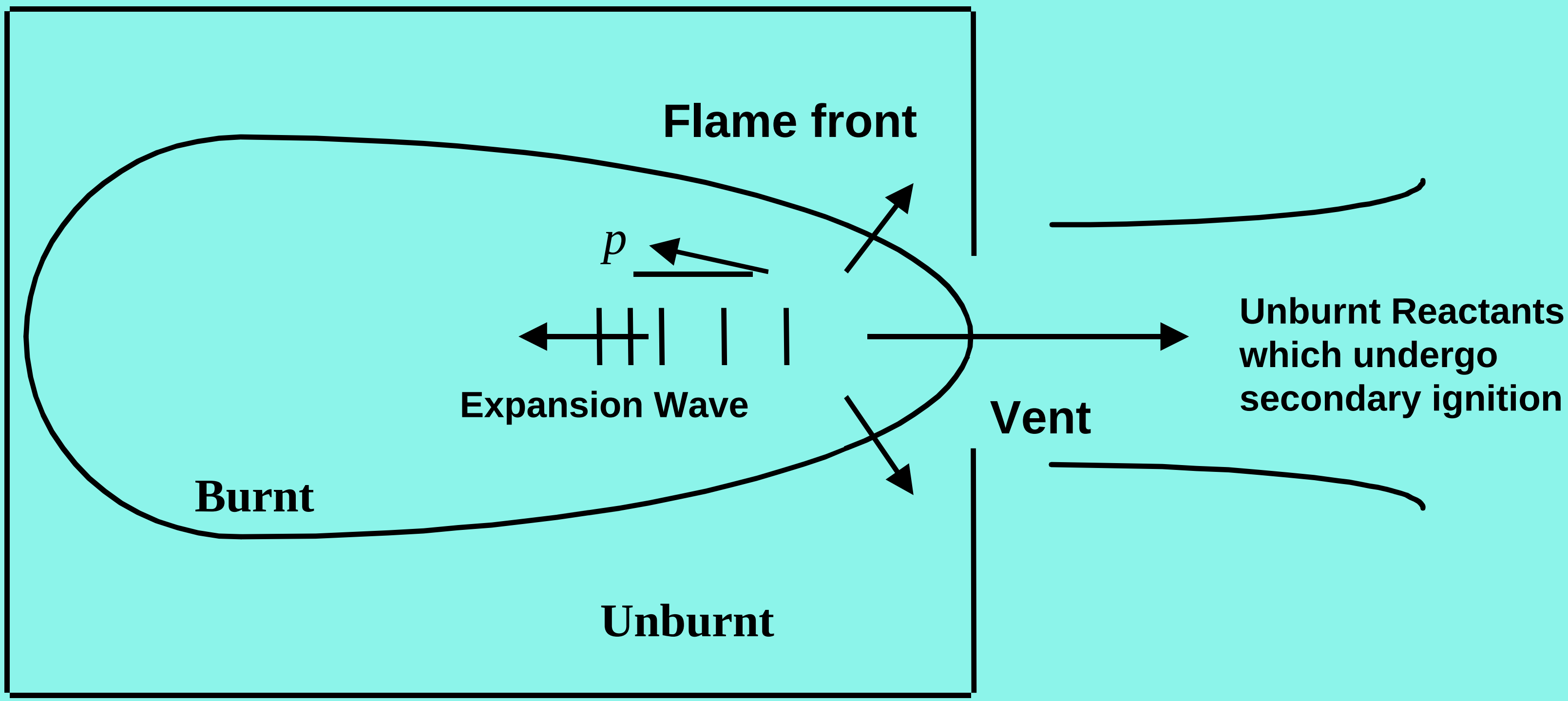
Average Burning Velocity — Original Area



Schematic of Effect of Surface Area

Flames, burning velocity and surface area

Pressure Waves and Premixed Flames



Schematic of vented explosion showing the flame front near the vent and decompression wave passing through.

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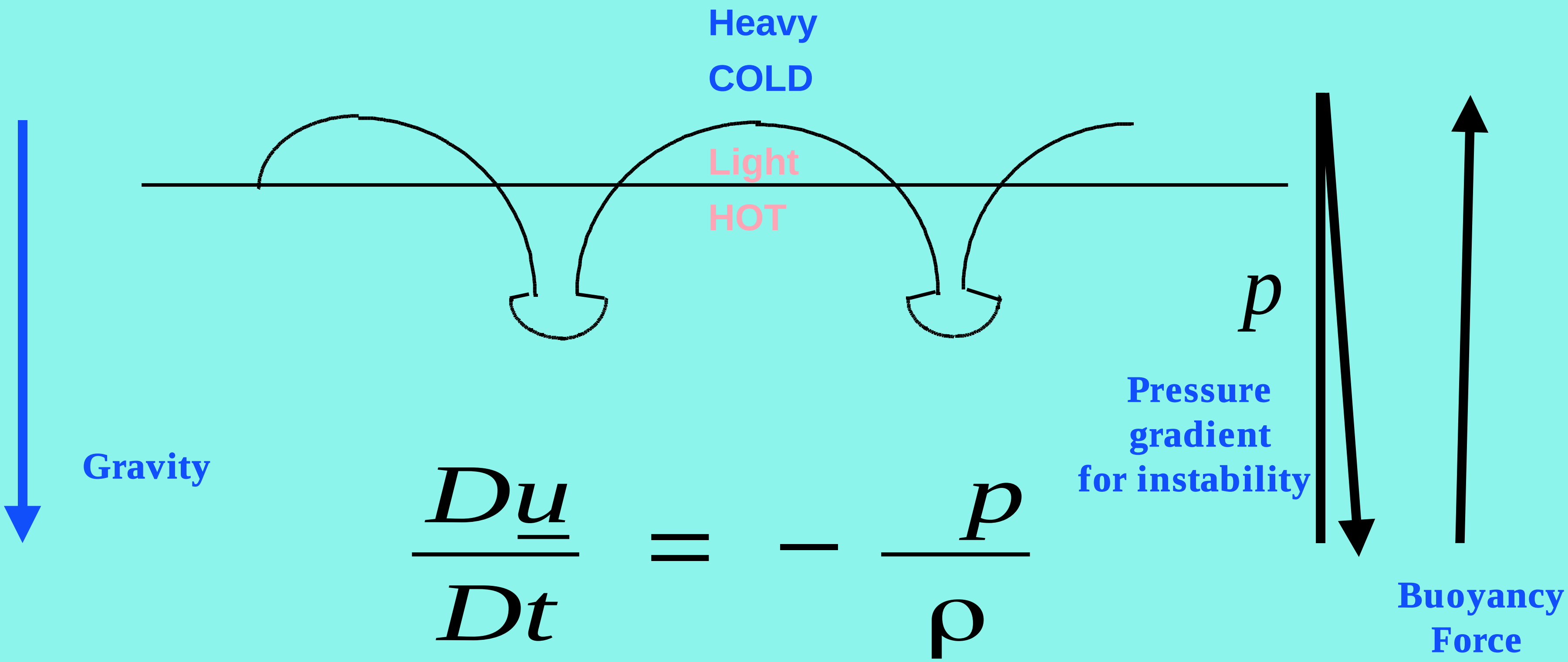
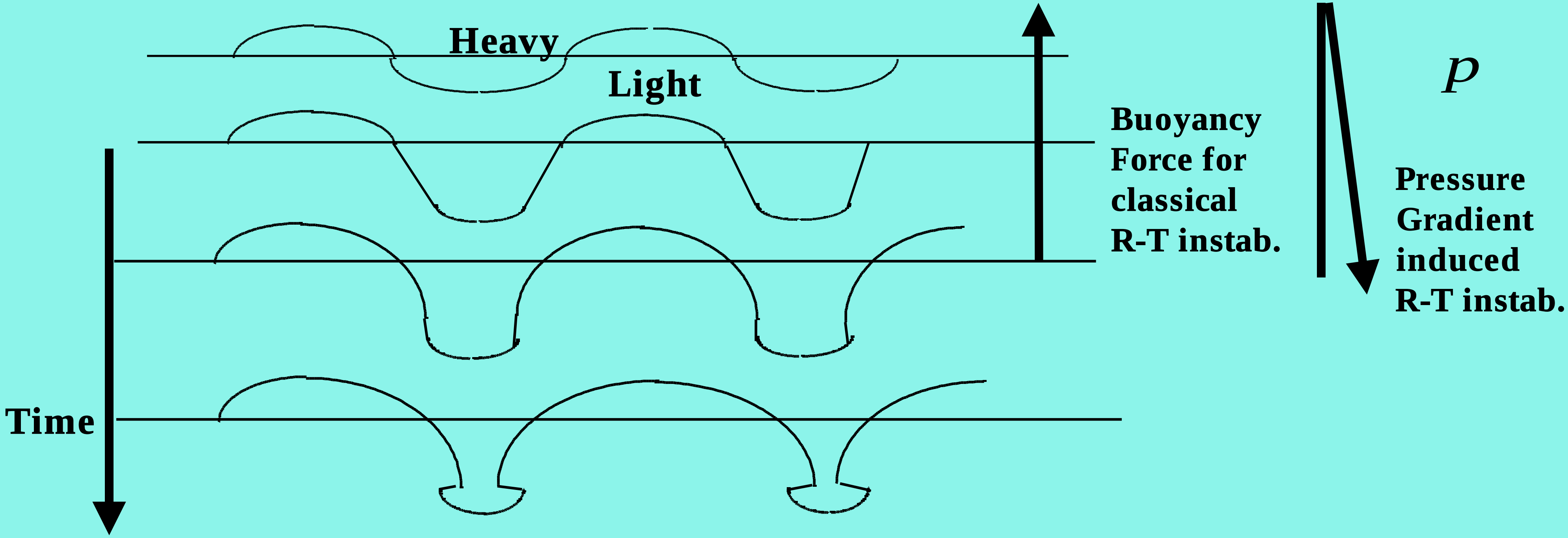


$$\frac{D\underline{u}}{Dt} = - \frac{p}{\rho}$$

Acceleration Pressure gradient

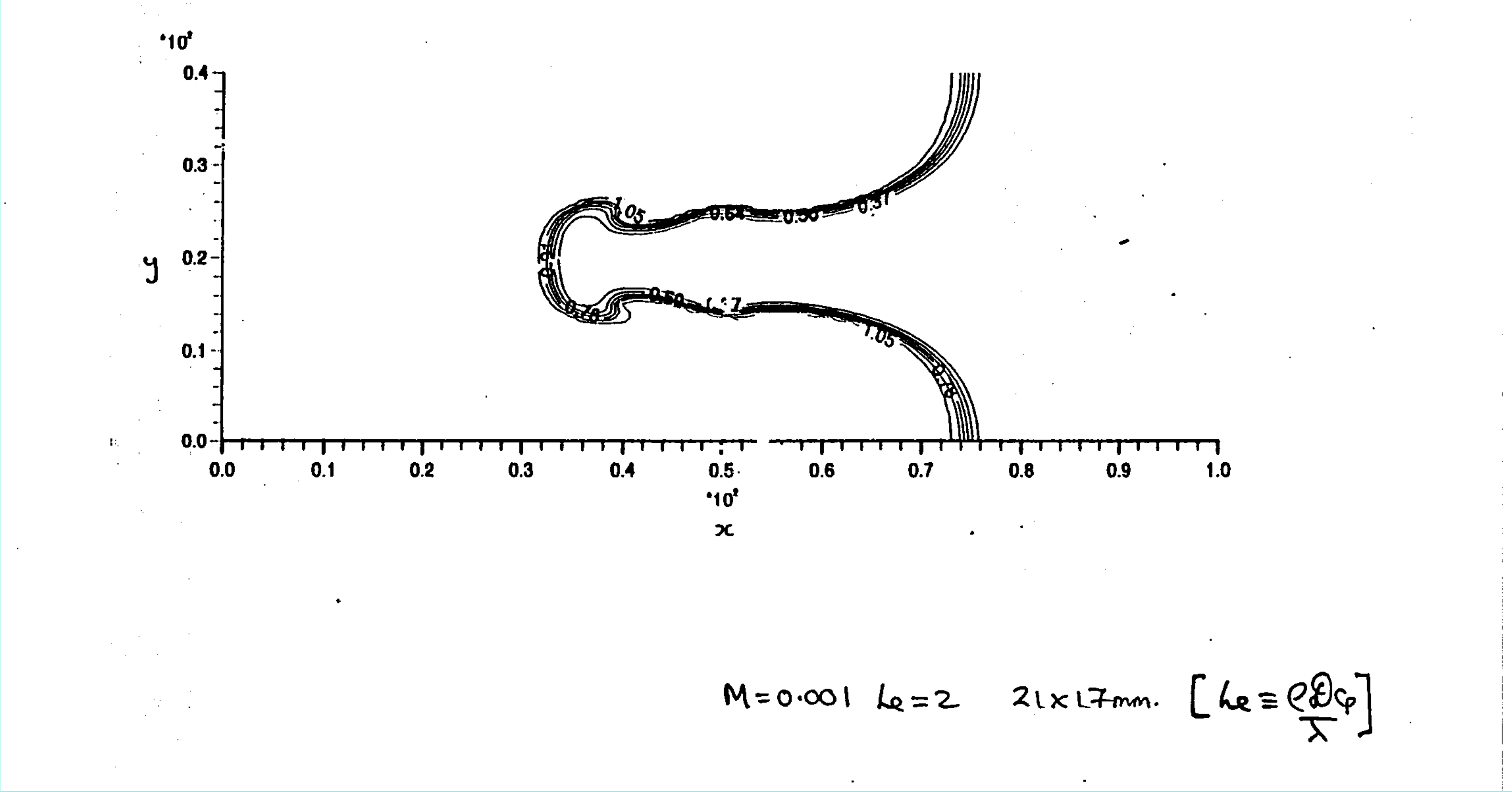
(u, p, ρ, t and space coord. are here dimensional)

Pressure Waves and Premixed Flames



Schematic of the Rayleigh-Taylor Instability applied to Premixed Flames

Pressure Waves and Premixed Flames



Temperature contours of a rippled flame undergoing interaction with a 1 atm m^{-1} pressure gradient, after 0.98ms (Edwards, McIntosh and Brindley, CST 99,179-200, 1994).

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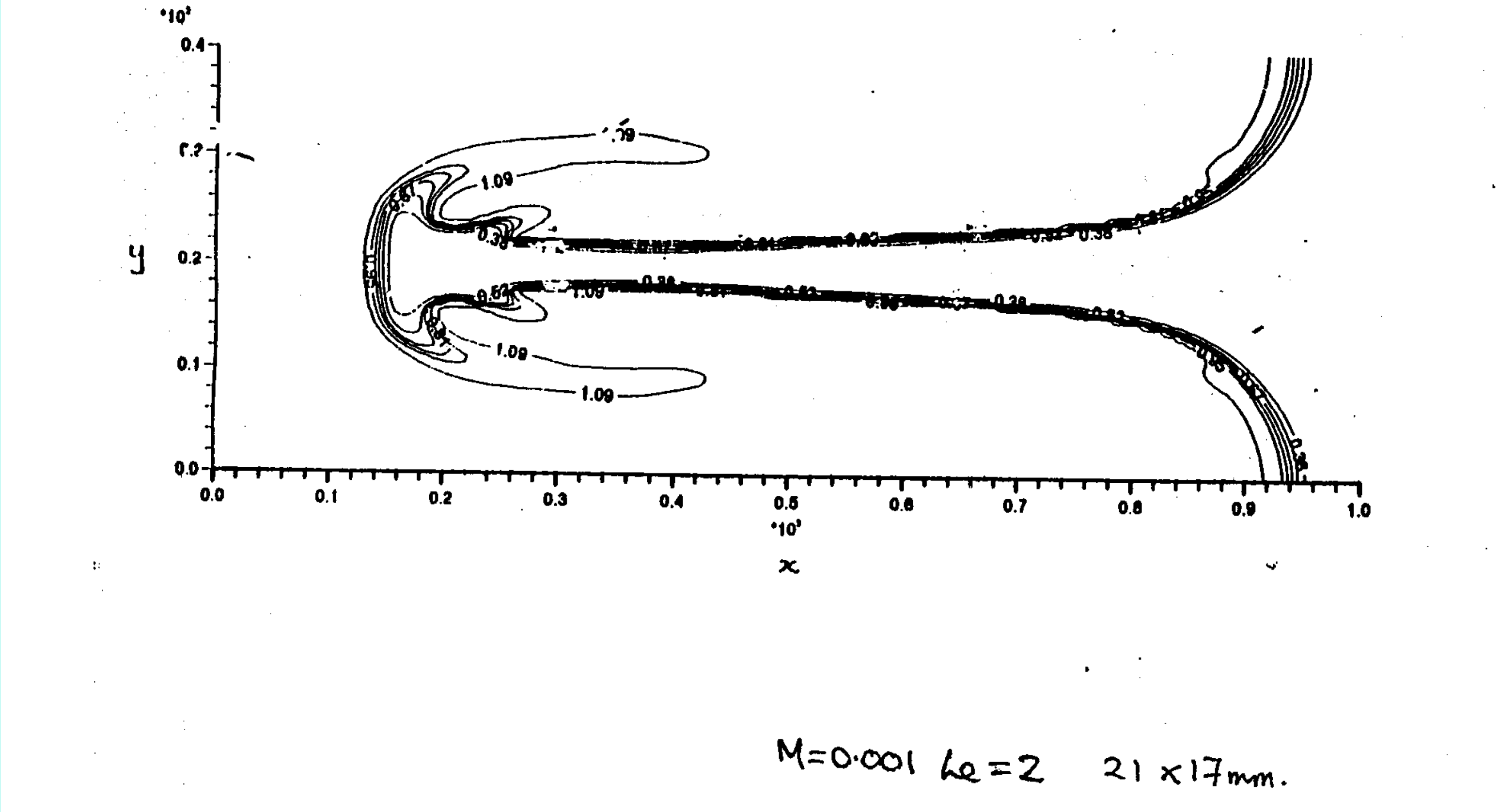
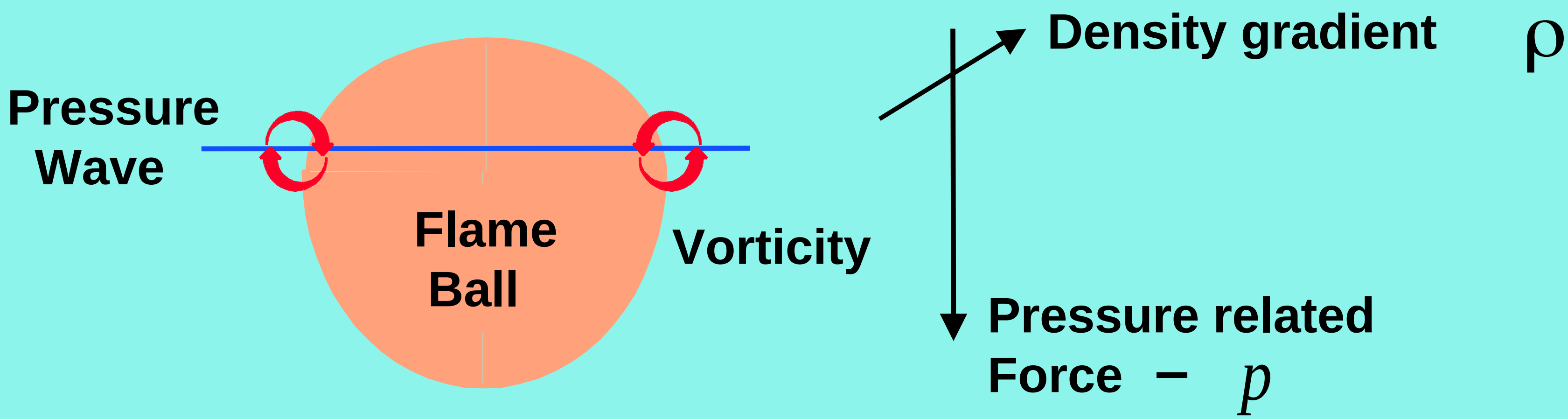


Temperature contours of a rippled flame undergoing interaction with a 1 atm m^{-1} pressure gradient, after 1.28ms (Edwards, McIntosh and Brindley, CST 99,179-200, 1994).

- **2-D Interactions**

Shock-Waves & Generation of Turbulence

Pressure Waves and Premixed Flames



$$\frac{D(\omega/\rho)}{Dt} = \frac{\nabla\rho \times \nabla p}{\rho^3} + \frac{\omega}{\rho}\cdot\nabla u + \nu\nabla^2\frac{\omega}{\rho}$$

Baroclinic generation of vorticity due to a shock wave passing through a curved flame.

Pressure Waves and Premixed Flames

General 2-D Equations including viscosity

$$p = \frac{\rho T}{T_{01}}$$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \underline{u}) = 0$$

$$\rho \frac{\partial \underline{u}}{\partial t} + \rho \underline{u}.\nabla \underline{u} = -\frac{1}{\gamma M^2} \nabla p + \frac{\Pr}{Le} \nabla.(\mu \underline{\underline{S}})$$

$$\rho \frac{\partial T}{\partial t} + \rho \underline{u}.\nabla T - \frac{1}{Le} \nabla.(\rho D \nabla T) = \rho Q R$$

$$+(1-\gamma^{-1}) \frac{\partial p}{\partial t} + \underline{u}.\nabla p + \gamma M^2 \frac{\Pr}{Le} \mu [\underline{\underline{S}}] : [\nabla \underline{u}]$$

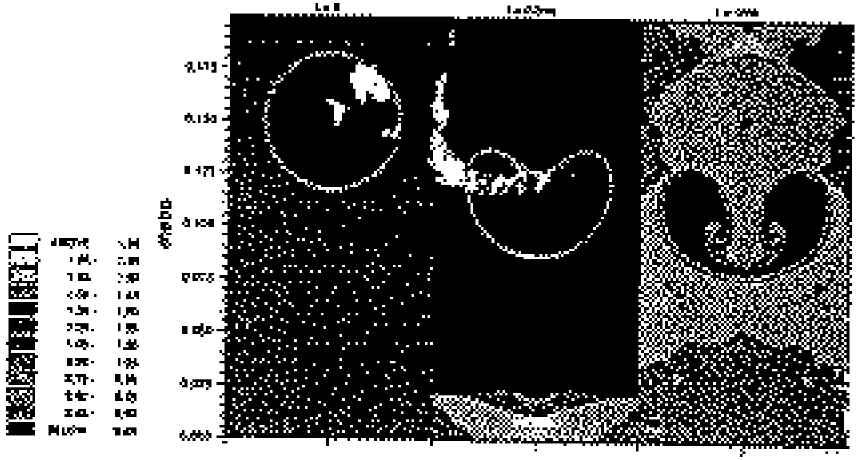
$$\rho \frac{\partial C}{\partial t} + \rho \underline{u}.\nabla C - \nabla.(\rho D \nabla C) = - \rho R \; ; \; R \quad A C e^{-\theta/T}$$

Assume (as in 1-D)

$\rho \boldsymbol{D} \propto \boldsymbol{T}$ ***(as λ).***

2nd order Godunuv scheme used to find numerical solution.

Pressure Waves and Premixed Flames



igure 0: The Initial Evolution of The Density Distribution During Th Interaction Between a Positive Pressure Step with Fractional Amplitu .3 and A Cylindrical Flame Ball of Radius 3cm. Grid Size dx = dy .4mm

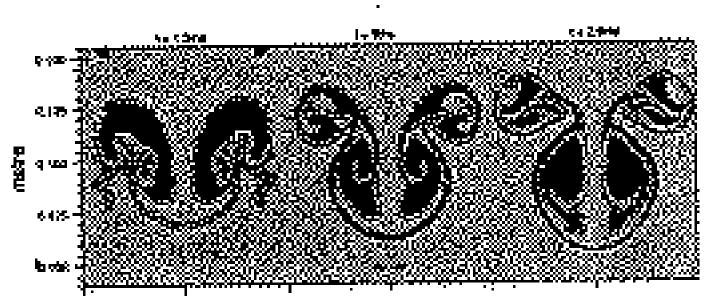


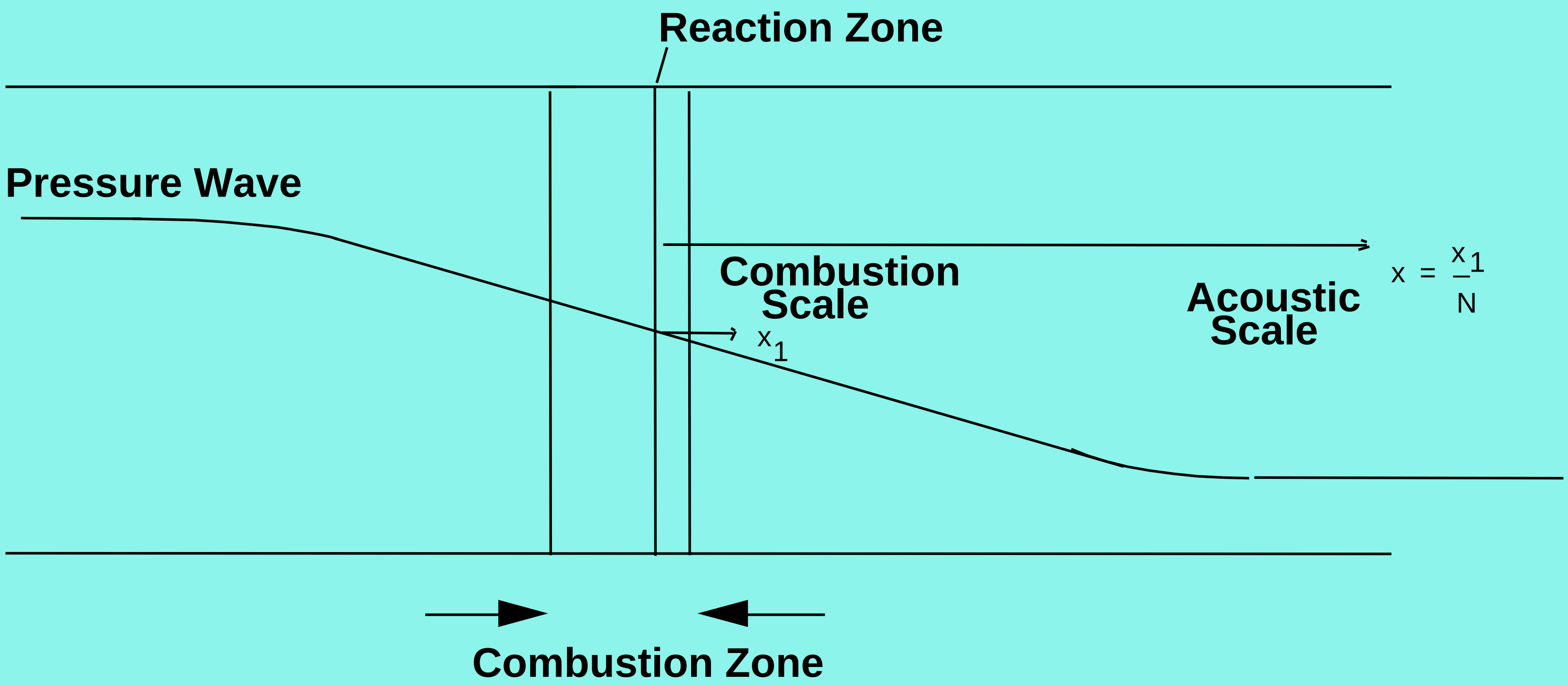
Figure 6: The Evolution of The Density Distribution between t = 1.5ms and t = 2.5ms during run 1, which Simulated the Behaviour of the Slowest Flame with Low Viscosity (Pr = 1). Grid Size dx = dy = 0.4mm

The effect of a shock wave of strength 0.3 bar propagating a flame ball with local laminar flame speed 1 ms^{-1}. The flame is in a tube closed at one end. Proc. Roy. Soc. A452, 199-221, (1996).

- **1-D Interactions**

General Considerations & Important Ratios

Pressure Waves and Premixed Flames



N ~ number of typical combustion lengths in one acoustic length.

τ ~ ratio of typical combustion (difusion time) to acoustic time.

Typical length and time-scales for pressure interactions with flames.

Pressure Waves and Premixed Flames

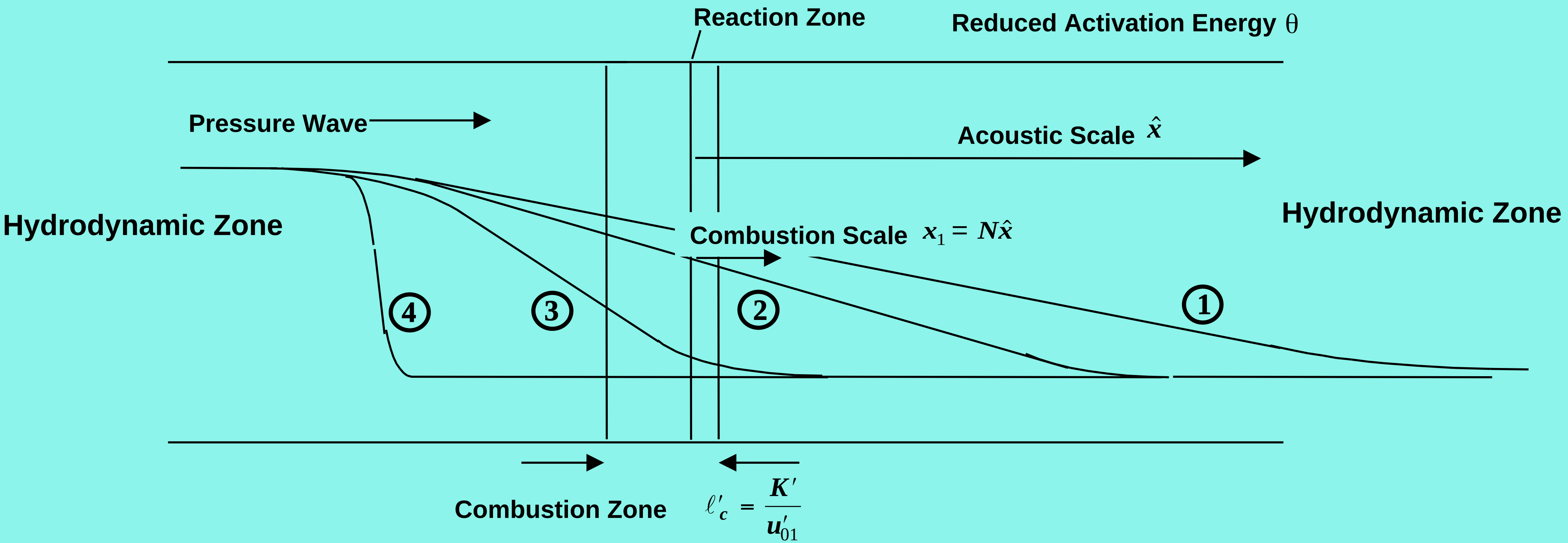
• Three Important Ratios

$$\tau \qquad \frac{\textit{diffusion time}}{\textit{acoustic time}} = \frac{K' / u'^{2}_{01}}{l'_{a} / a'_{01}}$$

$$N \qquad \frac{\textit{acoustic wavelength}}{\textit{diffusion length}} = \frac{l'_{a}}{K' / u'_{01}}$$

$$M \qquad \frac{u'_{01}}{a'_{01}} \qquad ? \quad \tau = \frac{1}{NM}$$

Pressure Waves and Premixed Flames



x_1 : mass-weighted coordinate following the flame.

$\hat{x}$: re-scaled coordinate to describe effects driven by spatial pressure gradients (usually away from the combustion zone).

N ~ number of diffusion “lengths“ in one typical length of pressure wave (usually large) $= \ell'_a / \ell'_c$

$\tau = (NM)^{-1}$ i.e. ratio of tdiffusion “time” ℓ'_c / u'_{01} to typical time ℓ'_a / a'_{01} of pressure wave. $(M \quad u'_{01}/a'_{01})$.

① **Long length-scale**
$N >> M^{-1}, \ \tau << 1$

② **Medium length-scale**
$N = M^{-1}, \ \tau = 1$

③ **Short length-scale**
$N = \theta^{-2} M^{-1}, \ \tau = \theta^2$

④ **Ultra-short length-scale**
$N = 1, \ \tau = M^{-1}$

Pressure Waves and Premixed Flames

4 Main Cases :

- **Long length-scale :** $N \quad M^{-1}, \tau \quad 1$

  Relevant to thin fast flame with low frequency acoustics : Flame totally quasi-steady.

- **Medium length-scale :** $N = M^{-1}, \tau = 1$

  Relevant to a wide range of flames - Audible Acoustics. Combustion zone unsteady, reaction zone quasi-steady.

- **Short length-scale :** $N = \theta^{-2} M^{-1}, \tau = \theta^{2}$

  Sharp pressure changes. Combustion zone and reaction zone both unsteady.

- **Ultra-short length-scale :** $N = 1, \tau = M^{-1}$

  Shocks. Two separate events; Shock passage followed by severe distortion of the flame - possible extinction.

Pressure Waves and Premixed Flames

$$p=\frac{\rho T}{T_{01}}\ \lambda \quad T;Sc^{¾}\ \frac{\Pr}{Le}√=1$$

$$\frac{1}{NM}\frac{f\rho}{f\hat{t}}+\frac{m_0}{N}\frac{f\rho}{f\hat{x}}+\frac{\rho^2}{NM}\frac{fu^a}{f\hat{x}}=0$$

$$\frac{1}{NM}\frac{fT}{f\hat{t}}+\frac{m_0}{N}\frac{fT}{f\hat{x}}-\frac{p}{N^2 Le}\frac{f^2 T}{f\hat{x}^2}=QR$$

$$+T_{01}(1-\gamma^{-1})\ \frac{1}{NM\rho}\frac{fp}{f\hat{t}}+\frac{m_0}{N\rho}\frac{fp}{f\hat{x}}+\frac{4\gamma}{3N^2}\ ¾\frac{fu^a}{f\hat{x}}√^{2}$$

$$\frac{1}{NM}\frac{fC}{f\hat{t}}+\frac{m_0}{N}\frac{fC}{f\hat{x}}-\frac{p}{N^2}\frac{f^2 C}{f\hat{x}^2}=-R\,;R\quad ACe^{-\theta/T}$$

$$\frac{1}{NM^2}\frac{fu^a}{f\hat{t}}+\frac{m_0}{NM}\frac{fu^a}{f\hat{x}}-\frac{1}{\gamma NM^2}\frac{fp}{f\hat{x}}=\frac{4}{3N^2 M}\frac{f^2 u^a}{f\hat{x}^2}$$

$\hat{x}$ is acoustic distance $=\frac{x_1}{N}$ where x_1 is combustion distance

$x_1 \quad \int_0^{\hat{x}} \rho\, dx$ ***i.e. mass-weighted.***

$\hat{t}$ is acoustic time $=\frac{t}{NM}$ where t is combustion time.

N : number of "diffusion lengths" in 1 characteristic pressure length-scale.

M : Mach Number of flame propagation $\frac{u'_{01}}{a'_{01}}$

m_0 ***: mass burning rate*** $=\frac{\rho_0}{M}\left(u_0+v^a\right)$

- **1-D Interactions**

Acoustics and Flames

Medium wavelength $N = M^{-1}, \tau = 1$
small amplitude

Pressure Waves and Premixed Flames

Practical Applications :

- **Flame Buzz - very much geometry dependent**

- **Domestic Boilers/ Power Station Furnaces**

Pressure Waves and Premixed Flames

Low amplitude Acoustics and Flames

$$p = 1 + Mp_u^a \quad ; \quad M \ldots \frac{u'_{01}}{a'_{01}} \quad ; \; u^a \ldots \frac{u'}{a'_{01}} \quad ; \; u \ldots \frac{u'}{u'_{01}} \quad \therefore u^a = Mu$$

$$T = T_s + MT_u^a$$

$$\rho = \rho_s + M\rho_u^a$$

$u^a = u_s^a(O(M)) + Mu_u^a \sim Mu$, order of combustion zone velocit

i.e. $u = u_s + u_u^a$ and $u_u^a \cup u_u$.

$$p = \frac{\rho T}{T_{01}} \lambda \quad T ; Sc = 1$$

$$\frac{1}{NM}\frac{f\rho}{f\hat{t}} + \frac{m_0}{N}\frac{f\rho}{f\hat{x}} + \frac{\rho^2}{NM}\frac{fu^a}{f\hat{x}} = 0$$

$$\frac{1}{NM}\frac{fT}{f\hat{t}} + \frac{m_0}{N}\frac{fT}{f\hat{x}} - \frac{p}{N^2 Le}\frac{f^2T}{f\hat{x}^2} = QR$$

$$+T_{01}(1-\gamma^{-1}) \; \frac{1}{NM\rho}\frac{fp}{f\hat{t}} + \frac{m_0}{N\rho}\frac{fp}{f\hat{x}} + \frac{4\gamma}{3N^2} \, ¾ \frac{fu^a}{f\hat{x}} √^{2}$$

$$\frac{1}{NM}\frac{fC}{f\hat{t}} + \frac{m_0}{N}\frac{fC}{f\hat{x}} - \frac{p}{N^2}\frac{f^2C}{f\hat{x}^2} = -R ; R \quad ACe^{-\theta/T}$$

$$\frac{1}{NM^2}\frac{fu^a}{f\hat{t}} + \frac{m_0}{NM}\frac{fu^a}{f\hat{x}} - \frac{1}{\gamma NM^2}\frac{fp}{f\hat{x}} = \frac{4}{3N^2M}\frac{f^2u^a}{f\hat{x}^2}$$

Thus the small combustion velocities ($\sim 0.3\ ms^{-1}$) regarded as a flow velocity perturbation of a low-amplitude acoustic field.

This leads to Wave Equations:

Upstream & Downstream

$$\frac{f^2 p_u^a}{f\hat{t}^2} = \frac{f^2 p_u^a}{f\hat{x}^2} \qquad\qquad \frac{f^2 p_u^a}{f\hat{t}^2} = \frac{1}{T_{01}}\frac{f^2 p_u^a}{f\hat{x}^2}$$

Pressure Waves and Premixed Flames

COMBUSTION EQUATIONS - Asymptotic matching of combustion zones to acoustic zones

$$\frac{f}{f\hat{x}} \quad N \frac{f}{f x_1}$$

$$\frac{fT}{ft} + m_0 \frac{fT}{fx_1} - \frac{1}{Le}\frac{f^2T}{fx_1^2} = QR$$

$$\frac{fC}{ft} + m_0 \frac{fC}{fx_1} - \frac{f^2C}{fx_1^2} = -R \quad ; \quad R \ldots ACe^{-\theta/T}$$

with continuity integrated across the combustion region:

$$\frac{f\rho}{ft} + m_0 \frac{f\rho}{fx_1} + \rho^2 \frac{fu}{fx_1} = 0$$

Combustion Zone expansions :

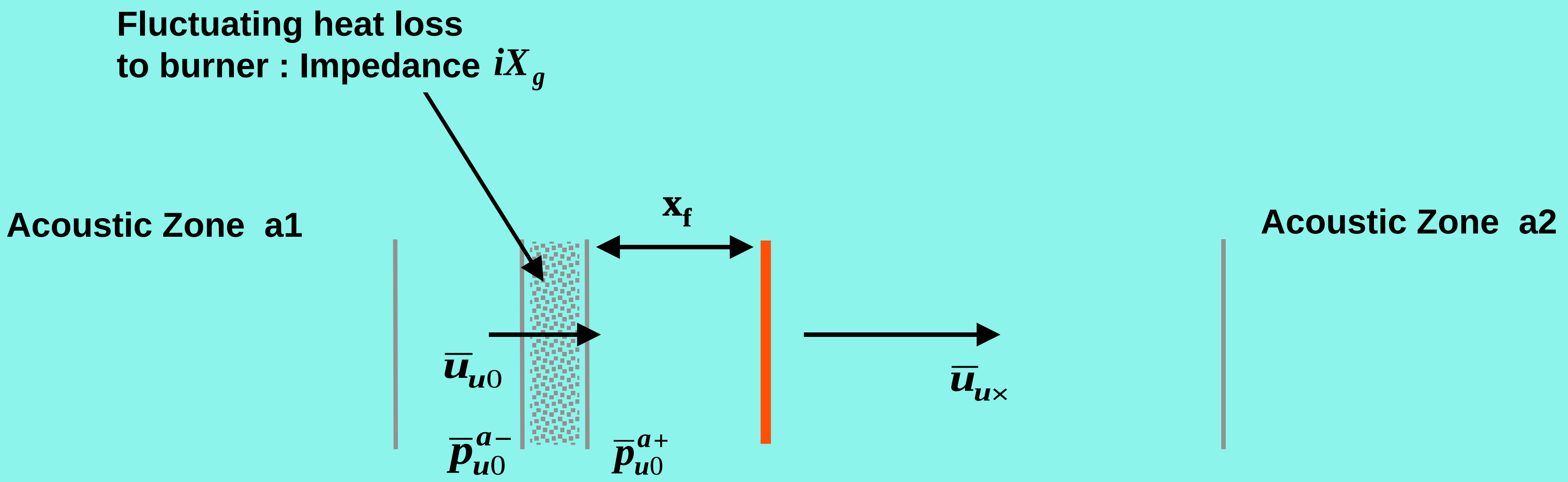
$$p = 1 + Mp_u$$

$$T = T_s + T_u$$

$$\rho = \rho_s + \rho_u$$

$$u = u_s + u_u \; (= u_s + u_u^a)$$

Pressure Waves and Premixed Flames



$$\bar{u}_{u\times} = -\frac{\bar{u}_{u0}}{T_0} V$$

$$\bar{p}^{a+}_{u0} - \bar{p}^{a-}_{u0} = \gamma\, iX_g \bar{u}^{a-}_{u0}$$

Pressure Waves and Premixed Flames

Wave equation upstream

$$\frac{f^2 \boldsymbol{p_u^a}}{f\hat{\boldsymbol{t}}^2} = \frac{f^2 \boldsymbol{p_u^a}}{f\hat{\boldsymbol{x}}^2}$$

Wave equation downstream

$$\frac{f^2 \boldsymbol{p_u^a}}{f\hat{\boldsymbol{t}}^2} = \frac{1}{\boldsymbol{T}_{01}} \frac{f^2 \boldsymbol{p_u^a}}{f\hat{\boldsymbol{x}}^2}$$

Assume Harmonic disturbances : $p_u^a = \overline{p}_u^a e^{\omega t}$ ....*etc*

Jump conditions $\overline{p}_{u0}^{a+} - \overline{p}_{u0}^{a-} = \gamma\, iX_g \overline{u}_{u0}^{a-}$

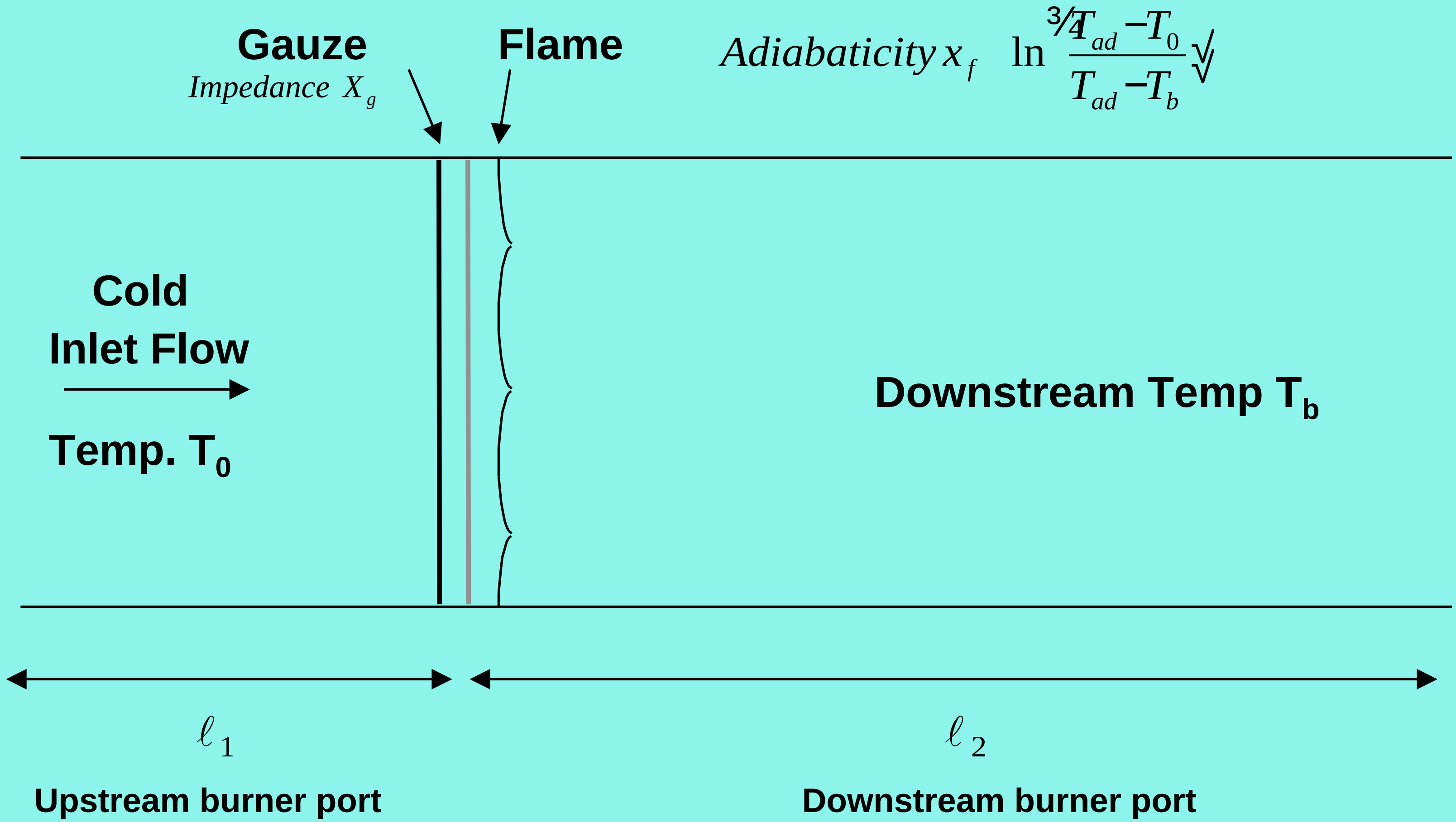
$$\overline{u}_{u0}^{a+} = - \frac{\overline{u}_{u0}^{a-}}{T_0} V$$

where $$V \quad -T_0 - \frac{(1-T_0)(\frac{1}{2}+r)e^{-(\frac{1}{2}+r)x_f}}{2 \quad r e^{-2rx_f} + \frac{\omega}{\theta(1-T_0)}} \; ; r \quad \sqrt{\omega + \frac{1}{4}}$$

comes from asymptotic analysis of small combustion zone diffusion-driven disturbances. (T_0 **is the ratio of upstream to downstream temp.)**

$x_f \quad \ell n \dfrac{T_{ad} - T_0}{T_{ad} - T_b}$ **is the "adiabaticity".** θ **is the activation energy (non-dim.)**

Pressure Waves and Premixed Flames



Simple Model for Flame in Rijke Tube

Pressure Waves and Premixed Flames

$$t_{ac} = \frac{\ell_{ac}}{c} \qquad t_{diff} = \frac{D}{u_0^2}$$

$$t_{ac} = t_{diff} \; ? \quad \ell_{ac} = \frac{cD}{u_0^2}$$

Similarity of time scales for Burner Resonance

Pressure Waves and Premixed Flames

$$\bar{\boldsymbol{u}}_{\boldsymbol{u}0}^{+} = -\frac{\bar{\boldsymbol{u}}_{\boldsymbol{u}0}^{-}}{\boldsymbol{T}_0}\boldsymbol{V} \qquad \text{Adiabaticity}: x_f \quad \ln ¾ \frac{T_{ad}-T_0}{T_{ad}-T_b} √$$

$$V \quad -T_0 - \frac{(1-T_0)\left(\frac{1}{2}+r\right)e^{-\left(\frac{1}{2}+r\right)x_f}}{2\; r e^{-2rx_f} + \frac{\omega}{\theta(1-T_0)}} \;; r \quad \sqrt{\omega+\frac{1}{4}}$$

Velocity Transfer Function for Burner Resonance

Pressure Waves and Premixed Flames

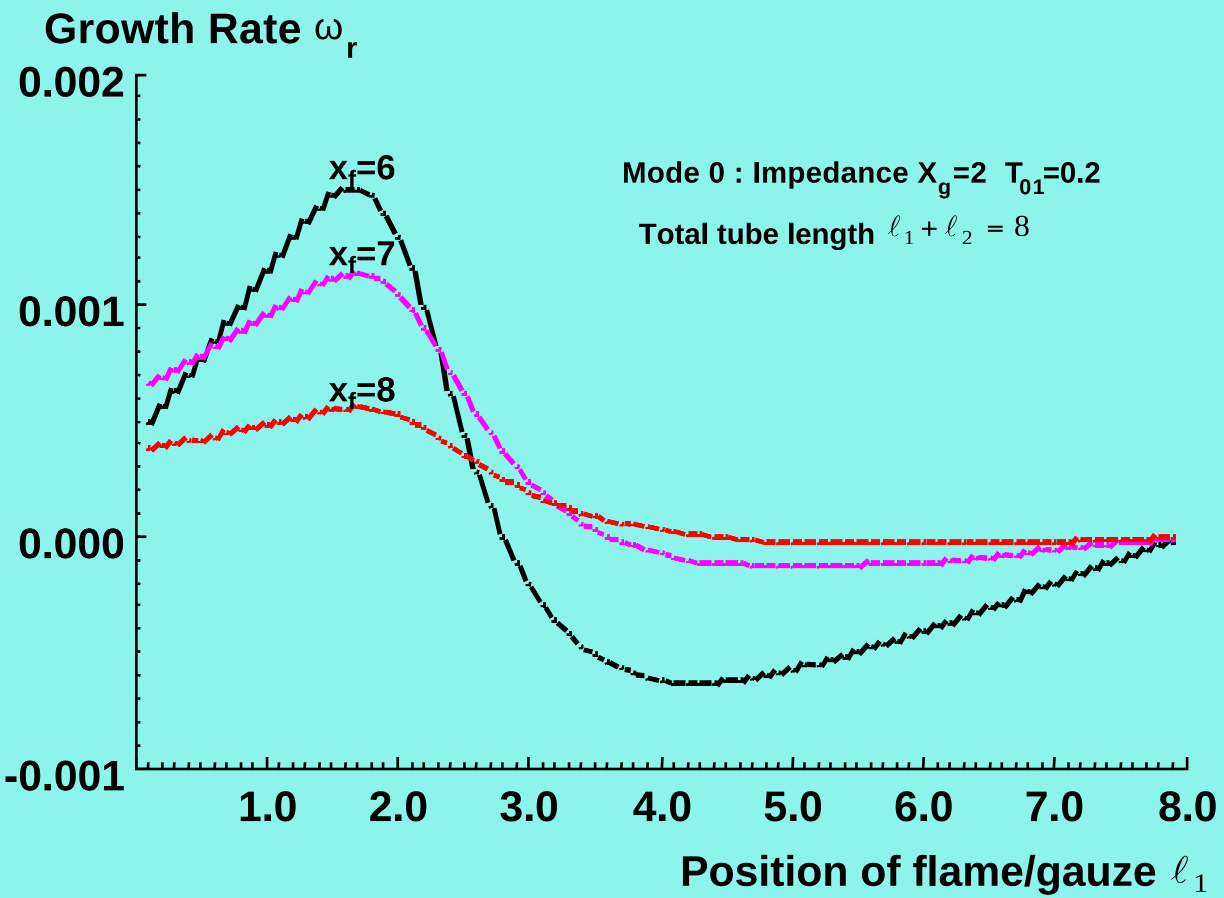
FREQUENCY CONDITION : TUBE OPEN BOTH ENDS

$$V \qquad \frac{\cosh(\omega\ell_1)}{\sinh(\omega\ell_1)} - \sqrt{T_{01}}\,\frac{\cosh\left(\omega\ell_2\sqrt{T_0}\right)}{\sinh\left(\omega\ell_2\sqrt{T_0}\right)} + \sqrt{T_0}\, iX_g\,\frac{\cosh(\omega\ell_1)\cosh\left(\omega\ell_2\sqrt{T_0}\right)}{\sinh(\omega\ell_1)\sinh\left(\omega\ell_2\sqrt{T_0}\right)} = 0$$

FREQUENCY CONDITION : TUBE CLOSED UPSTREAM

$$V - \sqrt{T_0} \qquad \frac{\cosh\left(\omega\ell_2\sqrt{T_0}\right)\cosh(\omega\ell_1)}{\sinh\left(\omega\ell_2\sqrt{T_0}\right)\sinh(\omega\ell_1)} + \sqrt{T_0}\, iX_g\,\frac{\cosh\left(\omega\ell_2\sqrt{T_0}\right)}{\sinh\left(\omega\ell_2\sqrt{T_0}\right)} = 0$$

Pressure Waves and Premixed Flames



Growth rate of gauze tone as a function of flame/gauze position for simple model of flame in Rijke tube.

Pressure Waves and Premixed Flames

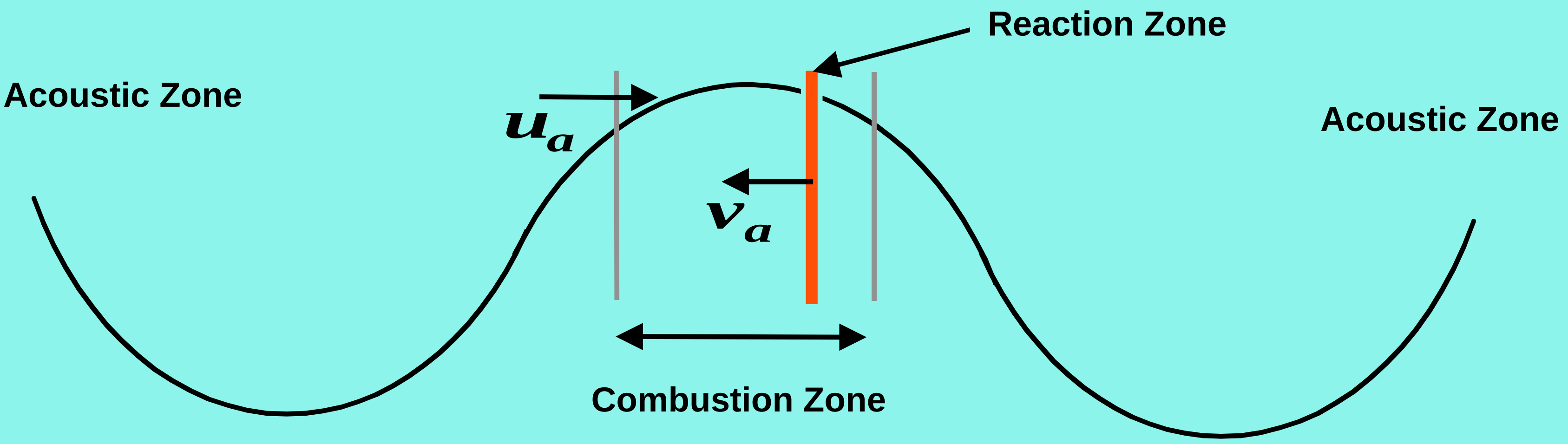
- **1-D Interactions**

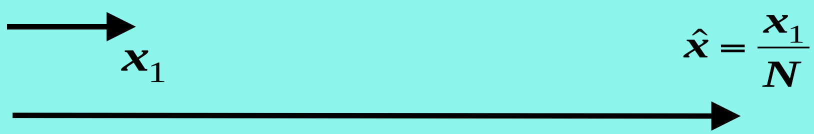
Acoustics and Flames

Medium wavelength $N = M^{-1}, \tau = 1$

Medium amplitude - Free Flames

Pressure Waves and Premixed Flames





Propagation mass flux

$$m_0 = \frac{\rho_0}{M}\left(u_0^a + v^a\right) << O\left(M^{-1}\right)$$

Equations to solve:

$$p_0\left(\hat{t}\right) = \frac{\rho T}{T_{01}}$$

$$\frac{\partial T}{\partial \hat{t}} + m_0 \frac{\partial T}{\partial x_1} - \frac{p_0}{Le}\frac{\partial^2 T}{\partial x_1^2} = QR + \frac{T_{01}}{\rho}\left(1 - \gamma^{-1}\right)\frac{dp_0}{d\hat{t}}$$

$$\frac{\partial C}{\partial \hat{t}} + m_0 \frac{\partial C}{\partial x_1} - p_0 \frac{\partial^2 C}{\partial x_1^2} = -R\,;\, R\quad ACe^{-\theta/T}$$

Pressure Waves and Premixed Flames

Using asymptotically derived jump conditions across the flame equations become :

$$\frac{\partial T}{\partial \hat{t}} + m_0 \frac{\partial T}{\partial x_1} - \frac{p_0}{Le}\frac{\partial^2 T}{\partial x_1^2} = \left(1-\gamma^{-1}\right)\frac{T}{p_0}\frac{dp_0}{d\hat{t}}$$

$$\frac{\partial C}{\partial \hat{t}} + m_0 \frac{\partial C}{\partial x_1} - p_0 \frac{\partial^2 C}{\partial x_1^2} = 0 \; (x_1<0); \; C = 0 \; (x_1>0)$$

with jump conditions :

$$[T]_{0+}^{0-} = 0; [C]_{0+}^{0-} = 0$$

$$\left.\frac{\partial T}{\partial x_1}\right|_{0+}^{0-} = \frac{LeQe^{\frac{1}{2}\theta\left(T_f - 1\right)}}{\sqrt{p_0}}; \quad \left.\frac{1}{Le}\frac{\partial T}{\partial x_1} + Q\frac{\partial C}{\partial x_1}\right|_{0+}^{0-} = 0$$

Harmonic solution for perturbations

$$m_0 = 1 + \varepsilon \overline{m}_{u0} e^{\omega \hat{t}}; p_0 = 1 + \varepsilon \overline{p}_{u0} e^{\omega \hat{t}} ¾ = 1 + \varepsilon \frac{\overline{p}_{u0}^{(1)}}{\theta} e^{\omega \hat{t}} √$$

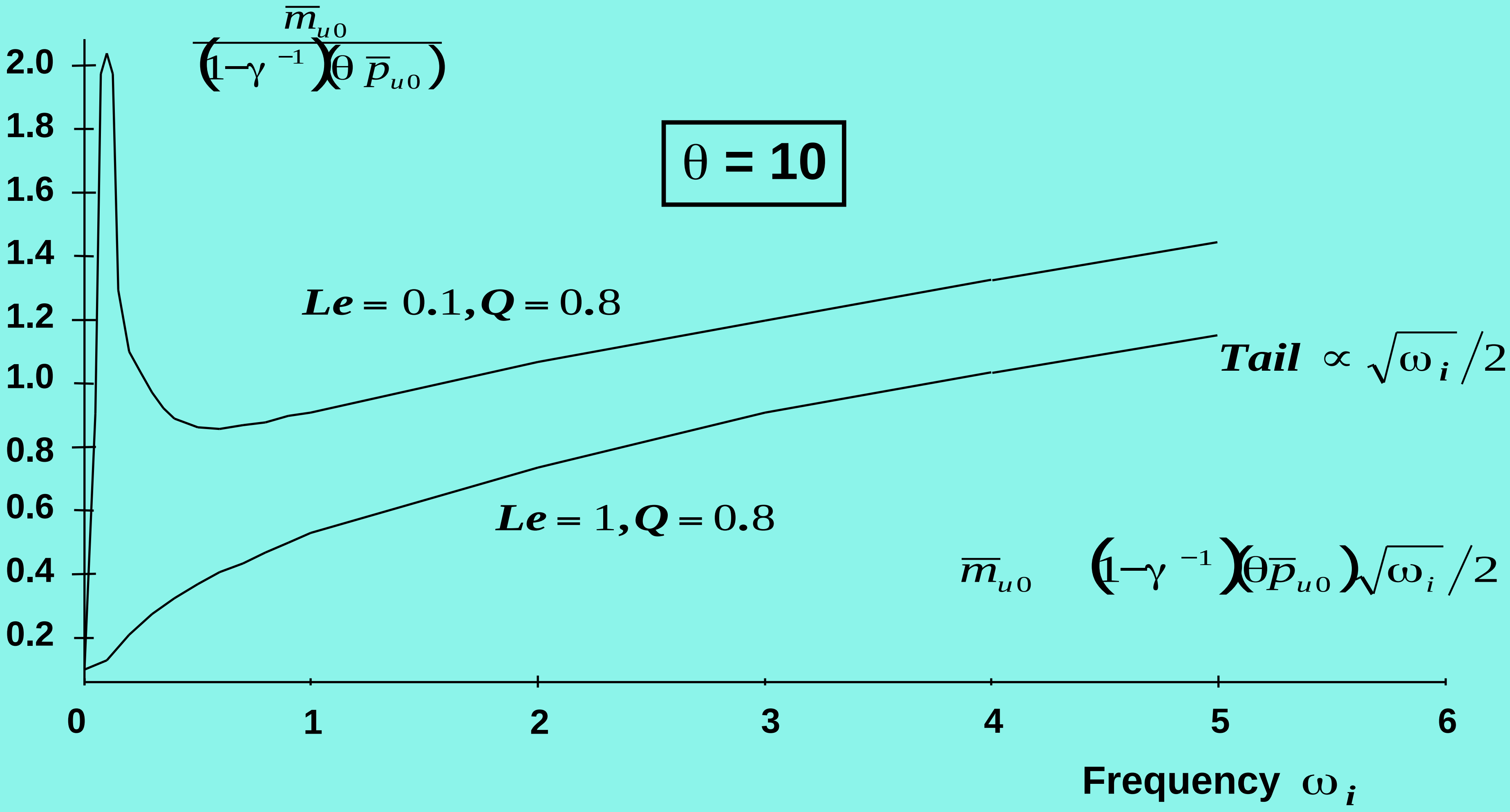
$$\overline{m}_{u0} = \frac{\left(1-\gamma^{-1}\right)\left(\theta\, \overline{p}_{u0}\right)\left(2s - Q\right)}{\frac{\theta Q}{\omega}\left[-Le\left(\frac{1}{2} - s\right) + \left(\frac{1}{2} - r\right)\right] - \frac{4s}{\omega}\left(\frac{1}{2} - r\right)}$$

where

$$\boldsymbol{r} \ldots \sqrt{\omega + \frac{1}{4}}$$

$$\boldsymbol{s} \ldots \sqrt{\frac{\omega}{Le} + \frac{1}{4}}$$

Pressure Waves and Premixed Flames



Amplitude of mass flux perturbation as a function of frequency

Pressure Waves and Premixed Flames

- **1-D Interactions**

Acoustics and Flames

Short wavelength $N = \theta^{-2} M^{-1}, \tau = \theta^{2}$

Medium amplitude - Free Flames

Pressure Waves and Premixed Flames

Short wavelength $N = \theta^{-2} M^{-1}, \; \tau = \theta^{2}$

Equations to solve:

$$p_0(\hat{t}) = \frac{\rho T}{T_{01}}$$

$$\theta^2 \frac{fT}{f\hat{t}} + m_0 \frac{fT}{fx_1} - \frac{p_0}{Le} \frac{f^2 T}{fx_1^2} = QR + \theta^2 \frac{T_{01}}{\rho}\left(1 - \gamma^{-1}\right)\frac{dp_0}{d\hat{t}}$$

$$\theta^2 \frac{fC}{f\hat{t}} + m_0 \frac{fC}{fx_1} - p_0 \frac{f^2 C}{fx_1^2} = -R ; R \quad ACe^{-\theta/T}$$

Now have larger mass flux response :

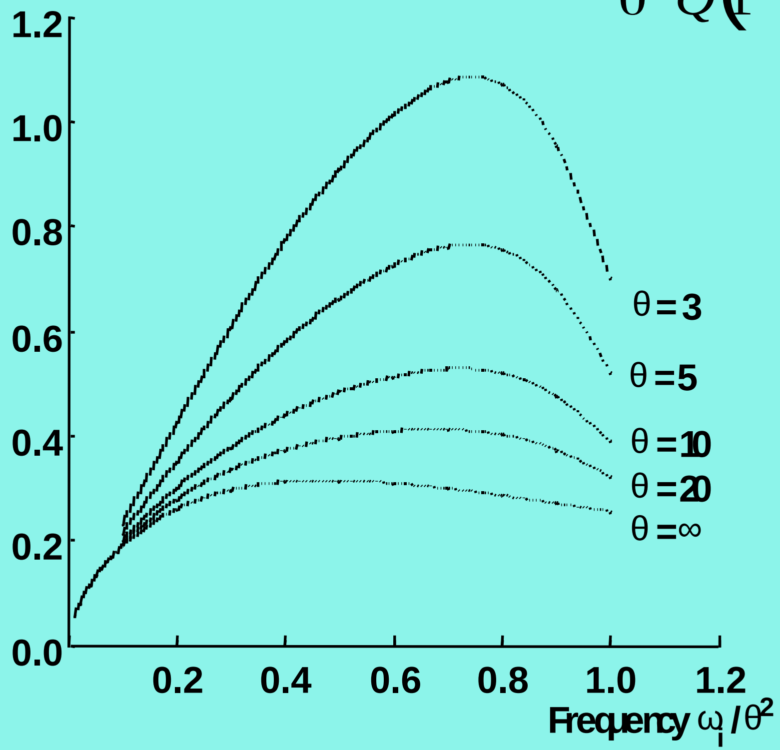
$$m_0 = \theta m_0^{(0)} + m_0^{(1)} + \ldots$$

Typical time scale of pressure wave now on a par with typical time of reaction zone

Pressure Waves and Premixed Flames

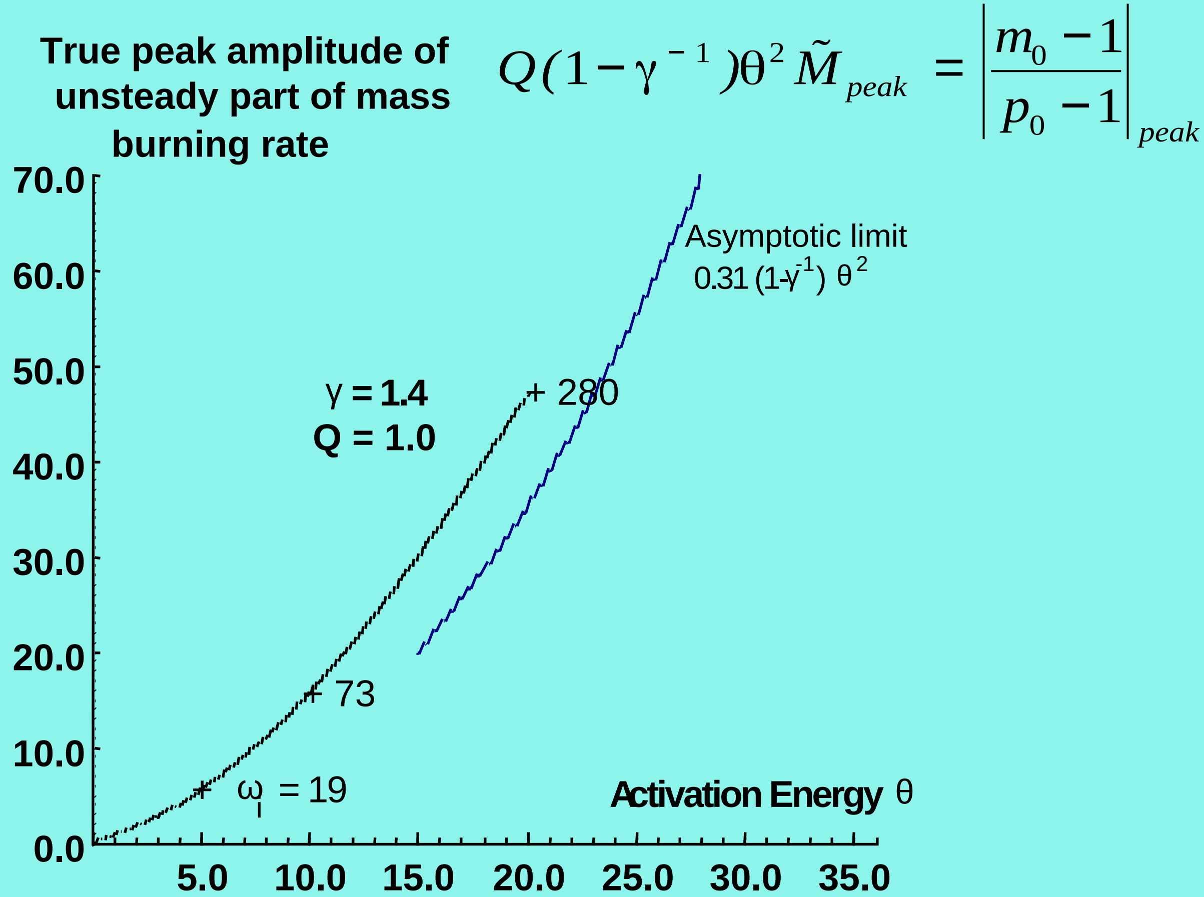
Amplitude of unsteady part of mass burning rate

$$\tilde{M} \quad \frac{\bar{m}_{u0}}{\theta^{2} Q\left(1-\gamma^{-1}\right)\bar{p}_{u0}}$$



Variation of amplitude of mass burning rate ratio with frequency and activation energy.

Pressure Waves and Premixed Flames



Harmonic oscillations of burning rate. True variation of the peak of unsteady fluctuations plotted against activation energy . Peak frequency is also marked.

Pressure Waves and Premixed Flames

There is thus a natural <u>high frequency</u> resonant mode for a given premixed flame and one can predict a dimensional high frequency natural resonance at approximately

$$\omega' \quad \left(0.7 \blacklozenge \; 0.8\right)^{3/4} \frac{E'_A}{R' T'_b} \sqrt{\;}^{2} \frac{\kappa'}{u'_{01}} \quad Hz$$

for a 1-step overall reaction, where θ **is the activation energy of overall reaction,** R' **is the Universal gas constant,** T_b' **is the flame temperature,** K' **is the thermal diffusion and** u_{01}' **is the steady burning velocity.**

This is awaiting verification by experiment.

Pressure Waves and Premixed Flames

- **1-D Interactions - Flat flame, normal flow**

Effect of Pressure Pulse

Short length scale - Medium length scale

$$N = \theta^{-2} M^{-1} \blacklozenge \ M^{-1}, \tau = \theta^{2} \blacklozenge \ 1$$

Medium amplitude - Free Flames : Numerical

Pressure Waves and Premixed Flames

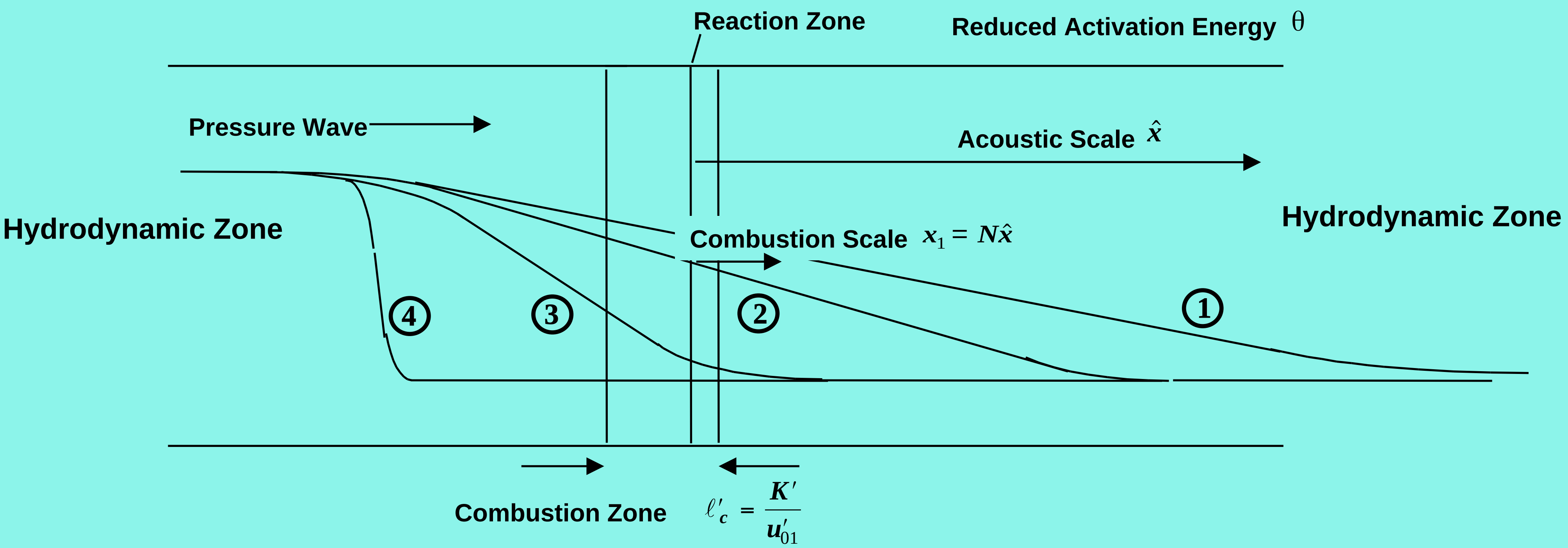
• Three Important Ratios

$$\tau \qquad \frac{\textit{diffusion time}}{\textit{acoustic time}} = \frac{K' / {u'_{01}}^2}{l'_a / a'_{01}}$$

$$N \qquad \frac{\textit{acoustic wavelength}}{\textit{diffusion length}} = \frac{l'_a}{K' / u'_{01}}$$

$$M \qquad \frac{u'_{01}}{a'_{01}} \qquad ? \quad \tau = \frac{1}{NM}$$

Pressure Waves and Premixed Flames



x_1 : mass-weighted coordinate following the flame.

$\hat{x}$: re-scaled coordinate to describe effects driven by spatial pressure gradients (usually away from the combustion zone).

N ~ number of diffusion "lengths" in one typical length of pressure wave (usually large) $= \ell'_a / \ell'_c$

$\tau = (NM)^{-1}$ i.e. ratio of tdiffusion "time" ℓ'_c / u'_{01} to typical time ℓ'_a / a'_{01} of pressure wave. $(M \quad u'_{01}/a'_{01})$.

① **Long length-scale**
$N >> M^{-1}, \ \tau << 1$

② **Medium length-scale**
$N = M^{-1}, \ \tau = 1$

③ **Short length-scale**
$N = \theta^{-2} M^{-1}, \ \tau = \theta^2$

④ **Ultra-short length-scale**
$N = 1, \ \tau = M^{-1}$

Pressure Waves and Premixed Flames

Suppose *N*=1, i.e. $\ell'_a = \dfrac{\kappa'}{u'_{01}}$

Then length scale of pressure disturbance is of the order of the flame thickness.

Define $t \quad \dfrac{t'}{\kappa' / u'^{2}_{01}}$ **(time with respect to "diffusion time")**

Then during passage of pressure wave

$$t \quad \frac{t'_{ac}}{\kappa' / u'^{2}_{01}} = \frac{\ell'_a / a'_{01}}{\kappa' / u'^{2}_{01}} = \frac{\kappa' / u'_{01} a'_{01}}{\kappa' / u'^{2}_{01}}$$

i.e. $t \quad O(M)$

Pressure Waves and Premixed Flames

On this time scale no diffusion effects take place. Combustion effects take place on longer time scale ,

with $t \quad O\left(\theta^{-2}\right)$ ♦ $t \quad O(1)$

($\theta \quad \dfrac{E'_a}{R' T'_b}$ **non-dimensional activation energy).**

Pressure Waves and Premixed Flames

For $N = \theta^{-2} M^{-1}$, i.e. $\ell_a' = \dfrac{\kappa'}{u_{01}'} \theta^{-2} M^{-1}$

Then length scale of pressure disturbance is small, but greater than the order of the flame thickness.

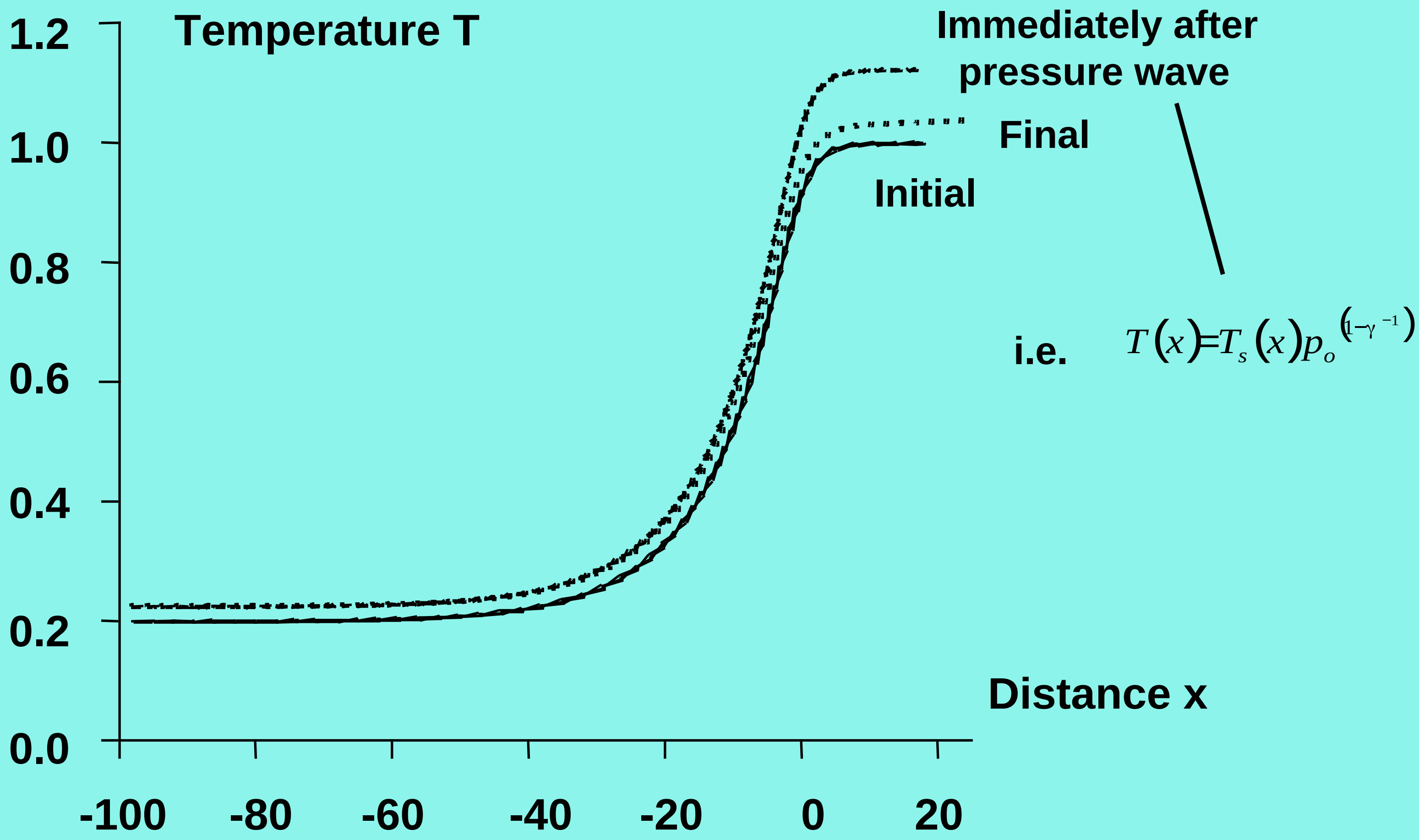
Again using $t \quad \dfrac{t'}{\kappa'/u_{01}'^{2}}$ **(time with respect to "diffusion time"), then during passage of pressure wave, the time is n**

$$t \quad \frac{t_{ac}'}{\kappa'/u_{01}'^{2}} = \frac{\ell_a'/a_{01}'}{\kappa'/u_{01}'^{2}} = \frac{\theta^{-2} M^{-1} \kappa'/u_{01}' a_{01}'}{\kappa'/u_{01}'^{2}}$$

i.e. $t \quad O\left(\theta^{-2}\right)$

Combustion effects take place on longer time scale $O(1)$

Pressure Waves and Premixed Flames



The effect of a pressure wave on temperature profiles of a premixed flame (Le=0.9, θ=10, p_0=1.5)

Pressure Waves and Premixed Flames

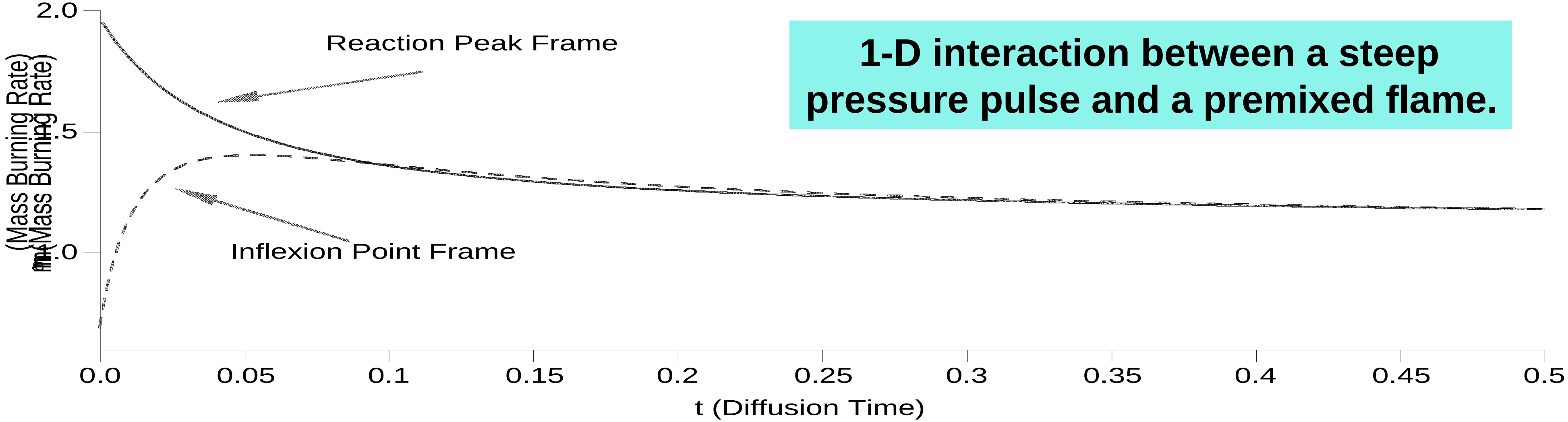
"Residual" 1-D Equations after pressure wave has passed through

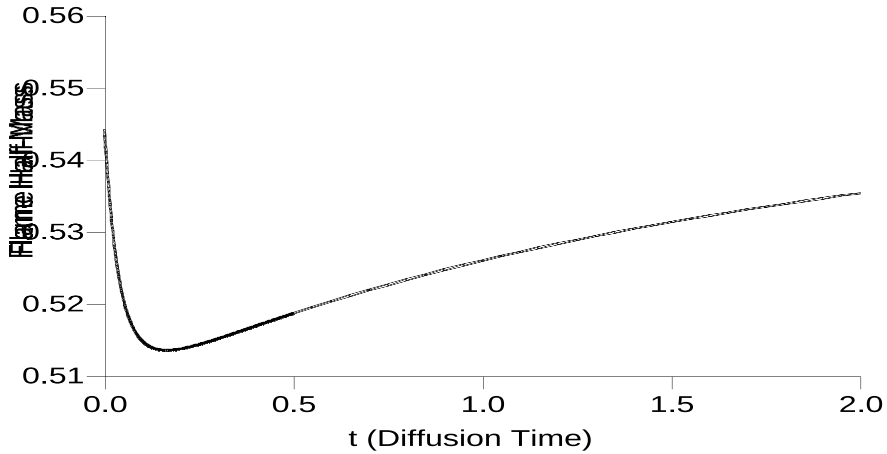
$$\frac{\partial T}{\partial t}+m_0\frac{\partial T}{\partial x_1}-\frac{p_0}{Le}\frac{\partial^2 T}{\partial x_1^2}=QR+\left(1-\gamma^{-1}\right)\frac{T}{p_0}\frac{dp_0}{dt}$$

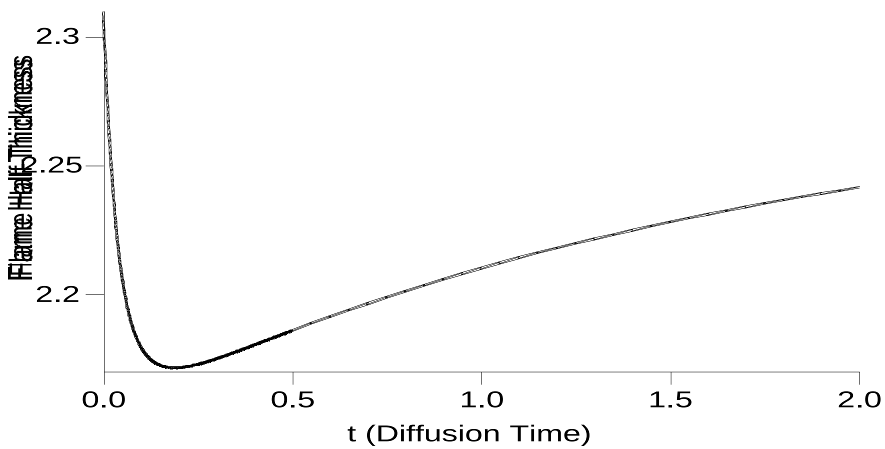
$$\frac{\partial C}{\partial t}+m_0\frac{\partial C}{\partial x_1}-p_0\frac{\partial^2 C}{\partial x_1^2}=-R\,;\,R\quad ACe^{-\theta/T}\quad;\quad \lambda,\rho D \text{ prop. to } T?\ \rho\lambda \text{ prop. to } p$$

Pressure Waves and Premixed Flames

1-D interaction between a steep pressure pulse and a premixed flame.







Pressure Waves and Premixed Flames

- **1-D Interactions - Flat flame, normal flow**

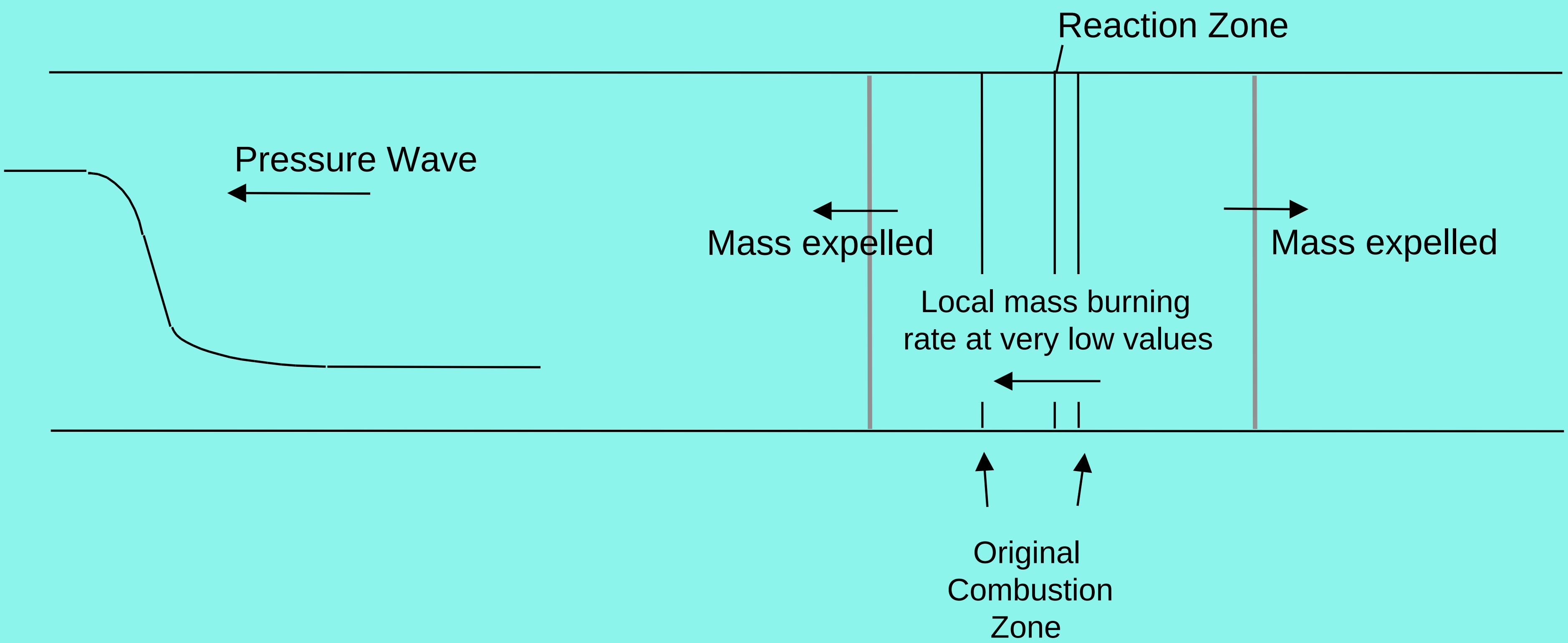
Effect of Pressure Drop - Extinction Behaviour

Short length scale - Medium length scale

$$N = \theta^{-2} M^{-1} \blacklozenge M^{-1}, \tau = \theta^{2} \blacklozenge 1$$

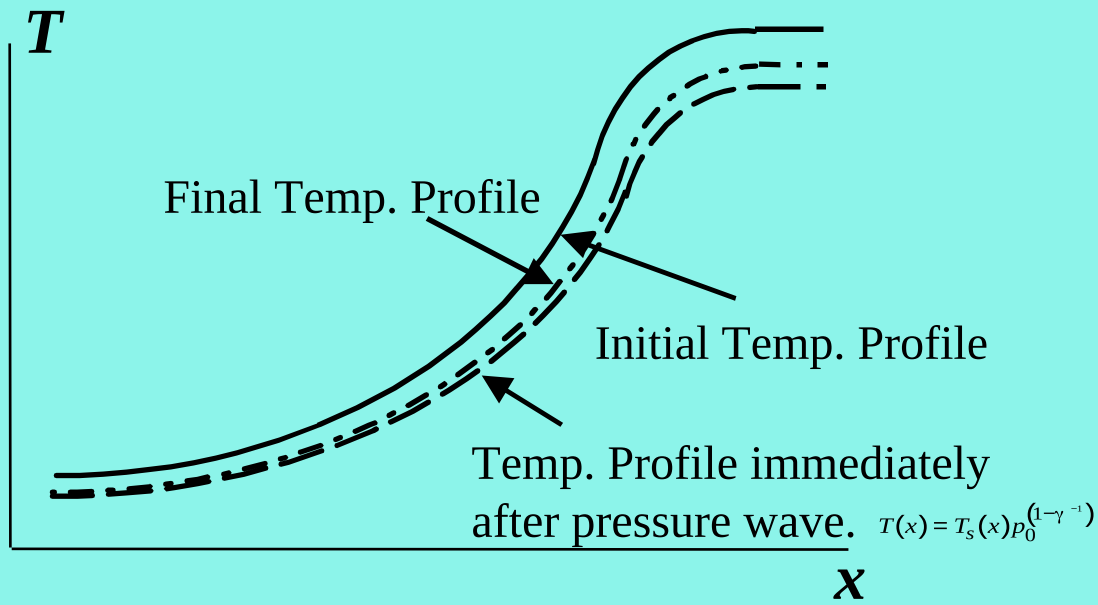
Medium amplitude - Free Flames : Numerical

Pressure Waves and Premixed Flames



Transient flame dilation after a sharp pressure drop.

Pressure Waves and Premixed Flames

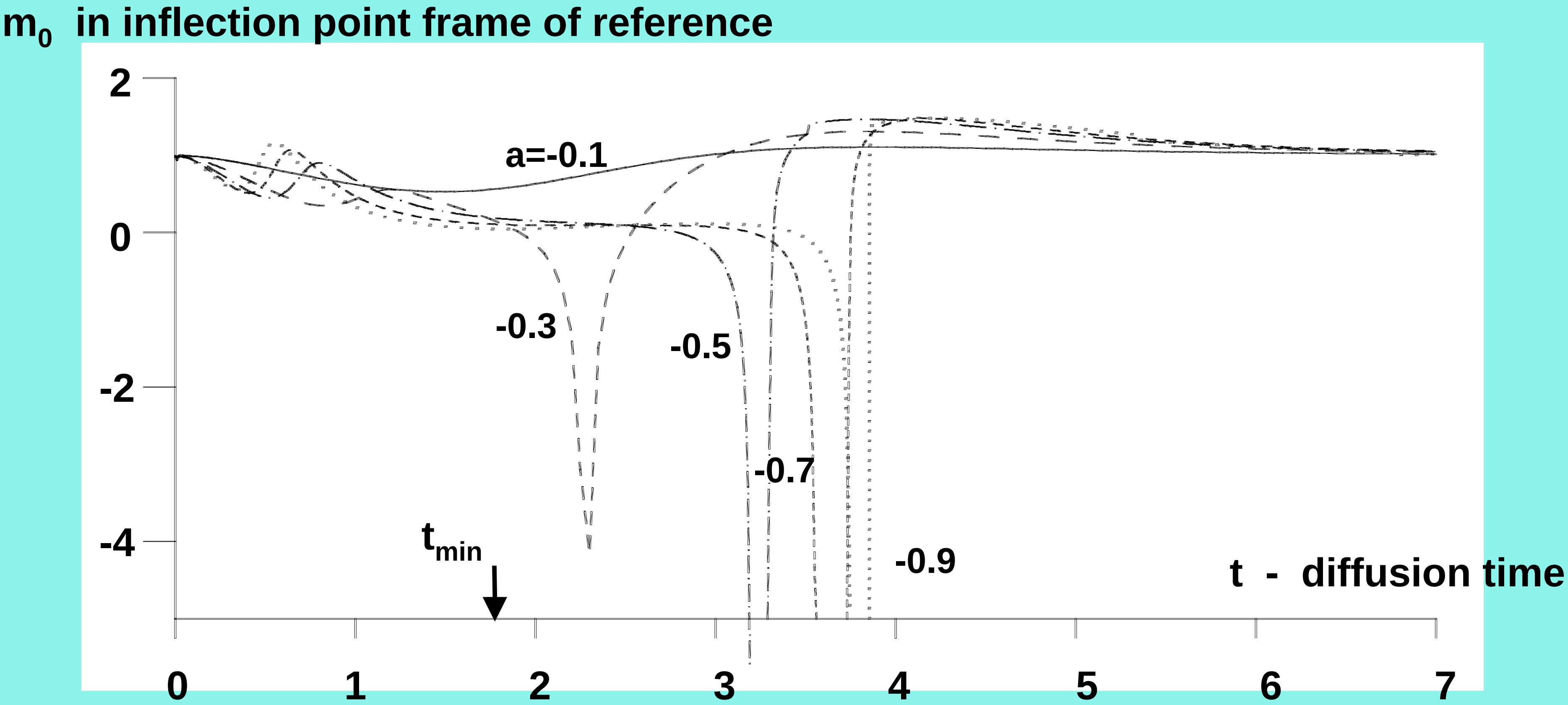


Schematic of the effect of a pressure change on the temperature profile of a premixed flame

Starting condition for residual combustion time-scale investigation :

$$T(x) = T_s(x) p_0^{\left(1-\gamma^{-1}\right)}$$

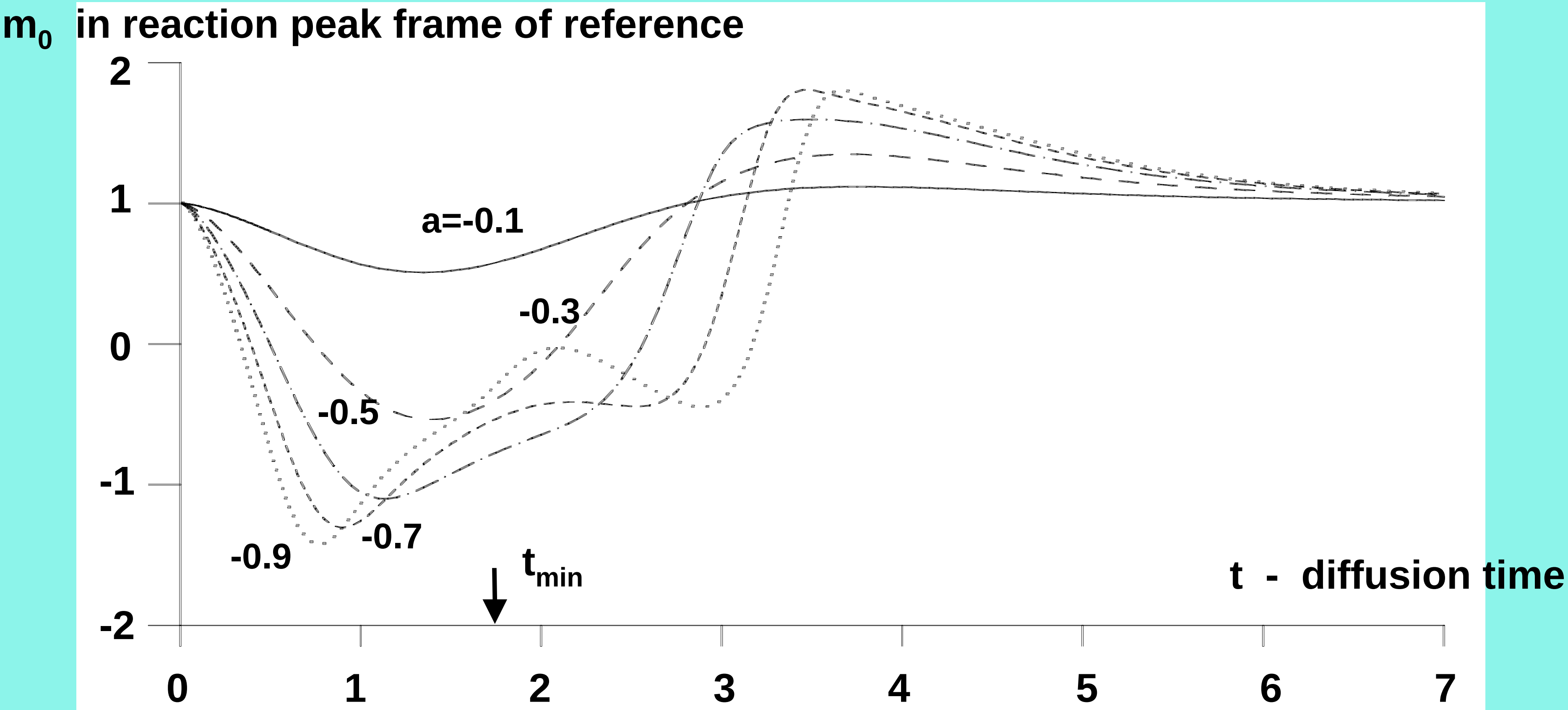
Pressure Waves and Premixed Flames



The evolution of the inflection point mass flux of a flame with activation energy θ=30 and Lewis number Le=1.0, during negative pulses of the form

$$p = 1 + 4a\left(2^{-bt^2} - 2^{-2bt^2}\right)$$ **with b=0.3.**

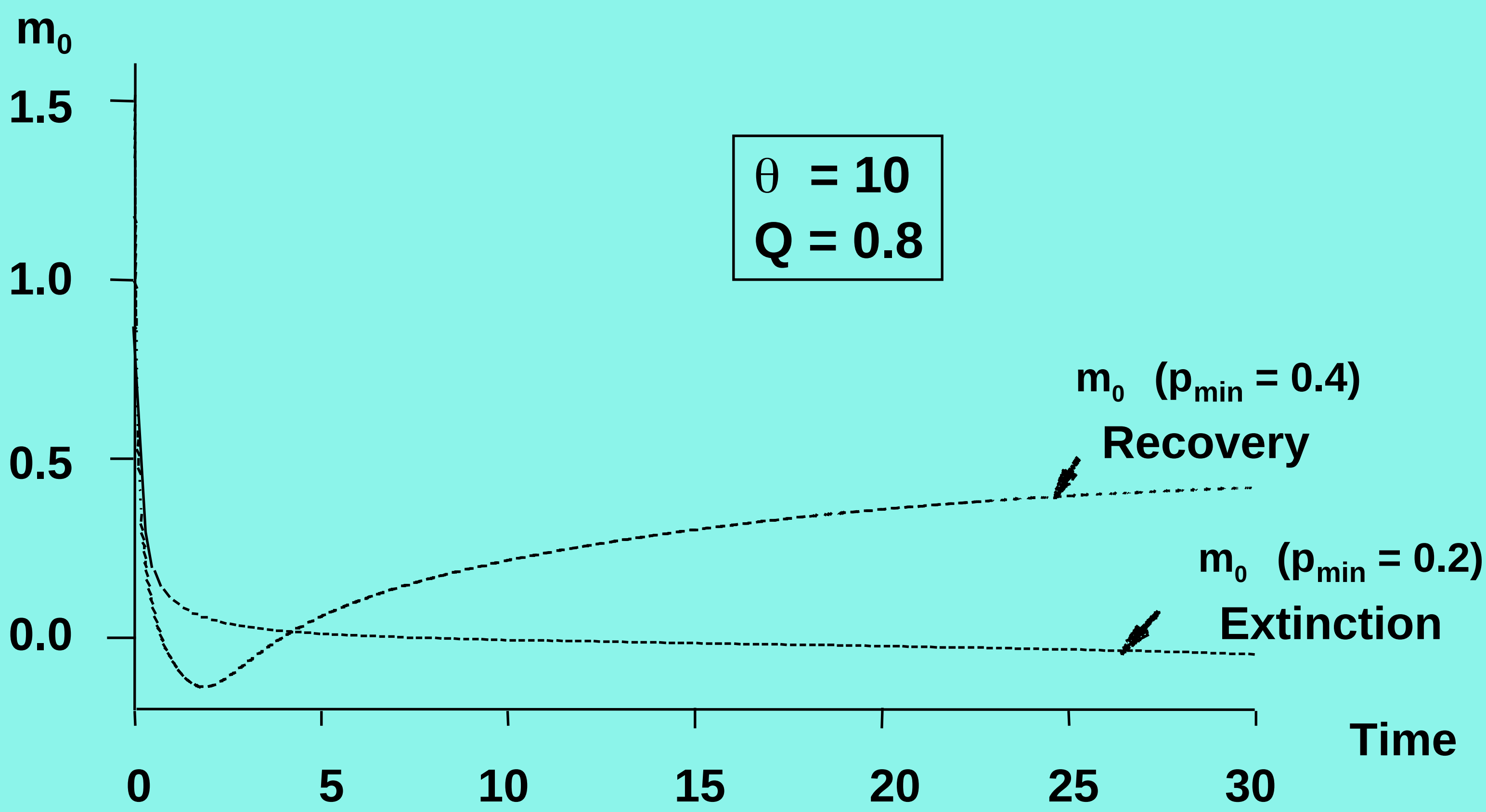
Pressure Waves and Premixed Flames



The evolution of the reaction peak mass flux of a flame with activation energy θ=30 and Lewis number Le=1.0, during negative pulses of the form

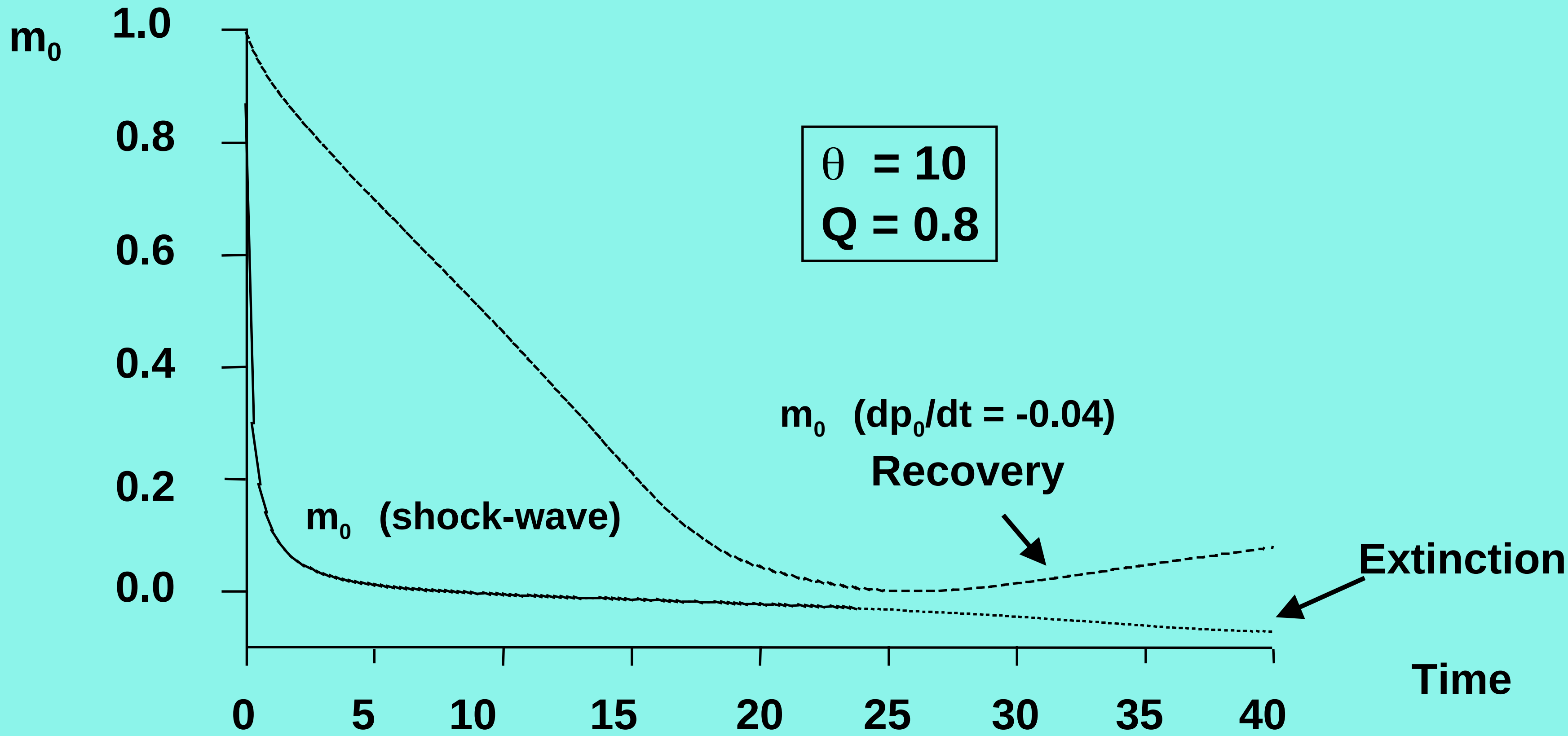
$$p = 1 + 4a\left(2^{-bt^2} - 2^{-2bt^2}\right)$$ **with b=0.3.**

Pressure Waves and Premixed Flames



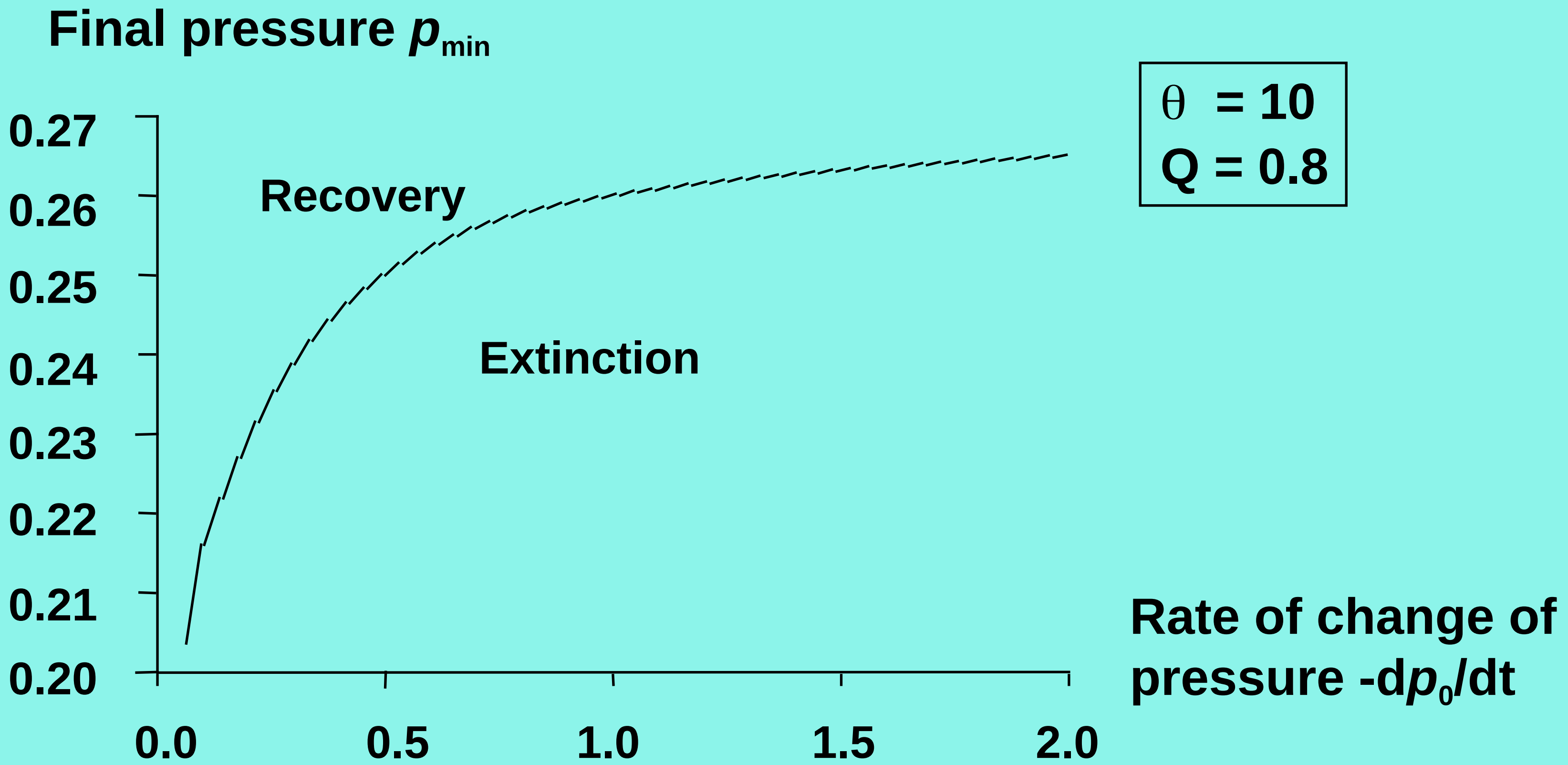
The mass flux response of a premixed flame to a sudden decrease in pressure.

Pressure Waves and Premixed Flames



The mass flux response of a premixed flame to a decrease in pressure - the effect of time rate of change.

Pressure Waves and Premixed Flames



The boundary between extinction and recovery in (p_{min} , dp_0/dt) parameter space.

Pressure Waves and Premixed Flames

- **1-D Interactions**

Accelerative effects

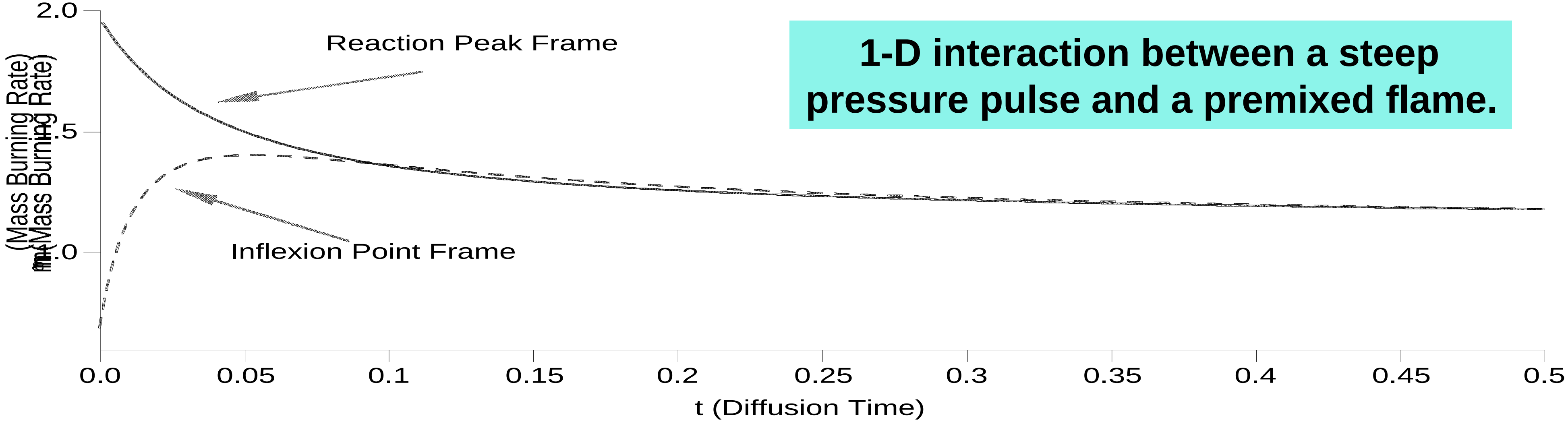
Short length scale - Medium length scale

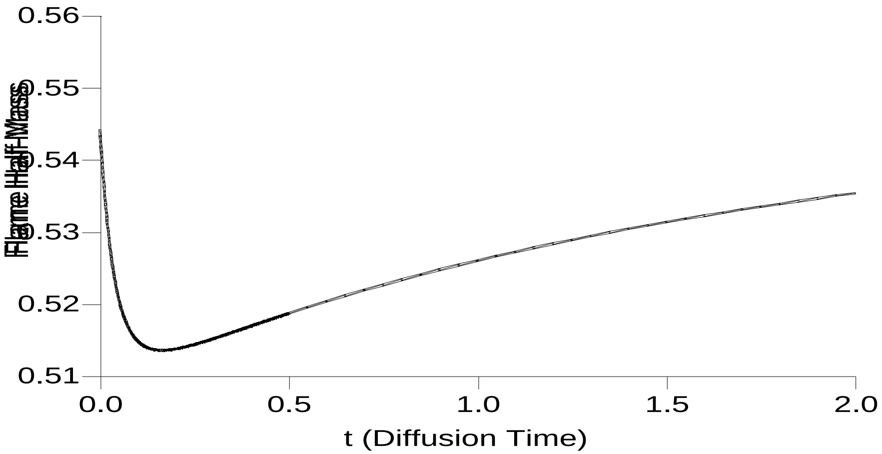
$$N = \theta^{-2} M^{-1} \blacklozenge \ M^{-1}, \tau = \theta^{2} \blacklozenge \ 1$$

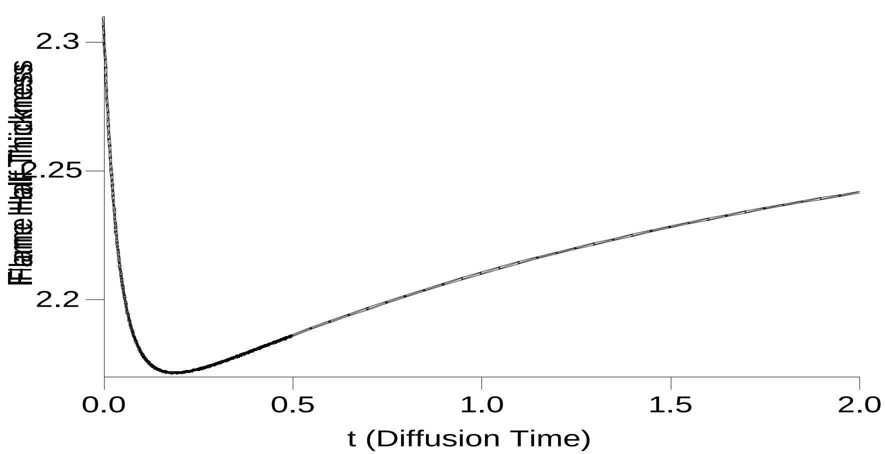
Medium amplitude - Free Flames : Numerical

Pressure Waves and Premixed Flames

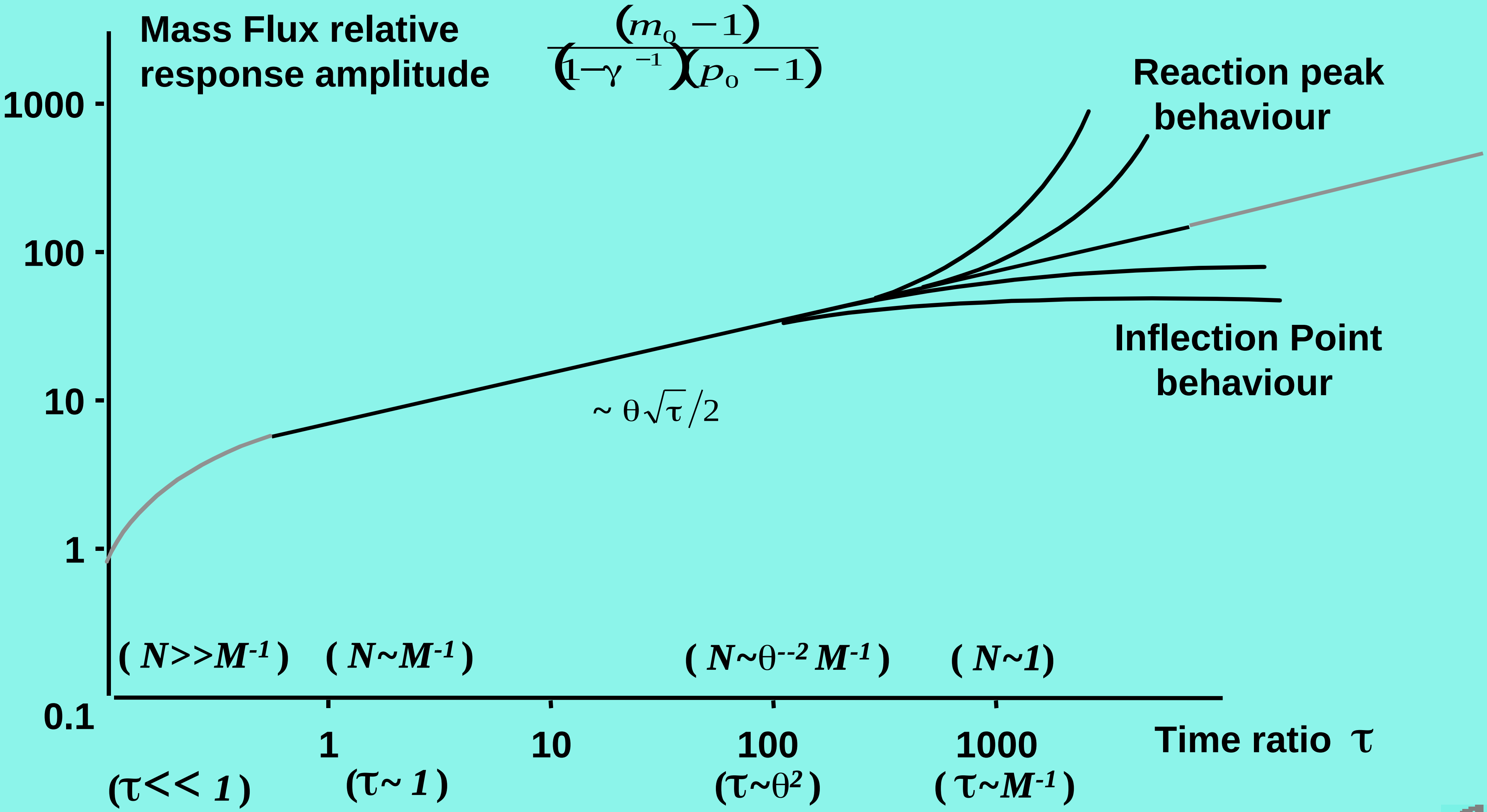
1-D interaction between a steep pressure pulse and a premixed flame.







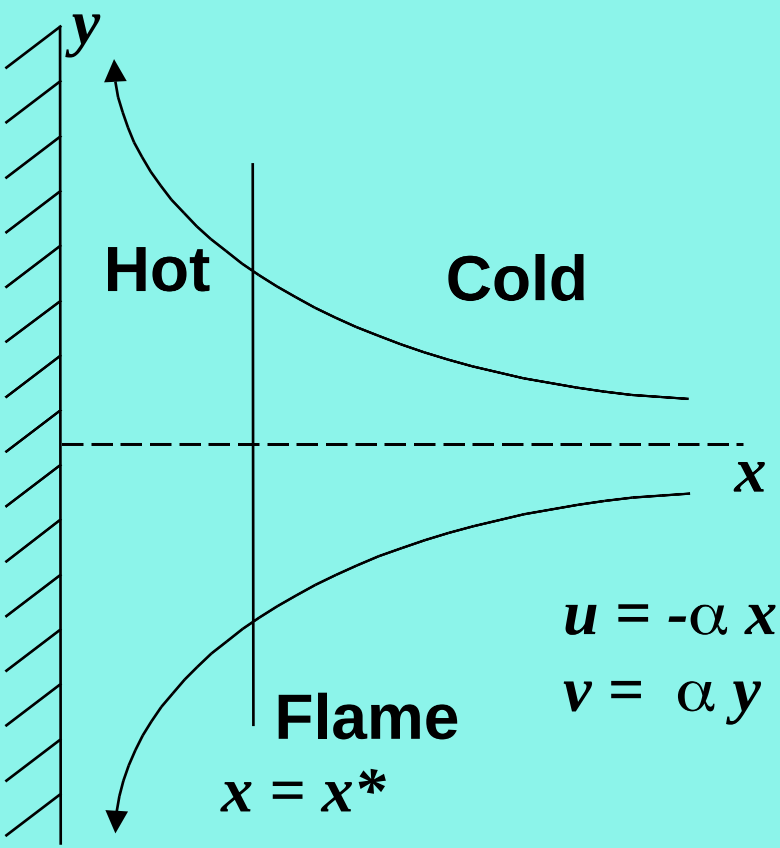
Pressure Waves and Premixed Flames



- **1-D Interactions - Flat flame, strained flow**

Effect of Pressure Drop

Pressure Waves and Premixed Flames



Strained Pre-mixed Flame

Pressure Waves and Premixed Flames

N=1, i.e. $\ell'_a = \dfrac{\kappa'}{u'_{01}}$

Length scale of pressure disturbance is of the order of the flame thickness.

Define $t \quad \dfrac{t'}{\kappa' / u'^2_{01}}$ **(time with respect to "diffusion time")**

Then during passage of pressure wave

$$t \quad \frac{t'_{ac}}{\kappa' / u'^2_{01}} = \frac{\ell'_a / a'_{01}}{\kappa' / u'^2_{01}} = \frac{\kappa' / u'_{01} a'_{01}}{\kappa' / u'^2_{01}} \quad \textbf{i.e.} \quad t \quad O(M)$$

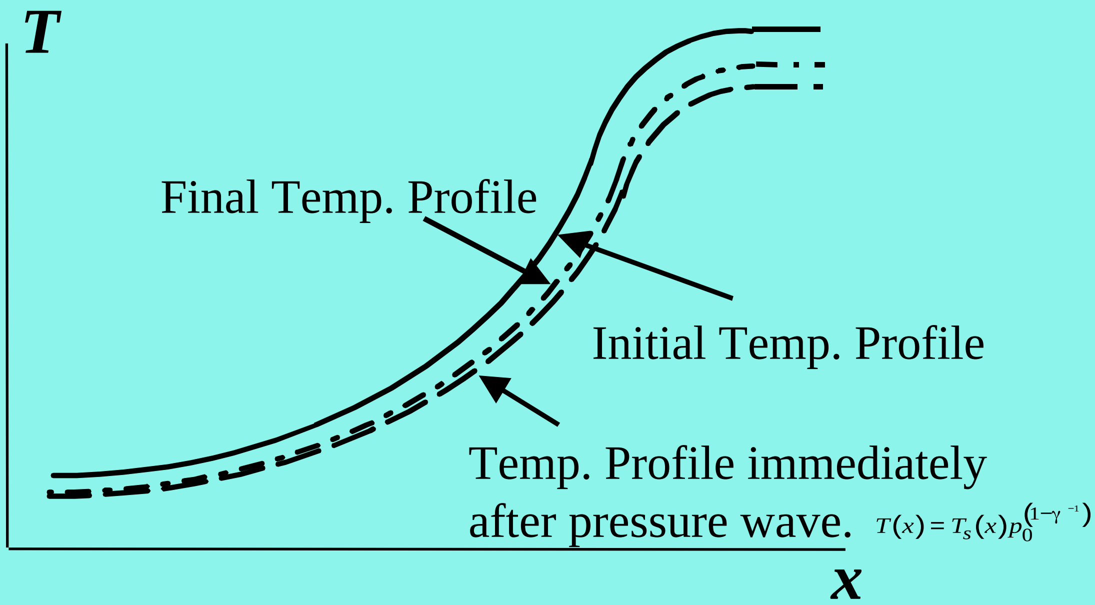
Pressure Waves and Premixed Flames

On this time scale no diffusion effects take place. Combustion effects take place on longer time scale ,

with $t \quad O\left(\theta^{-2}\right)$ ♦ $t \quad O(1)$

($\theta \quad \dfrac{E'_a}{R' T'_b}$ **non-dimensional activation energy).**

Pressure Waves and Premixed Flames



Schematic of the effect of a pressure drop on the temperature profile of a premixed flame

Starting condition for residual combustion time-scale investigation :

$$T(x) = T_s(x) p_0^{(1-\gamma^{-1})}$$

Pressure Waves and Premixed Flames

<u>Assumption</u> - Not only does the combustion take place on a longer time scale, but the natural time for the effect of the pressure drop on the strain is also considered to be on a time scale which is longer still. Thus

$$\frac{1}{\alpha'} >> \frac{\kappa'}{u_{01}'^{2}} \quad ? \quad \alpha \quad \frac{\alpha'}{u_{01}'^{2}/\kappa'} << 1$$

where α' has units of secs^{-1}.

Pressure Waves and Premixed Flames

Strained Flow with Prescribed Velocity U

If the velocity field $\mathbf{u} = \alpha \mathbf{U}$ is *prescribed* so that $\nabla . \mathbf{U} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$ (and $\frac{\partial U}{\partial x} = -1$, $\frac{\partial V}{\partial y} = 1$), we have

$$\frac{\partial \rho}{\partial t} + \alpha U \frac{\partial \rho}{\partial x} = 0,$$

$$\frac{\partial C}{\partial t} + \alpha U \frac{\partial C}{\partial x} - \frac{1}{\rho}\frac{\partial}{\partial x}(\rho D \frac{\partial C}{\partial x}) = -\mathcal{R},$$

$$\rho(\frac{\partial T}{\partial t} + \alpha U \frac{\partial T}{\partial x}) - \frac{1}{Le}\frac{\partial}{\partial x}(\lambda \frac{\partial T}{\partial x}) = \rho Q \mathcal{R} + T_{01}(\frac{\gamma - 1}{\gamma})\frac{\partial p}{\partial t},$$

where

$$Le \equiv \frac{\rho D c_p}{\lambda}, \quad T_{01} \equiv \frac{T_u'}{T_b'}.$$

Defining the new variable by

$$\tilde{x} = \int_{x^*(t)}^{x} \rho dx, \quad \tilde{t} = t,$$

$$m_0 = \alpha\rho(U - U^*)|_{(x^*(t),t)}, \quad u^* = \alpha U^* = \frac{dx^*}{dt},$$

we have

$$\partial_t = (m_0 - \alpha\rho U - \alpha\tilde{x})\partial_{\tilde{x}} + \partial_{\tilde{t}},$$

$$\partial_x = \rho\partial_{\tilde{x}}, \quad \frac{\partial}{\partial t} + \alpha U \frac{\partial}{\partial x} = \partial_{\tilde{t}} + (m_0 - \alpha\tilde{x})\partial_{\tilde{x}}.$$

Pressure Waves and Premixed Flames

With $\rho D = \lambda$ and $\rho\lambda \approx p$, the equations in the mass-weighted co-ordinates become

$$\frac{\partial C}{\partial \tilde{t}} + (m_0 - \alpha\tilde{x})\frac{\partial C}{\partial \tilde{x}} - p\frac{\partial^2 C}{\partial \tilde{x}^2} = -\mathcal{R}, \quad \mathcal{R} = \Lambda C e^{\theta(1-\frac{1}{T})},$$

$$\frac{\partial T}{\partial \tilde{t}} + (m_0 - \alpha\tilde{x})\frac{\partial T}{\partial \tilde{x}} - \frac{p}{Le}\frac{\partial^2 T}{\partial \tilde{x}^2} = Q\mathcal{R} + (1-\gamma^{-1})\frac{T}{p}\frac{dp}{d\tilde{t}}.$$

We will only consider the half-space $x < 0$ due to its symmetrical property of the counter flow. The related boundary conditions are

$$C = C_\infty, \quad T = T_u, \quad \text{for} \quad \tilde{x} < 0,$$

$$C = 0, \quad T = T_* \ (\text{ or } T_b \ \text{ for } \ \alpha = 0), \quad \text{for} \quad \tilde{x} \to \tilde{x}^*.$$

Pressure Waves and Premixed Flames

Perturbations and Stability

$$\frac{\partial C}{\partial \tilde{t}} + (m_0 - \alpha\tilde{x})\frac{\partial C}{\partial \tilde{x}} - p\frac{\partial^2 C}{\partial \tilde{x}^2} = 0,$$

$$\frac{\partial T}{\partial \tilde{t}} + (m_0 - \alpha\tilde{x})\frac{\partial T}{\partial \tilde{x}} - \frac{p}{Le}\frac{\partial^2 T}{\partial \tilde{x}^2} = (1-\gamma^{-1})\frac{T}{p}\frac{dp}{d\tilde{t}}.$$

$$[T]_+^- = 0, \quad [C]_+^- = 0,$$

$$[\frac{\partial T}{\partial \tilde{x}} + LeQ\frac{\partial C}{\partial \tilde{x}}]_+^- = 0, \quad [\frac{\partial T}{\partial \tilde{x}}]_+^- = \frac{LeQ}{\sqrt{p_\infty}}e^{\frac{\beta}{2}(T_*-1)}.$$

By assuming that $\epsilon \ll \alpha \ll 1$, the perturbations in term of ϵ give

$$T = T_s + \epsilon T^{(1)} + ..., \quad C = C_s + \epsilon C^{(1)} + ...,$$

$$m_0 = m_s + \epsilon m^{(1)} + ..., p = p_s + \epsilon p^{(1)} + ..., \quad (p_s = 1),$$

Pressure Waves and Premixed Flames

Effect of Pressure Perturbation

The pressure response is

$$m_u = \frac{(1-\gamma^{-1})(\theta p_u)(2s-Q)}{m_s^2\{\frac{\theta Q}{\omega}\frac{e^{\frac{\alpha Q l_1}{2}}}{m_s}[-Le(\frac{1}{2}-s)+(\frac{1}{2}-r)]-\frac{4s}{\omega}(\frac{1}{2}-r)\}},$$

$$m_s \approx 1-\alpha(1-\frac{Ql_1}{2}),$$

$$r \equiv \sqrt{\omega+\frac{1}{4}}, \quad s \equiv \sqrt{\frac{\omega}{Le}+\frac{1}{4}}.$$

Thus, the full expression for m_0 is

$$m_0 = m_s + \epsilon m_u, \quad \delta p = \epsilon p_u.$$

For $\alpha \ll 1$ and $\omega \to 0$ (or $r \to 1/2$), we have (for $t \to \infty$)

$$\boxed{m_0 = m_s + \frac{(1-\gamma^{-1})(\theta\delta p)(1-Q)}{2m_s^2}, \quad (\delta p = \epsilon p_u)}$$

i.e.

$$\boxed{m_0 \approx 1 + \frac{(1-\gamma^{-1})(\theta\delta p)(1-Q)}{2} - \alpha(1-\frac{Ql_1}{2})[1-(1-\gamma^{-1})(\theta\delta p)(1-Q)]}$$

Perturbation of mas burning rate due to pressure change, in presence of strain α.

Pressure Waves and Premixed Flames

Dispersion relation

The new dispersion relation is

$$\frac{4m_s s}{\omega}(r-\frac{1}{2})-\frac{\theta Q}{\omega}\exp[\frac{\alpha\theta Q(Le-1)}{2}][(r-\frac{1}{2})+Le(s-\frac{1}{2})]=0,$$

with

$$m_s = 1-\alpha(1-Ql_1/2),$$

$$r \equiv \sqrt{\omega+\frac{1}{4}}, \quad s \equiv \sqrt{\frac{\omega}{Le}+\frac{1}{4}}.$$

Rewritten in term of $l_1=\theta(Le-1)$, it becomes

$$8r^2[1-\alpha(1-Ql_1/2)]-Ql_1(\frac{1}{2}-r)e^{\alpha Q l_1/2}=0,$$

which becomes McIntosh's (1984) relation when $\alpha=0$

$$4s(\frac{1}{2}-r)+Le\theta Q(\frac{1}{2}-s)-\theta Q(\frac{1}{2}-r)=0,$$

or

SivashinskyRelation : $\quad 8r^2-l_1Q(\frac{1}{2}-r)=0.$

Pressure Waves and Premixed Flames

References

Sivashinsky G I, Law C K and Joulin G, On stability of premixed flames in stagnation-point flow, Comb. Sci. and Tech., **28**, 155-159 (1982).

Buckmaster J D and Ludford G S S, Lectures on Mathematical Combustion (Philadelphia, PA: SIAM), 1983.

McIntosh A C and Clarke J F, Second order theory of unsteady burner-anchored flames with arbitrary Lewis number, Comb. Sci. and Tech., **38**, 161-196 (1984).

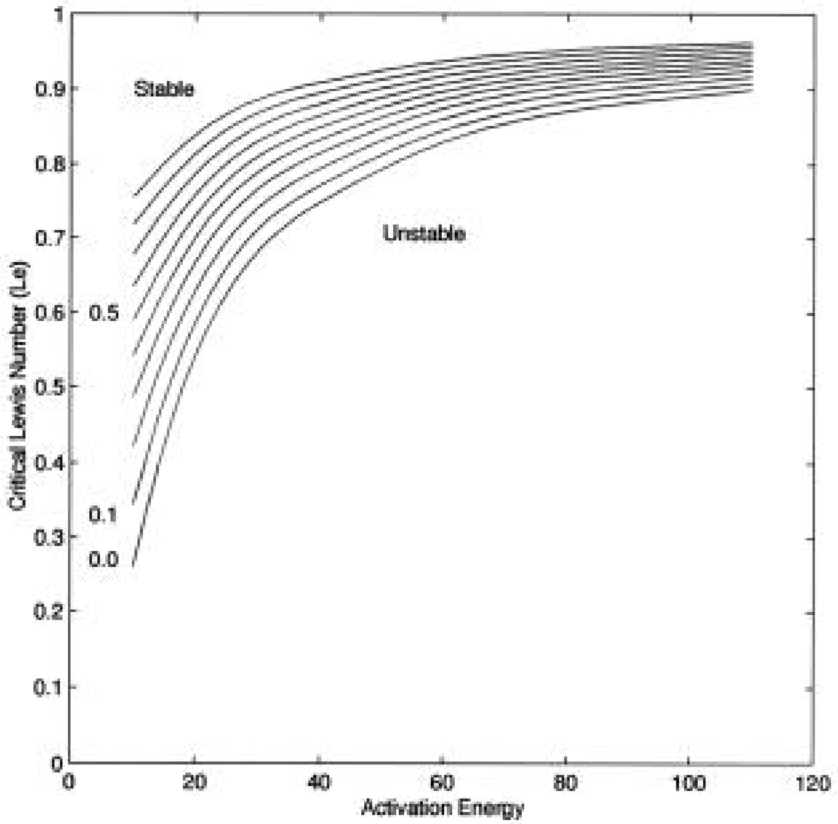
McIntosh A C, Pressure disturbances of different length scales interacting with conventional flames, Comb. Sci. and Tech., **75**, 287-309 (1991).

McIntosh A C, The linearised response of the mass burning rate of a premixed flame to rapid pressure changes, Comb. Sci. and Tech., **91**, 329-346 (1993).

Buckmaster J D, The effect of radiation on stretched flames, Comb. Theory and Modelling, **1**, 1-11 (1997).

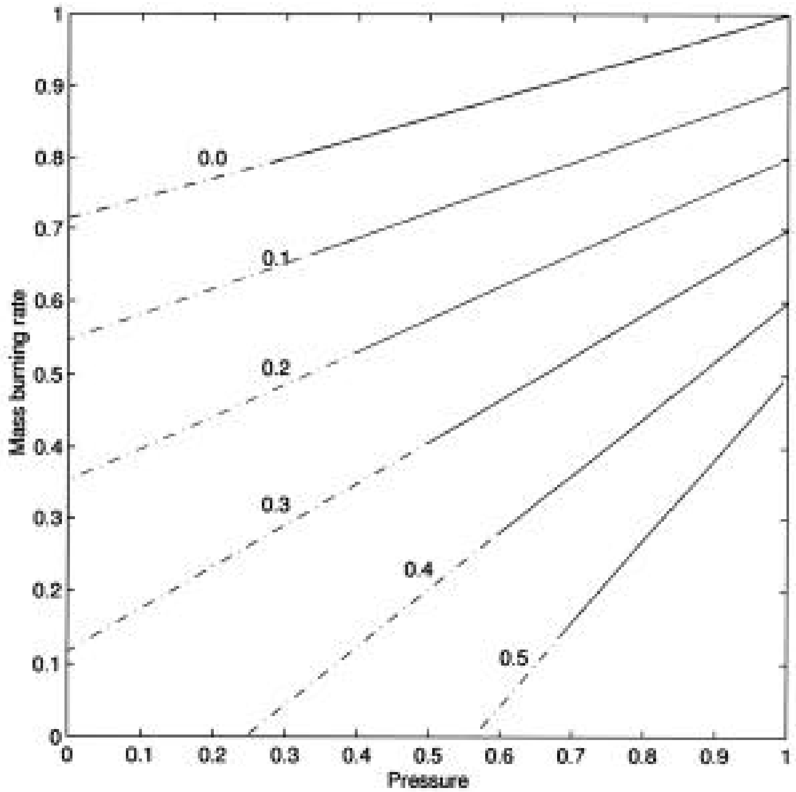
Buckmaster J D and M Short, Cellular instabilities, sublimit structures and edge-flames in premixed counterflows, Comb. Theory and Modelling, **3**, 199-241 (1999).

Pressure Waves and Premixed Flames



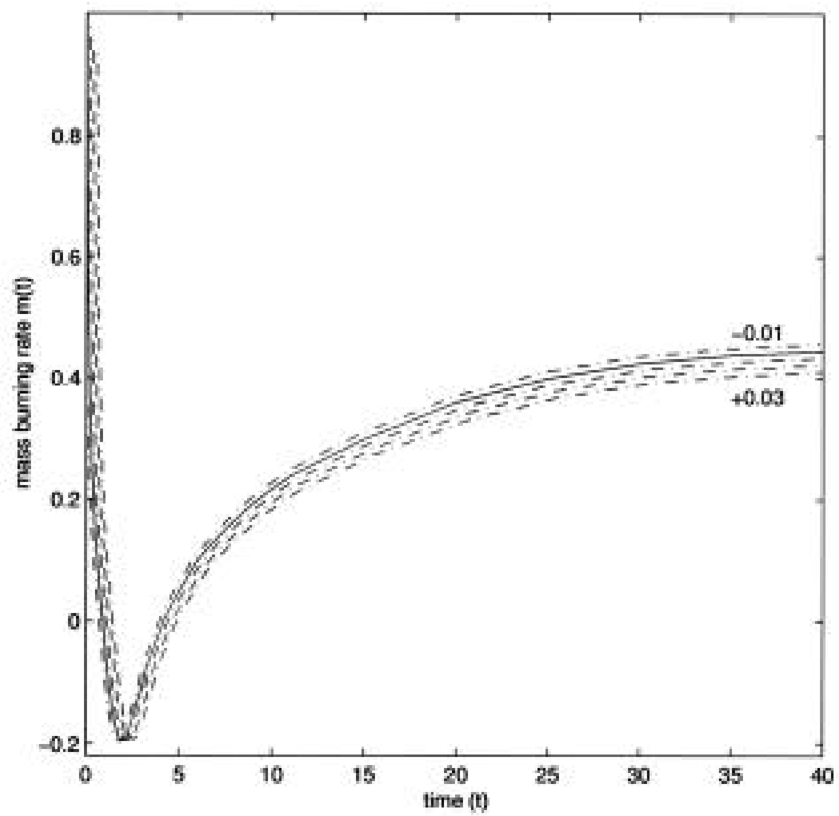
Neutral stability curve with strain present for $\alpha = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5$, $Q = 0.8$, $\theta = 10$, $\gamma = 1.4$.

Pressure Waves and Premixed Flames



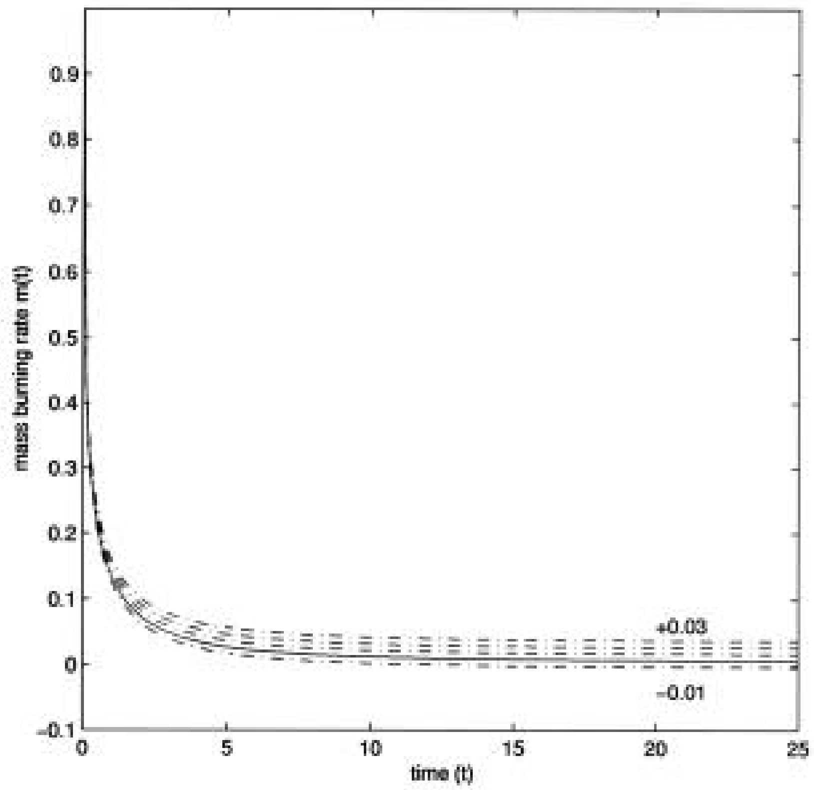
Perturbed Steady State ($Q = 0.8, \theta = 10, \gamma = 1.4$ and $\alpha = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5$). Dashed region is inaccessible.

Pressure Waves and Premixed Flames



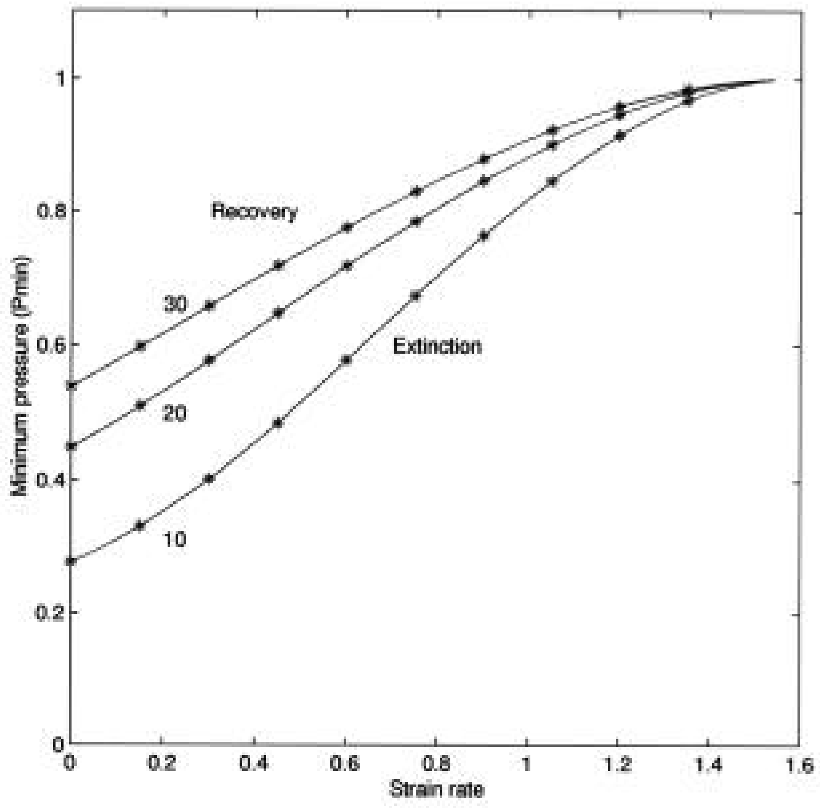
Recovery due to pressure change ($Q = 0.8, \theta = 10, \gamma = 1.4$, $\Delta\alpha = 0.01$) and $P_{min} = 0.4$.

Pressure Waves and Premixed Flames



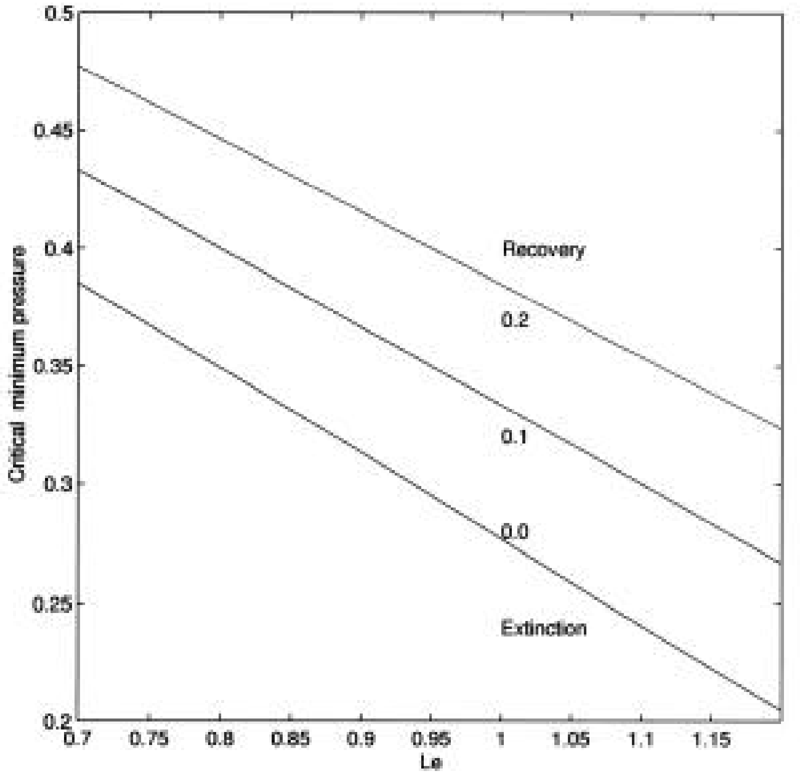
Extinction due to pressure change ($Q = 0.8, \theta = 10, \gamma = 1.4, \Delta\alpha = 0.01$) and $P_{min} = 0.2$.

Pressure Waves and Premixed Flames



Minimum pressure versus strain rate α and activation energy $\theta = 10, 20, 30$ for $Q = 0.8, \gamma = 1.4$

Pressure Waves and Premixed Flames



Effect of Lewis number (Le) on the extinction of a flat premixed flame in a strained flow with a pressure drop for $\alpha = 0.0, 0.1, 0.2$ (and $Q = 0.8, \theta = 10, \gamma = 1.4$).

Conclusions -

As strain rate increases, the susceptibility to extinction by a pressure drop increases.

Lewis number decreasing (or $\lambda'/\rho'D'c_p$ increasing) also causes the extinction boundary to be more accessible.

Pressure Waves and Premixed Flames

- **1-D Weakly compressible combustion fronts**

Acoustics and weakly compressible combustion fronts in a tube.

Pressure Waves and Premixed Flames

- Long wavelength acoustic oscillations interacting with a weakly compressible combustion front.

- The structure of the front is not à priori assigned.

- The combustion front front has a mass burning flux which is sensitive to pressure.

- Pioneering work of Alexander Ni :-

Ni, A.L. and Goel, B. (1995) “Stability of a deflagration front in compressible media”, Z. angew. Math. Phys. (ZAMP) 46, 328-338.

Pressure Waves and Premixed Flames

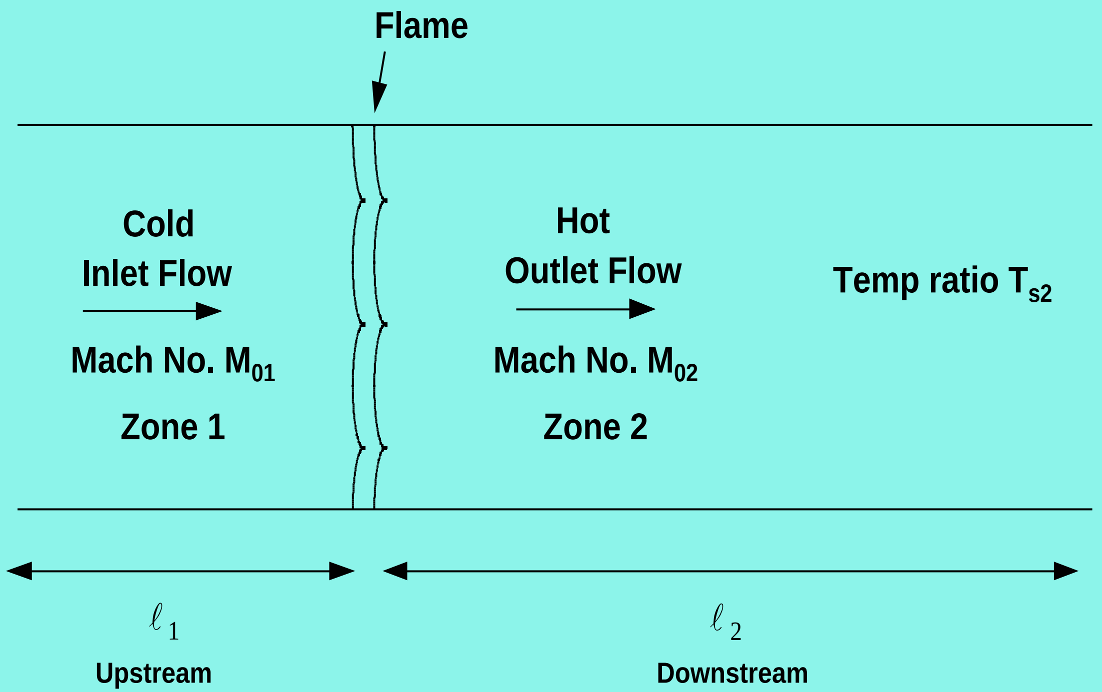


Fig. 1. Schematic of compressible combustion front within a tube which can undergo acoustic interactions.

Pressure Waves and Premixed Flames

The usual Rankine-Hugoniot relationships across a compressible front can be expressed as

$$p_2 = p_1 + \gamma\, m_0^{a\,2} \left(\mathrm{v}_1 - \mathrm{v}_2\right) \quad \text{Rayleigh Line}$$

$$p_1\mathrm{v}_1 - p_2\mathrm{v}_2 = \frac{1}{2}\left(1-\gamma^{-1}\right)\left(\mathrm{v}_1 + \mathrm{v}_2\right)\left(p_1 - p_2\right) - Q \qquad \text{Hugoniot curve}$$

$$Q \qquad Q' / c'_p T'_{s1}$$

$$\frac{u_1^a - \hat{\beta}_t}{\mathrm{v}_1} = \frac{u_2^a - \hat{\beta}_t}{\mathrm{v}_2} \qquad m_0^a \quad \text{Mass continuity}$$

$$\hat{\beta}_t \qquad \frac{f x_f}{f\, t}$$

In acoustic zones have

$$p_{u1} = A_1 e^{\omega x} + B_1 e^{-\omega x}$$

$$p_{u2} = A_2 e^{\omega x / \sqrt{T_{s2}}} + B_2 e^{-\omega x / \sqrt{T_{s2}}}$$

$$x \quad \frac{x'}{L'} \quad ; \quad t \quad \frac{t' a'_{01}}{L'} \quad ; \quad T_{s2} \quad \frac{T'_{s2}}{T'_{s1}}$$

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Jump conditions across combustion front :

$$\left(1-2M_{01}\chi\right)p_{u10}=p_{u20}$$

$$\left(1-M_{01}\chi\right)p_{u10}=p_{u20}-M_{01}\,\tfrac{3}{4}\frac{1}{\omega}\frac{dp_{u2}}{d\hat{x}}\bigg|_0-\frac{1}{\omega}\frac{dp_{u1}}{d\hat{x}}\bigg|_0\surd$$

For input from the hot side, the coefficients are :

$$K_R=\frac{\hat{M}-1+\chi}{\hat{M}+1-\chi}\qquad K_T=\frac{2\hat{M}}{\hat{M}+1-\chi}\qquad \hat{M}\quad\frac{M_{02}}{M_{01}}$$

For input from the cold side, the coefficients are :

$$L_R=\frac{\chi+1-\hat{M}}{\hat{M}-\chi+1}\qquad L_T=\frac{2}{\hat{M}+1-\chi}$$

All modes unstable

Pressure Waves and Premixed Flames

Multiple acoustic waves in tubes - open tubes

$$\frac{\cosh(\omega \ell_1)}{\sinh(\omega \ell_1)} + \hat{M}\frac{\cosh\left(\omega \ell_2 / \sqrt{T_{s2}}\right)}{\sinh\left(\omega \ell_2 / \sqrt{T_{s2}}\right)} = \chi$$

χ is the sensitivity of mass burning to pressure $\quad \chi \quad \dfrac{\gamma Q}{p_{s2}}\,\dfrac{m_u^a}{p_{u10}}$

Approximate form : $\quad \chi \quad \dfrac{Q' a'_{01}}{c'_p T'_{01}}\,\dfrac{m_u'^{\,a}}{p'_{u10}}$

Using $\quad \hat{M} = \dfrac{\sqrt{\rho_{s1} p_{s1}}}{\sqrt{\rho_{s2} p_{s2}}} = \dfrac{1}{\sqrt{\rho_{s2} p_{s2}}} = \dfrac{\sqrt{T_{s2}}}{p_{s2}} \quad \sqrt{T_{s2}}$ $\quad$ we then obtain

$$\frac{\cosh(\omega \ell_1)}{\sinh(\omega \ell_1)} + \sqrt{T_{s2}}\,\frac{\cosh\left(\omega \ell_2 / \sqrt{T_{s2}}\right)}{\sinh\left(\omega \ell_2 / \sqrt{T_{s2}}\right)} = \chi$$

Pressure Waves and Premixed Flames

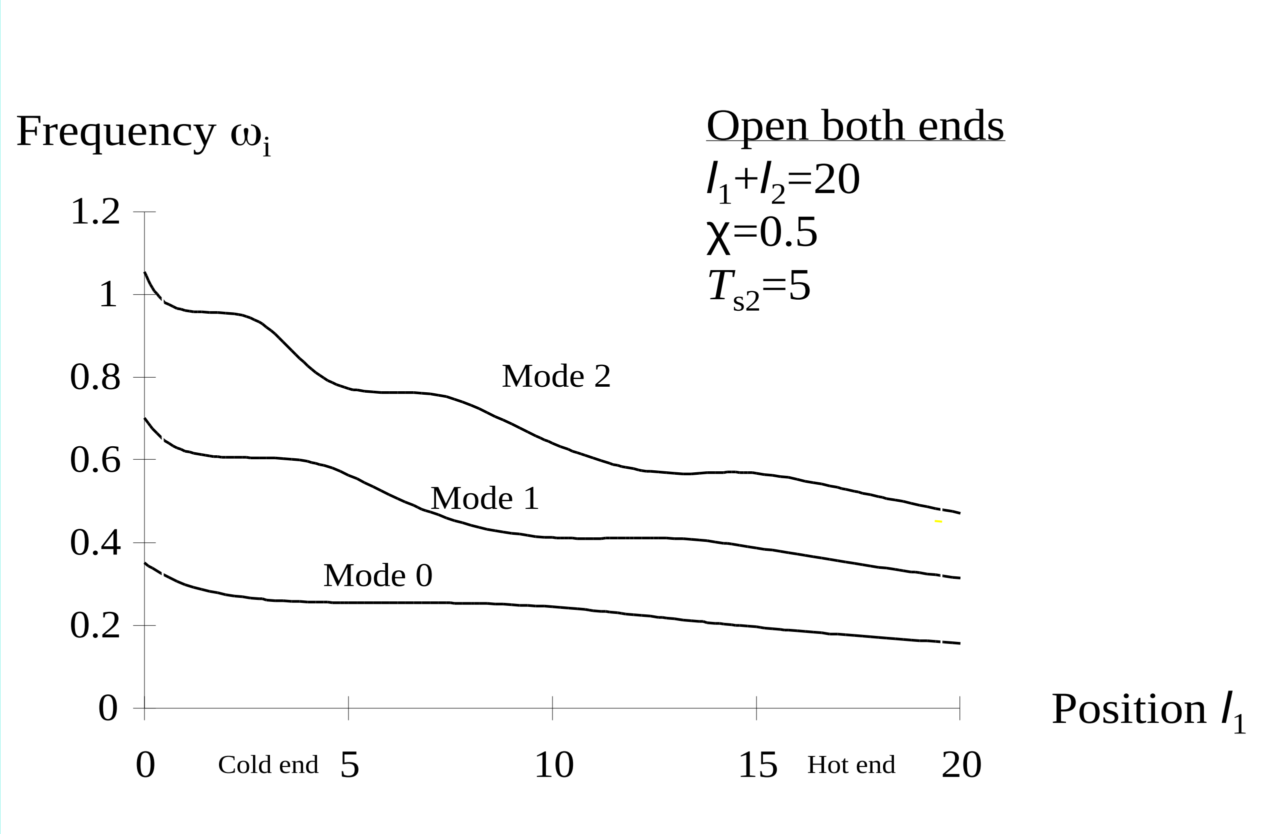

Fig. 2. Frequency variation for the first three modes of oscillation of a fast combustion front as it progresses down a tube length 20.

Pressure Waves and Premixed Flames

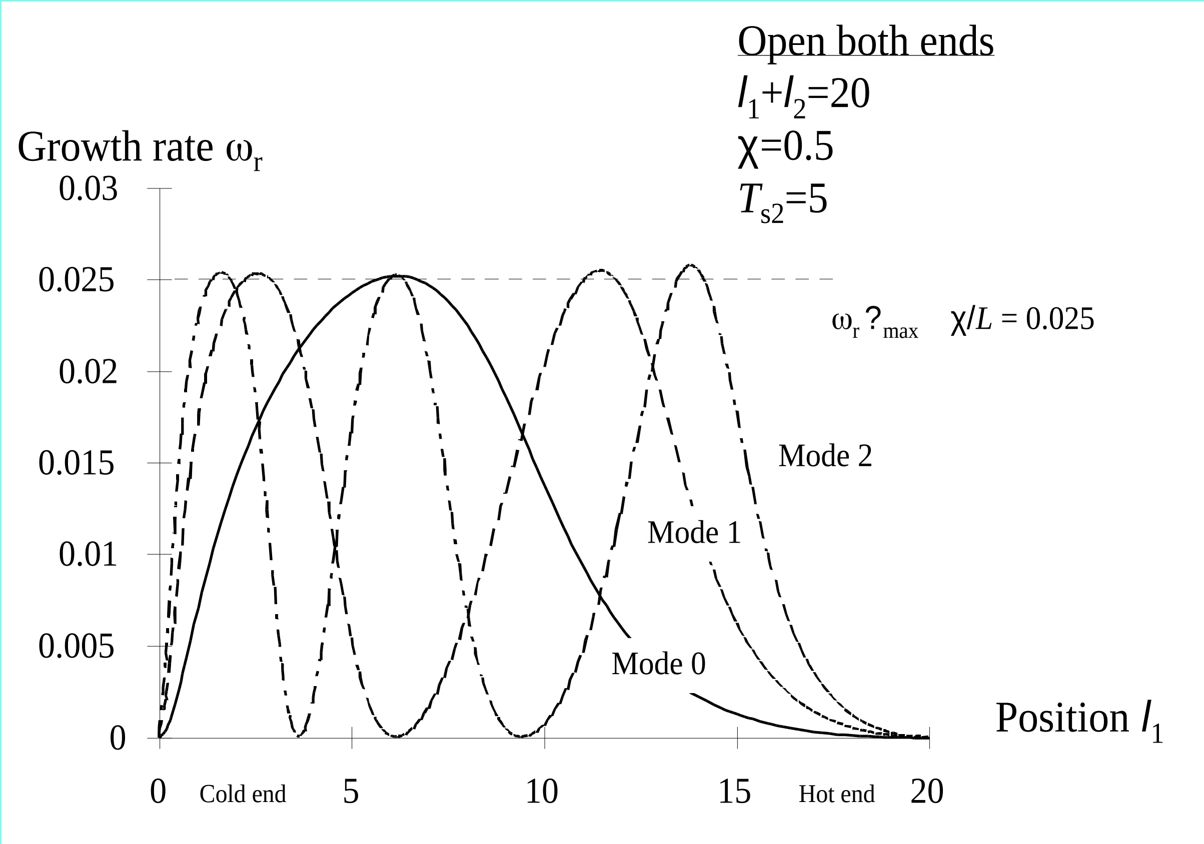

Fig. 3. Growth rate variation for the first three modes of oscillation of a fast combustion front as it progresses down a tube length 20.

Pressure Waves and Premixed Flames

For $\omega \quad i\omega_0 + \hat{\omega}$

$$\tan\left(\omega_0 \ell_2 / \sqrt{T_{s2}}\right) + \sqrt{T_{s2}}\tan\left(\omega_0 \ell_1\right) = 0 \qquad \hat{\omega} \quad \frac{\chi}{\dfrac{\ell_1}{\sin^2\left(\omega_0 \ell_1\right)} + \dfrac{\ell_2}{\sin^2\left(\omega_0 \ell_2 / \sqrt{T_{s2}}\right)}}$$

Minimum growth rate $\qquad \omega_0 \ell_1 = n_1 \pi \qquad \omega_0 \ell_2 / \sqrt{T_{s2}} = n_2 \pi$

$$\frac{\ell_1}{L - \ell_1} = \frac{n_1}{n_2 \sqrt{T_{s2}}}$$

Maximum growth rate $\qquad \omega_0 \ell_1 = \frac{\pi}{2} + n_1 \pi \qquad \omega_0 \ell_2 / \sqrt{T_{s2}} = \frac{\pi}{2} + n_2 \pi$

$$\frac{\ell_1}{L - \ell_1} = \frac{n_1 + \frac{1}{2}}{\left(n_2 + \frac{1}{2}\right)\sqrt{T_{s2}}} \qquad \omega_r\big|_{\max} \quad \frac{\chi}{L}$$

Pressure Waves and Premixed Flames

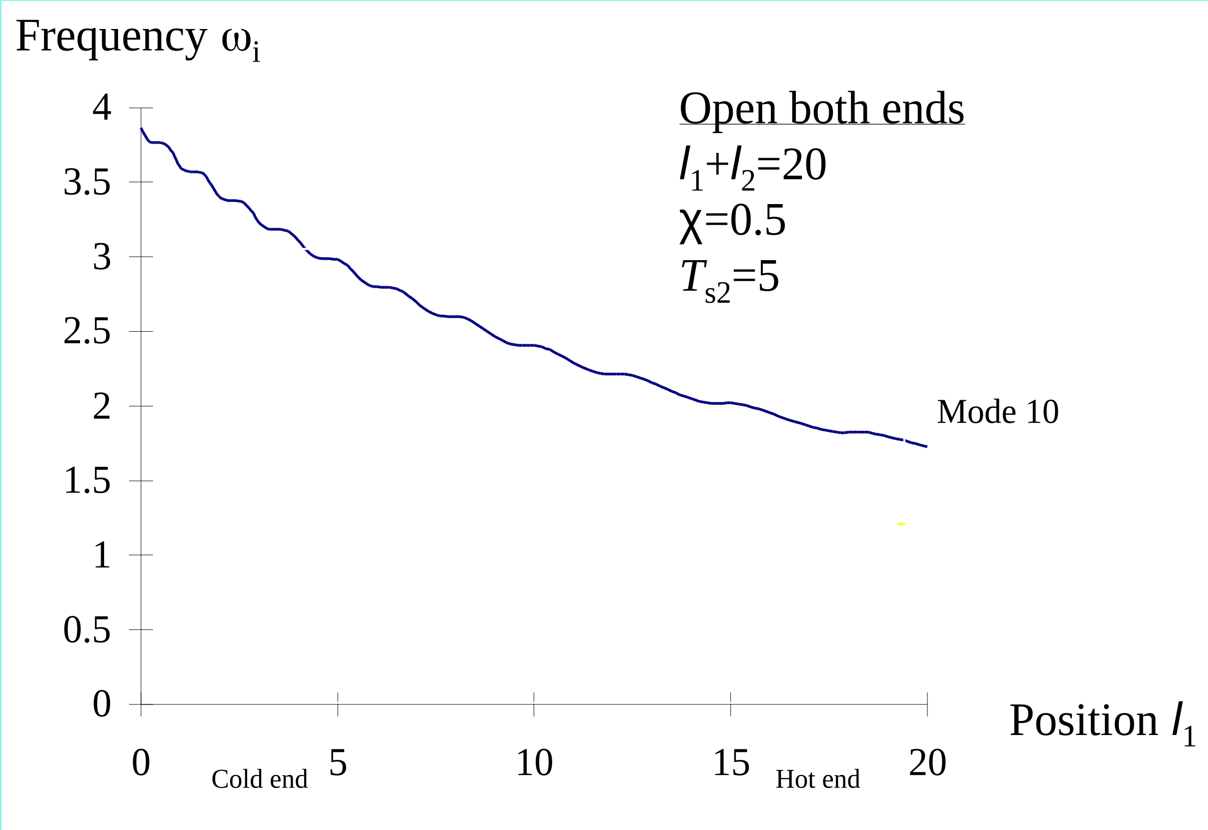

Fig. 4. Frequency variation for the 10th modes of oscillation of a fast combustion front as it progresses down a tube length 20.

Pressure Waves and Premixed Flames

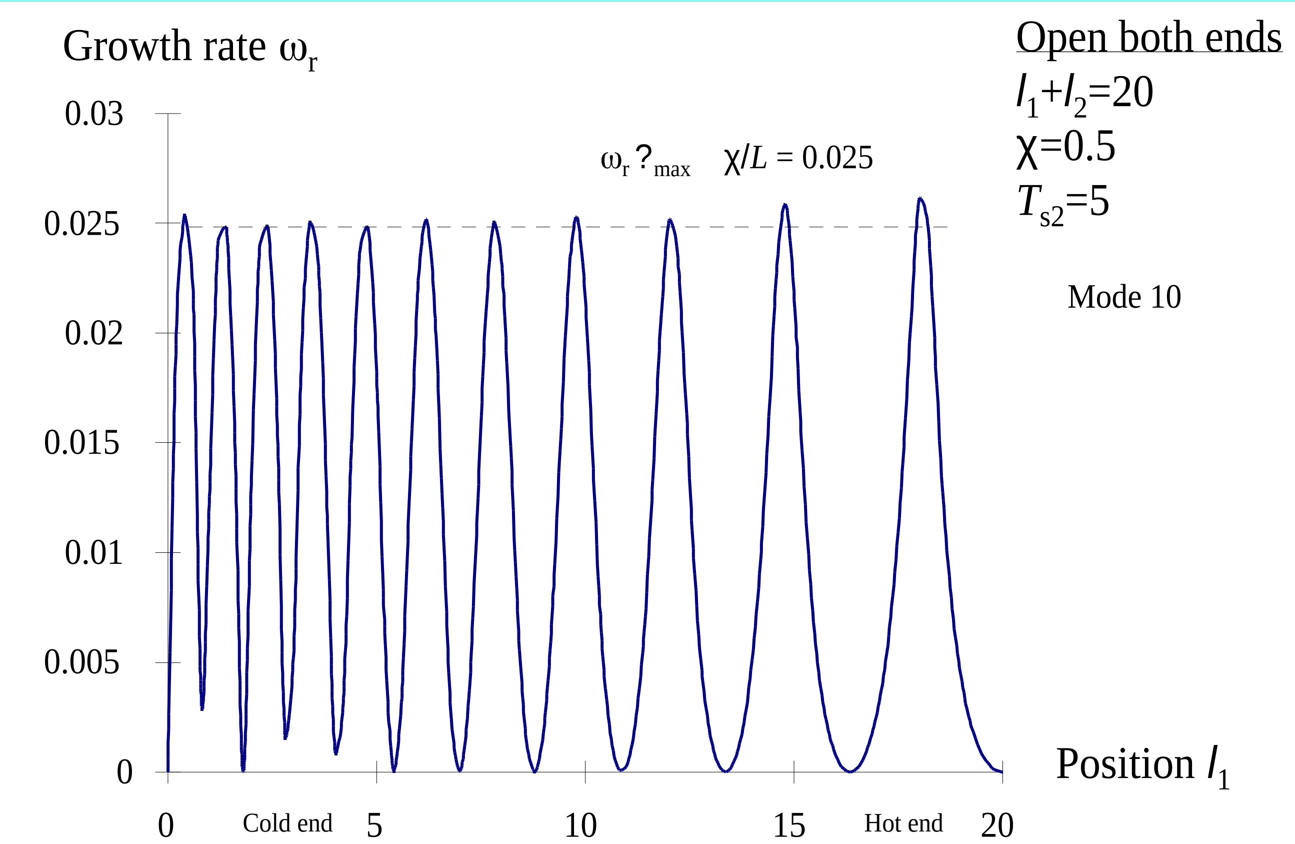

Fig. 5. Growth rate variation for the 10th mode of oscillation of a fast combustion front as it progresses down a tube length 20.

Pressure Waves and Premixed Flames

Multiple acoustic waves in tubes - hot end closed

$$\frac{\cosh(\omega\,\ell_1)}{\sinh(\omega\,\ell_1)} + \sqrt{T_{s2}}\,\frac{\sinh\left(\omega\,\ell_2\big/\sqrt{T_{s2}}\right)}{\cosh\left(\omega\,\ell_2\big/\sqrt{T_{s2}}\right)} = \chi$$

Analysis produces

$$\cot(\omega_0\ell_1) - \sqrt{T_{s2}}\tan\left(\omega_0\ell_2/\sqrt{T_{s2}}\right) = 0$$

$$\hat{\omega} \qquad \frac{\chi}{\dfrac{\ell_1}{\sin^2(\omega_0\ell_1)} + \dfrac{\ell_2}{\cos^2\left(\omega_0\ell_2/\sqrt{T_{s2}}\right)}}$$

Maximum growth rate $\quad \omega_0\ell_1 = \frac{\pi}{2} + n_1\pi \qquad \omega_0\ell_2/\sqrt{T_{s2}} = n_2\pi$

$$\frac{\ell_1}{L-\ell_1} = \frac{n_1 + \frac{1}{2}}{n_2\sqrt{T_{s2}}}$$

Pressure Waves and Premixed Flames

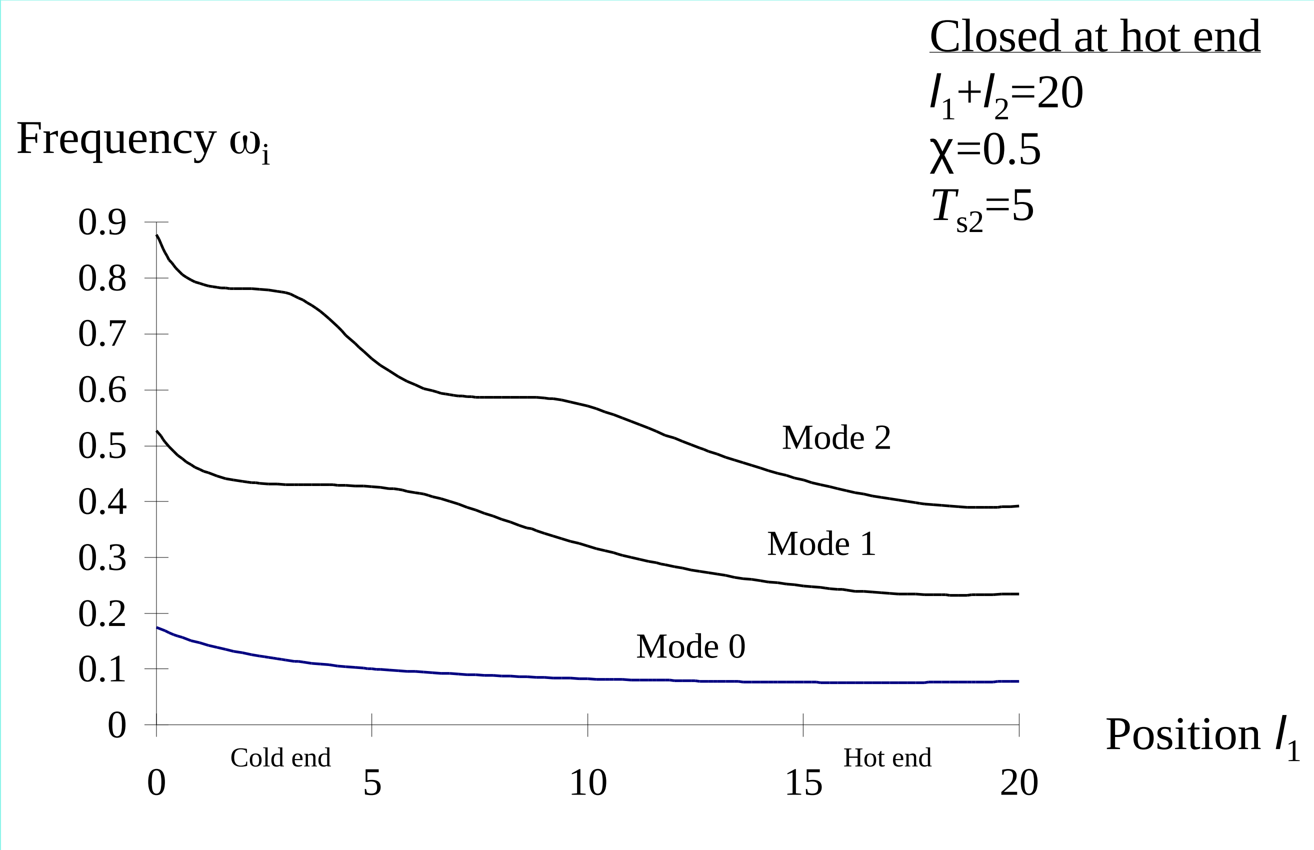

Fig. 6. Frequency variation for the first three modes of oscillation of a fast combustion front as it progresses down a tube length 20, closed at the hot end.

Pressure Waves and Premixed Flames

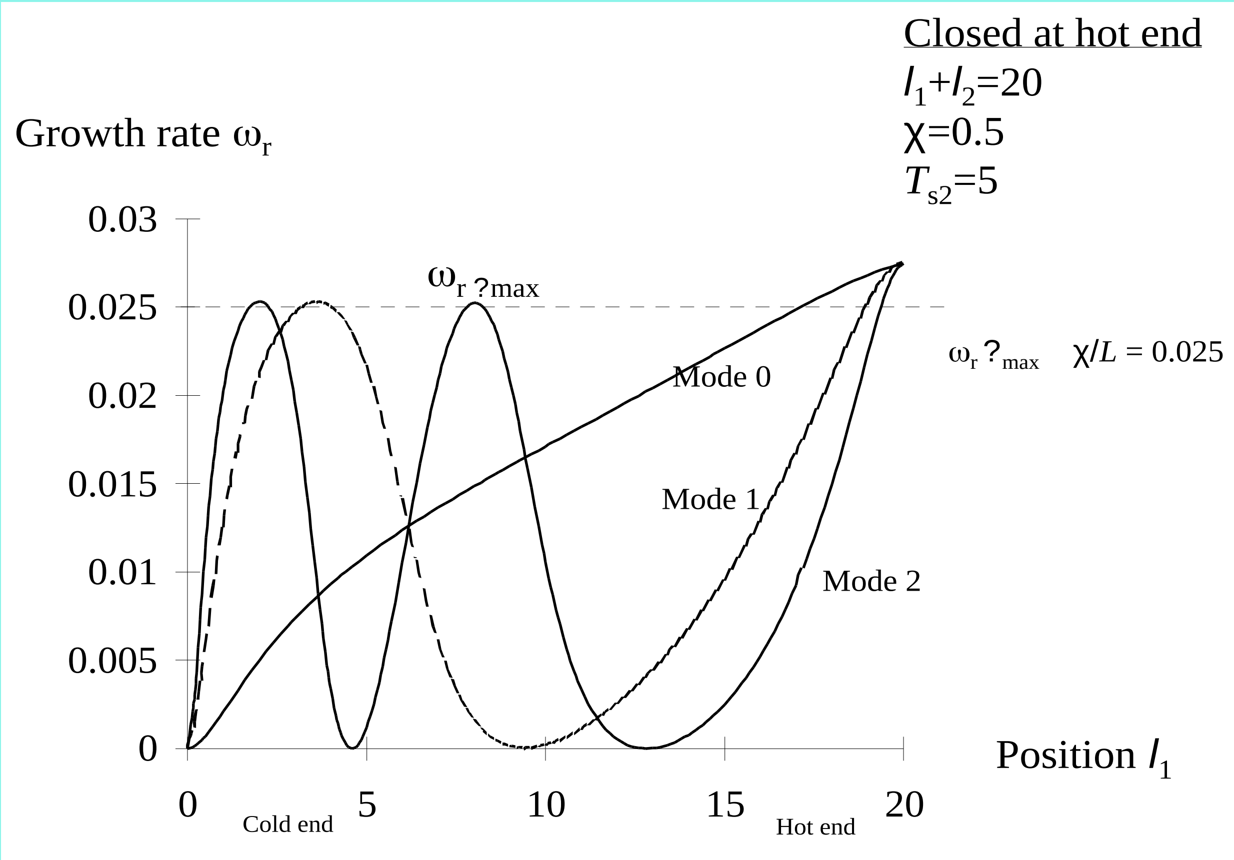

Fig. 7. Growth rate variation for the first three modes of oscillation of a fast combustion front as it progresses down a tube length 20, closed at the hot end.

Pressure Waves and Premixed Flames

Estimating the sensitivity of mass flux with pressure

$$\chi \quad \frac{\gamma Q}{p_{s2}} \frac{m_u^a}{p_{u10}}$$

$$\chi \quad \tfrac{1}{2}(\gamma \; - 1)(1 - 1/T_{s2})(\theta M_{01}) \frac{\left(\tau\omega + \left(\tfrac{1}{2} + r\right)/\,2\,/\,T_{s2}\right)}{r\,/\,T_{s2}}$$

from classical flame analysis

where $\quad r \quad \sqrt{\tau\omega + \frac{1}{4}} \qquad \tau \quad \frac{\text{diffusion time}}{\text{acoustic time}} = \frac{\kappa'\,/\,u_{01}'^{2}}{\ell'_{ac}\,/\,a'_{01}}$

Note $\tau = 0$ implies :

$$\chi = \tfrac{1}{2}(\gamma \; - 1)(1 - 1/T_{s2})M_{01}\frac{E_a'}{RT_b'}$$

Pressure Waves and Premixed Flames

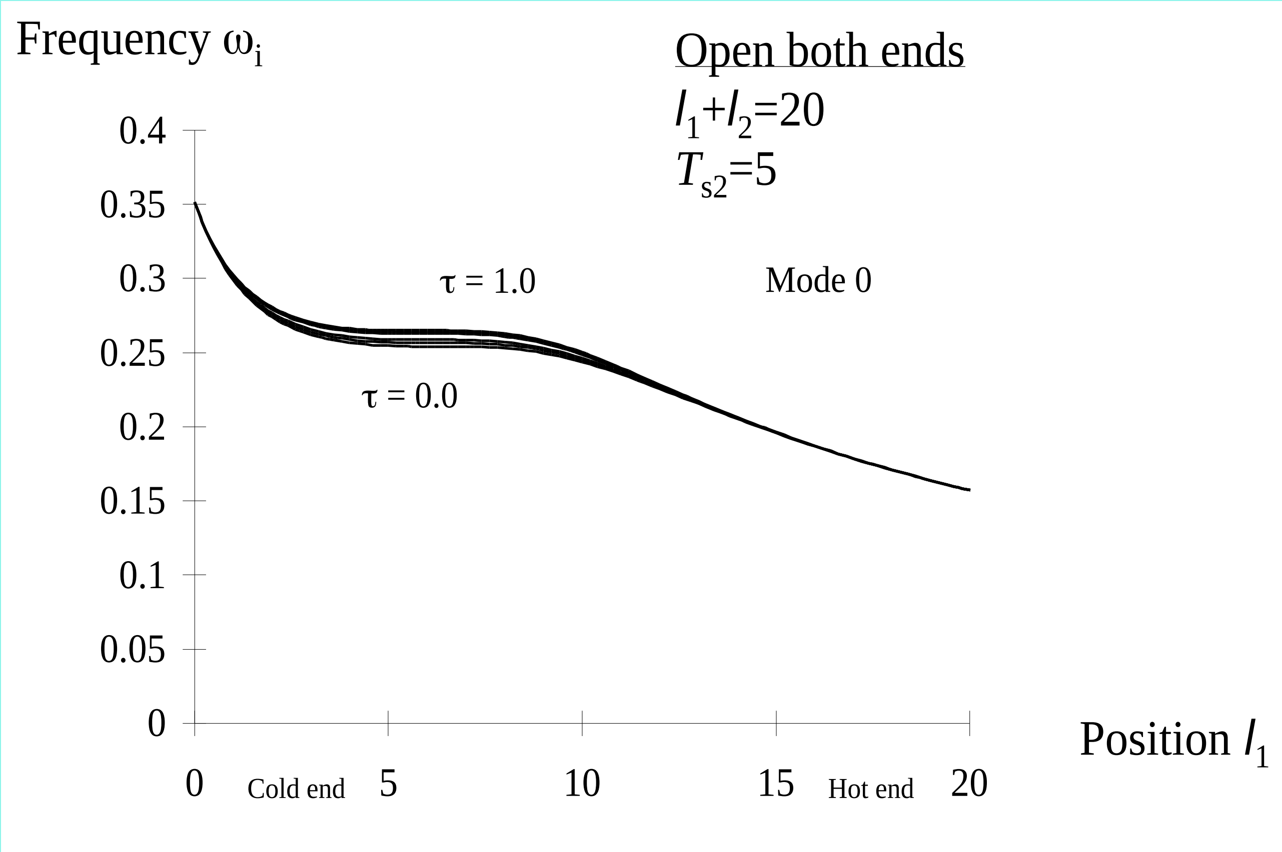

Fig. 8. Resonant frequency for a fast combustion front in a tube as a function of position. Effect of complex χ on fundamental mode.

Pressure Waves and Premixed Flames

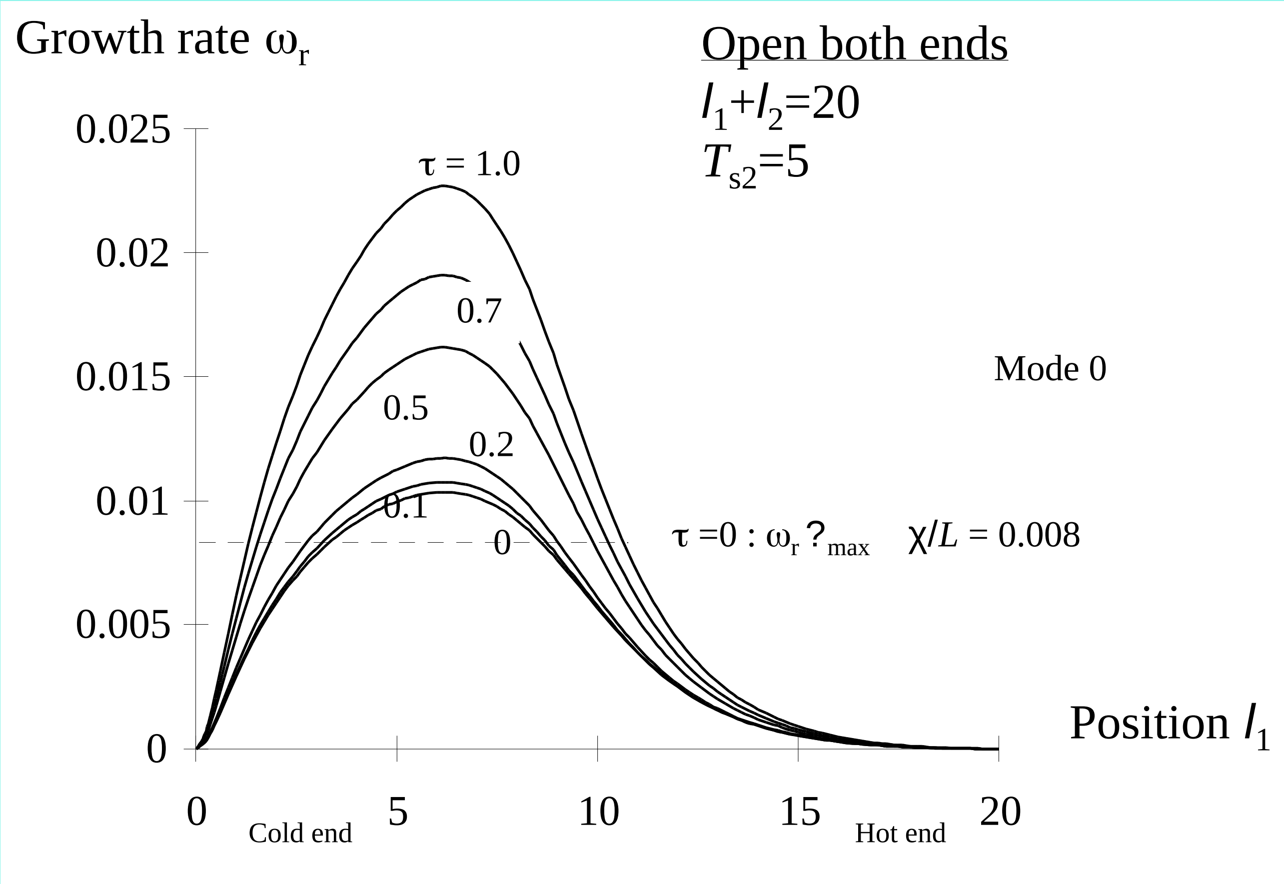

Fig. 9. Growth rate for resonance of a fast combustion front in a tube as a function of position. Effect of complex χ on fundamental mode.

Pressure Waves and Premixed Flames

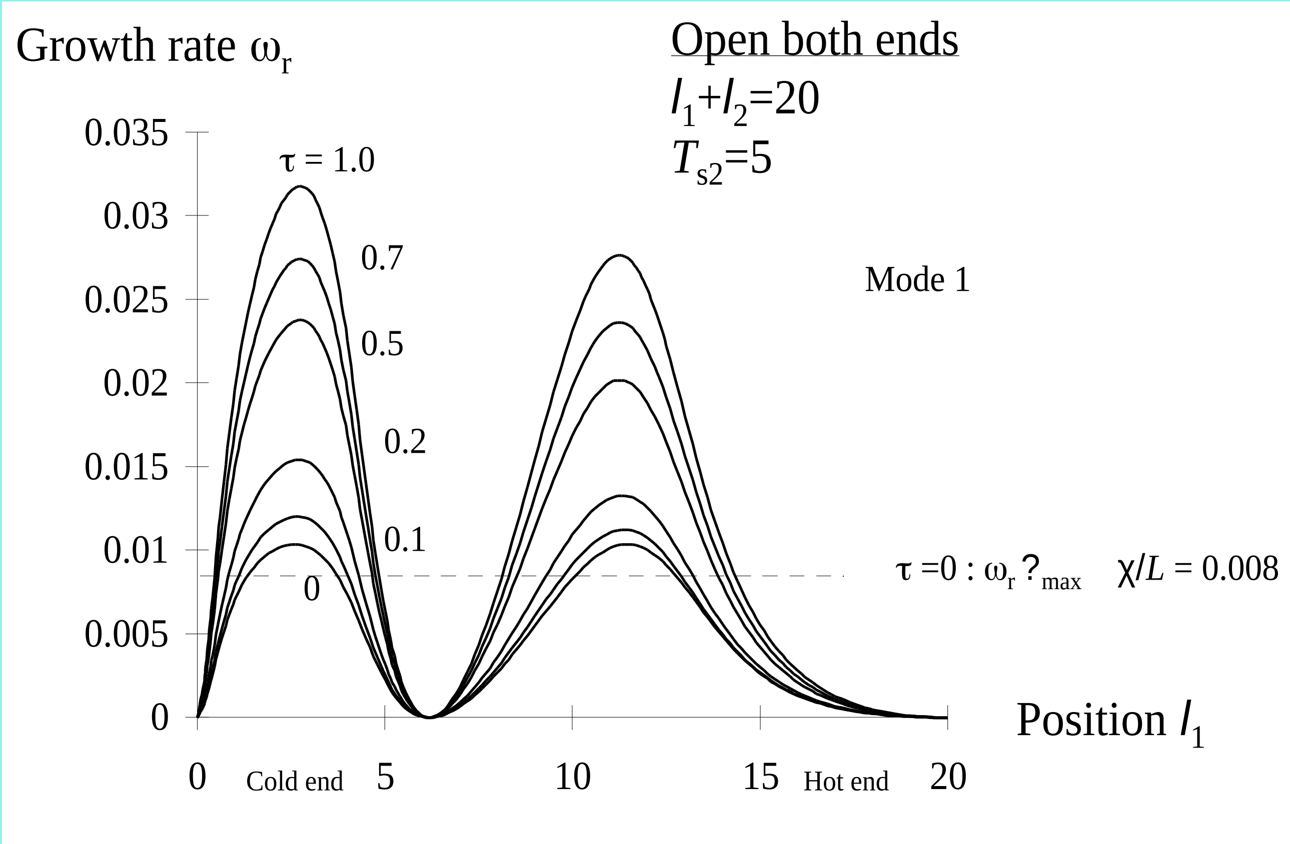

Fig. 10. Growth rate for resonance of a fast combustion front in a tube as a function of position. Effect of complex χ on 2nd mode.

Pressure Waves and Premixed Flames

Concluding remarks on growth rate

In reality for the turbulent flame, the value of χ will be substantially more than the laminar formula used here, but the form will be similar for the thick turbulent case.

Note $$\hat{M} = \frac{\sqrt{\rho_{s1} p_{s1}}}{\sqrt{\rho_{s2} p_{s2}}} = \frac{1}{\sqrt{\rho_{s2} p_{s2}}} = \frac{\sqrt{T_{s2}}}{p_{s2}} \quad \sqrt{T_{s2}}$$

and $$\omega_r\big|_{\max} \quad \frac{\chi}{L} \quad ? \quad \omega_r\big|_{\max} \quad \tfrac{1}{2}(\gamma - 1)(1 - 1/T_{s2}) M_{01} \frac{E'_a}{RT'_b}\frac{1}{L}$$

(by using the classical flame approximation)

Thus dimensionally $$\omega'_r\big|_{\max} \quad \frac{1}{2}(\gamma - 1)(1 - T'_{s1}/T'_{s2}) M_{01} \frac{E'_a}{RT'_b}\frac{a'_{01}}{L'}$$

i.e. $$\boxed{\omega'_r\big|_{\max} \quad \frac{1}{2}(\gamma - 1)(1 - T'_{s1}/T'_{s2}) \frac{E'_a}{RT'_{s2}}\frac{u'_{01}}{L'}}$$

Pressure Waves and Premixed Flames

The main conclusions are that

• For a given tube geometry, and acoustic interactions with a fast weakly compressible combustion front, there is near constant maximum growth rate of the oscillations.

• Depending on the mode, there are locations in the tube where the growth rate is zero.

• The maximum growth rate is inversely proportional to tube dimension, and directly proportional to the sensitivity of the mass flux of the combustion front to pressure.

• For a tube with the hot end closed, the higher modes become more dominant as the weakly compressible combustion front reaches the cold end of the tube.