Noise Analysis of RF Circuits

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Outline

- Existing approaches
- Stochastic differential equation based approach
  - open loop oscillator phase noise analysis (joint work with Alper Demir and Jaijeet Roychowdhury)
  - nonlinear nonautonomous circuit noise analysis
  - noise analysis of phase feedback systems (PLL and DLL)

Traditional Noise Analysis

Non-oscillatory circuits

- **based on linear perturbation analysis**
- **linearize circuit equations around large-signal steady-state response**

- **compute LPTV transfer function to obtain output noise in time and frequency domain**

Oscillator circuit

- **linear circuit operation assumed**
- **predicts infinite integrated noise power**

Stochastic Analysis Based Approach

- model circuit noise sources as time-varying white noise
- circuit represented as a system of differential equations
- inputs to the DE system are stochastic processes (circuit white noise sources)
- circuit equations: system of stochastic differential equations
- SDEs solved using techniques from stochastic calculus

For oscillators

- linear perturbation analysis is not valid
- developed new nonlinear perturbation analysis for oscillators

Preliminaries

Unperturbed (noiseless) circuit equations

$$\dot{x} = f(x) + b(t)$$

- $b(t)$, T-periodic input signal
- Noiseless T-periodic solution $x_s(t)$

Perturbed (noisy) circuit equations

$$\dot{x} = f(x) + b(t) + D(x)\xi(t)$$

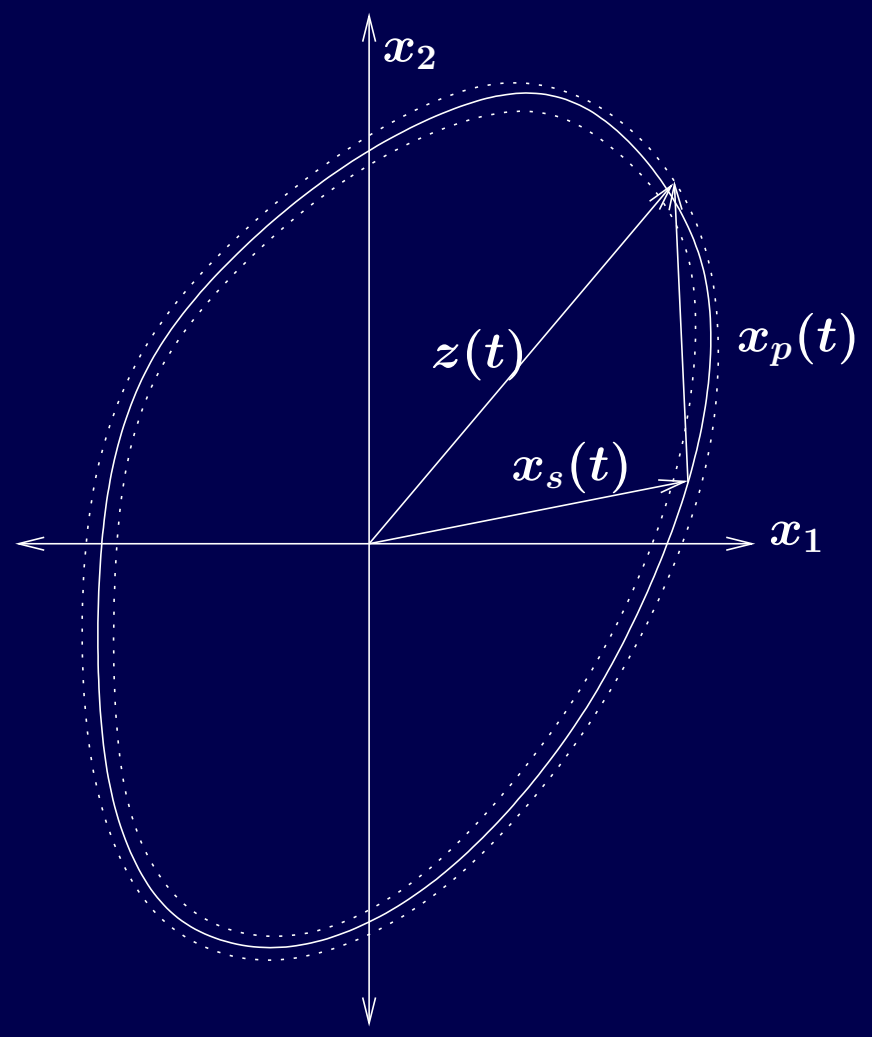
Perturbed response

$$z(t) = x_s(t) + x_p(t)$$

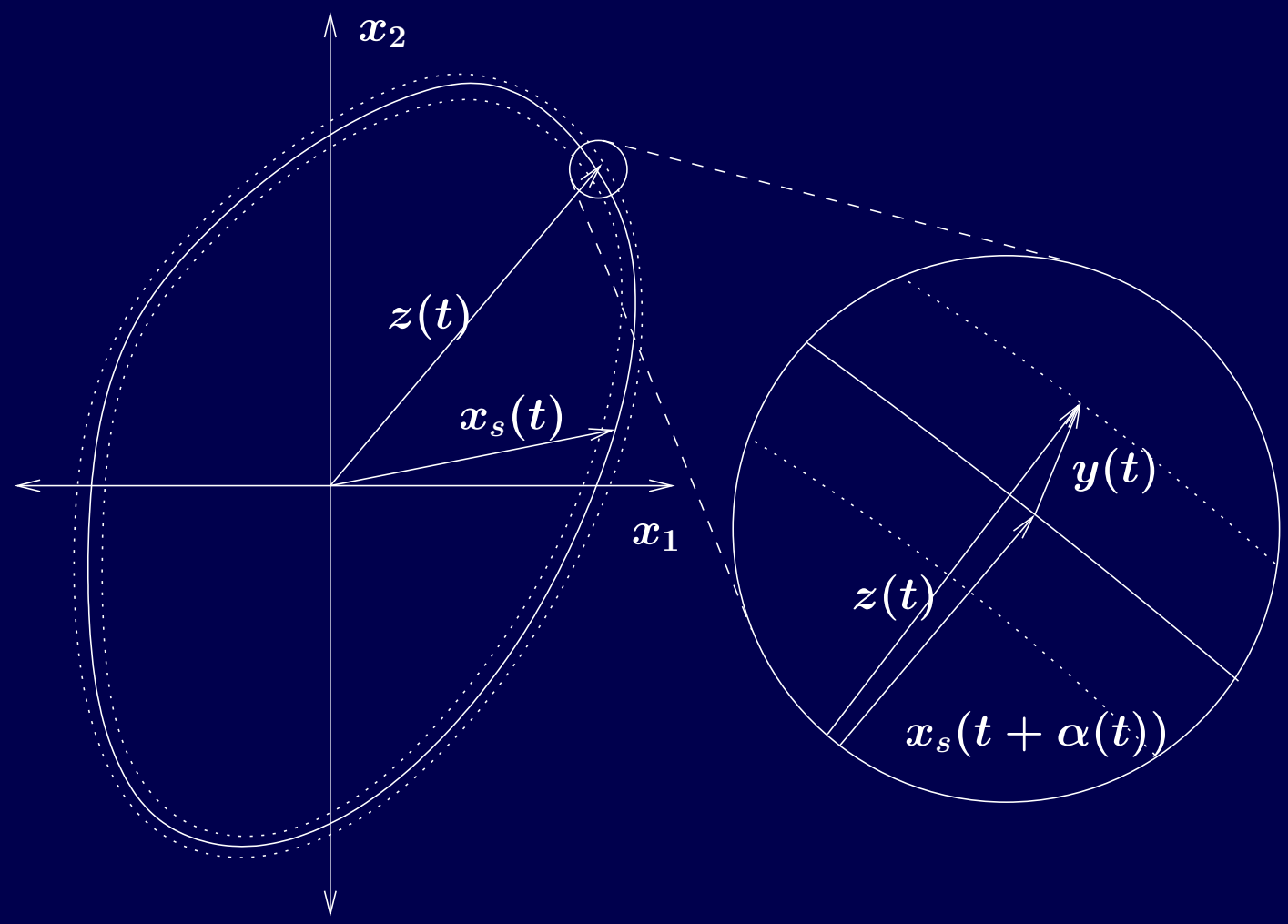
Perturbed Oscillator Output: Phase Plane View

Linear analysis predicts that $x_p(t)$ grows unbounded even for small perturbations

Linear analysis result not consistent!



Key Idea of Our Approach



Phase Error α

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = v^T(t + \alpha(t))\xi(t)$$

- $v^T(t) = v_1^T(t)D(x_s(t))$
- $D(x_s(t))$: noise source intensities at steady-state
- $v_1^T(t)$: Floquet basis vector for the LPTV system

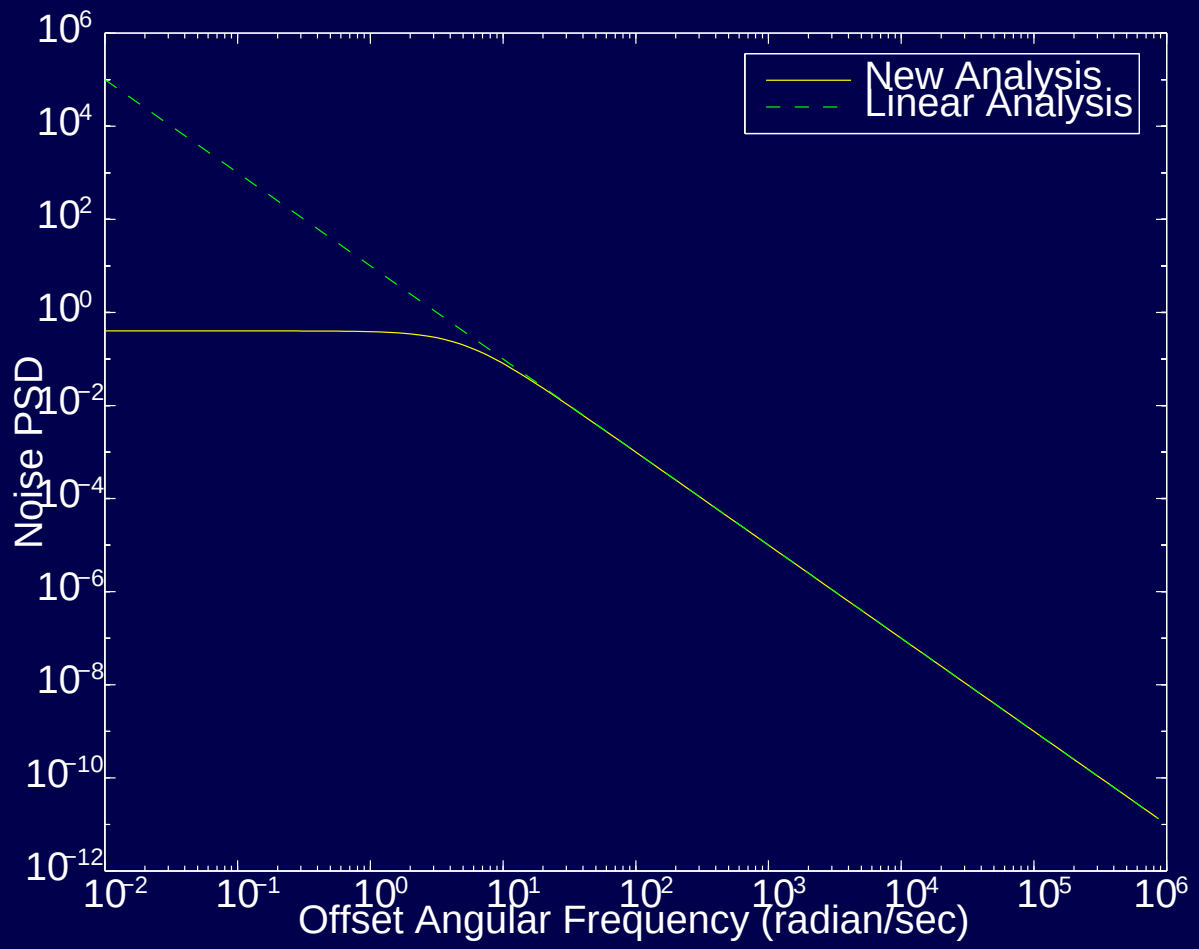
For $\xi(t)$ uncorrelated white noise processes

$$\mathbb{E}\left[\alpha(t_1)\alpha(t_2)\right] = c\min(t_1, t_2)$$

where c is the time-average of $v^T(t)v(t)$.
Oscillator output $x_s(t+\alpha(t))$ asymptotically wide sense stationary

$$S(\omega) = \sum_{i=-\infty}^{\infty} X_i X_i^* \frac{\omega_0^2 i^2 c}{\frac{1}{4}\omega_0^4 i^4 c^2 + (\omega + i\omega_0)^2}$$

Phase Noise Spectrum

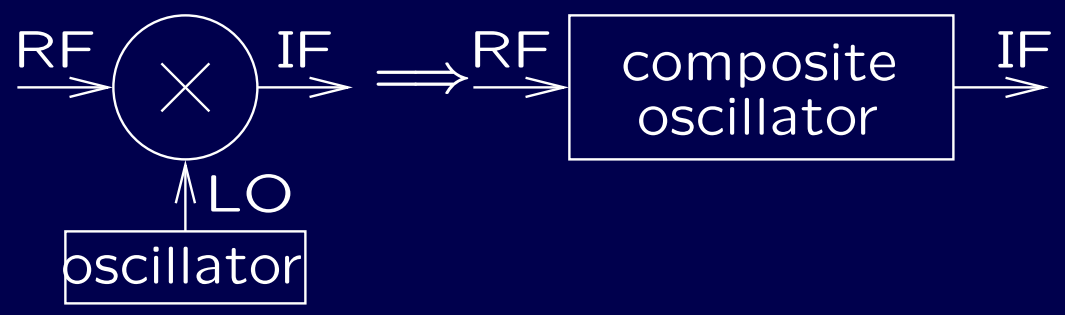


Results

- output process wide sense stationary
- the output spectrum splits as Lorentzian in presence of noise
- total noise power same as unperturbed oscillator output
- finite noise power density at the carrier
- the relative contribution of circuit noise sources to oscillator phase noise: useful to design low noise oscillators
- one scalar is sufficient to characterize phase noise: ideal for modelling
- efficient computation of phase noise in time and frequency domain

Noise Analysis of Non-Autonomous Circuits

- driving periodic signal from a real (noisy) oscillator
- view non-autonomous circuit and oscillator as composite oscillator



- Output noise asymptotically wide-sense stationary

Effect of Input Signal Phase Noise

- Circuit equations

$$\dot{x} = f(x) + b(t + \alpha_{in}(t)) + D(x)\xi_p(t)$$

where $\alpha_{in}(t) = \sqrt{c_{in}}B(t)$

- Solution of the form

$$x_s(t + \alpha(t)) + y(t)$$

- circuit output noise wide-sense stationary component of

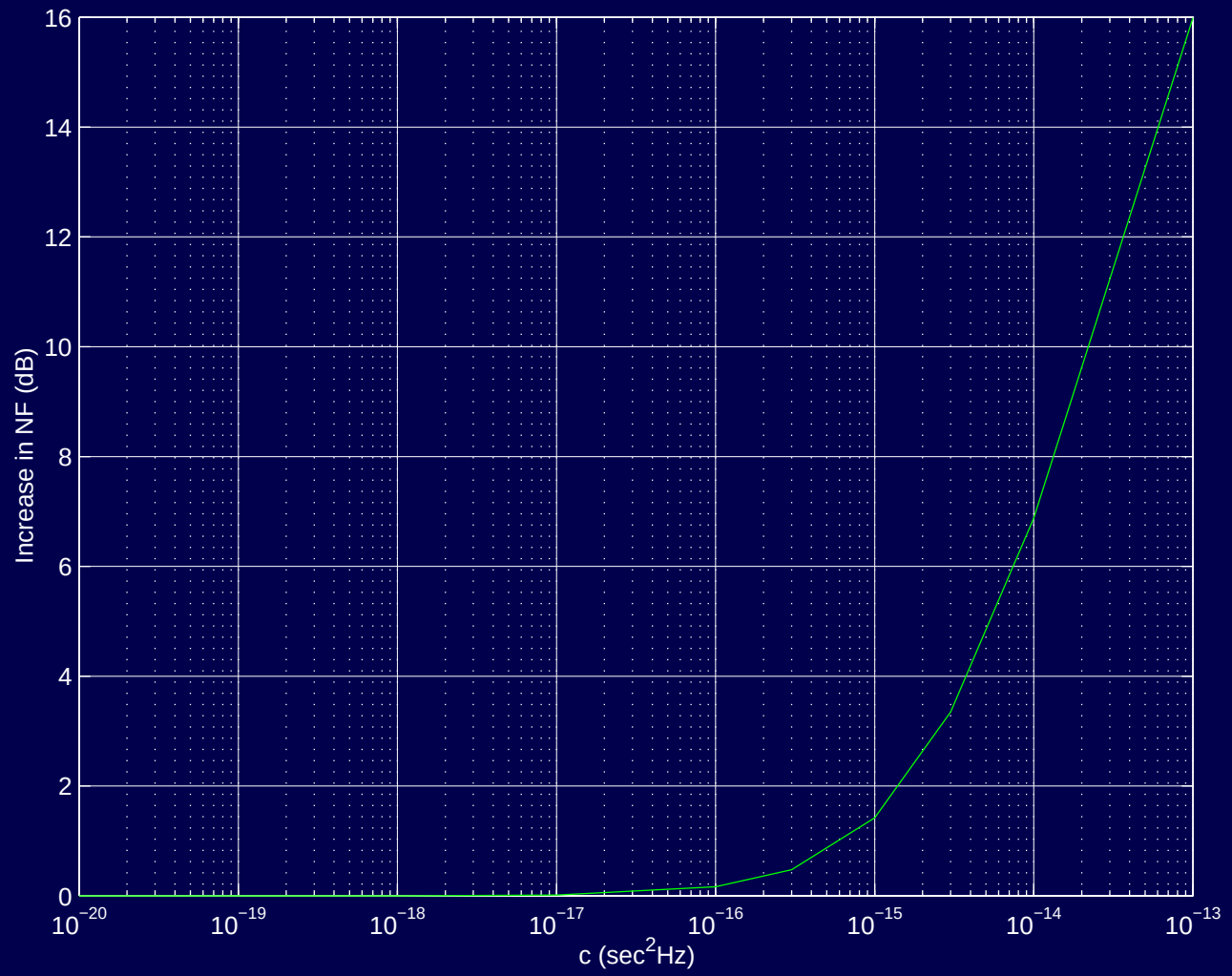
$$\mathrm{d}z = J(t)z(t)\mathrm{d}t + D(x_s(t))\mathrm{d}B_p(t) + \sqrt{c}\dot{x}_s(t)\mathrm{d}B_1(t)$$

where

$$J(t) = \left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x_s(t)}$$

- can use existing LPTV noise analysis techniques with extra noise source

Effect of Input Signal Phase Noise

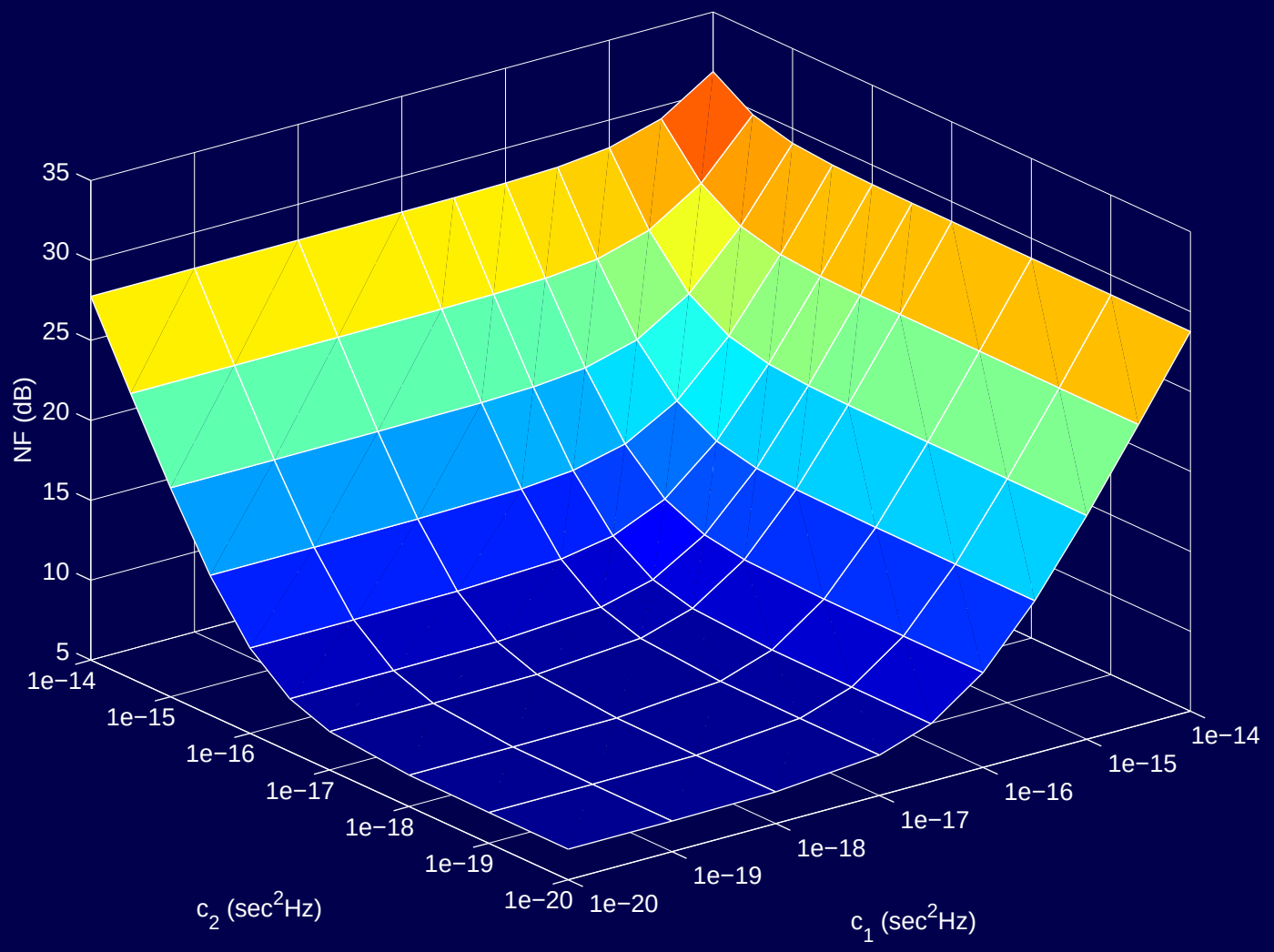


Extension to Circuits with Multi-Tone Excitations

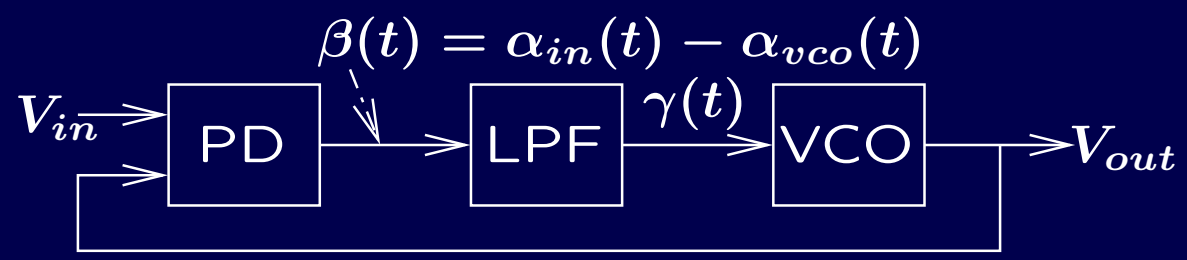
- circuit driven by two (or more) large periodic signals
- circuit response quasi-periodic $x(t) = \hat{x}(t_1, t_2)|_{t_1=t_2=t}$
- practical application: noise performance degradation in presence of blocker
- circuit output noise wide-sense stationary component of

$$\begin{aligned}
\mathrm{d}z(t) = & \sum_{i=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} J(i,k) \exp\big[\jmath(i\omega_1 + k\omega_2)t\big] z(t)\mathrm{d}t \\
& - \sqrt{c_1} \left.\frac{\partial \hat{x}_s(t_1,t_2)}{\partial t_1}\right|_{t_1=t_2=t} \mathrm{d}B_1(t) \\
& - \sqrt{c_2} \left.\frac{\partial \hat{x}_s(t_1,t_2)}{\partial t_2}\right|_{t_1=t_2=t} \mathrm{d}B_2(t) + D(x_s(t))\mathrm{d}B_p(t)
\end{aligned}$$

Two Tone Noise Analysis Results



Noise Analysis of Phase Feedback Systems: PLL



- VCO noise equation reformulated with phase feedback

$$\frac{\mathrm{d}\alpha_{vco}}{\mathrm{d}t} = v^T(t + \alpha_{vco}(t))\xi_p(t) + v_{control}(t + \alpha_{vco}(t))\gamma(t)$$

- V_{in} has Brownian motion phase error $\alpha_{in}(t)$
- Loop filter dynamics

$$G\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\beta(t)\\ \gamma(t)\\ \vdots\end{bmatrix} = E\begin{bmatrix}\beta(t)\\ \gamma(t)\\ \vdots\end{bmatrix} + F\xi_q(t)$$

q noise sources in LPF and PD

PLL Noise Analysis Results

- overall

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\beta(t)\\ \gamma(t)\\ \vdots\end{bmatrix} = -A\begin{bmatrix}\beta(t)\\ \gamma(t)\\ \vdots\end{bmatrix} + D\xi_{p+q+1}(t)$$

- $\beta(t)$ a component of an Ornstein-Uhlenbeck process and

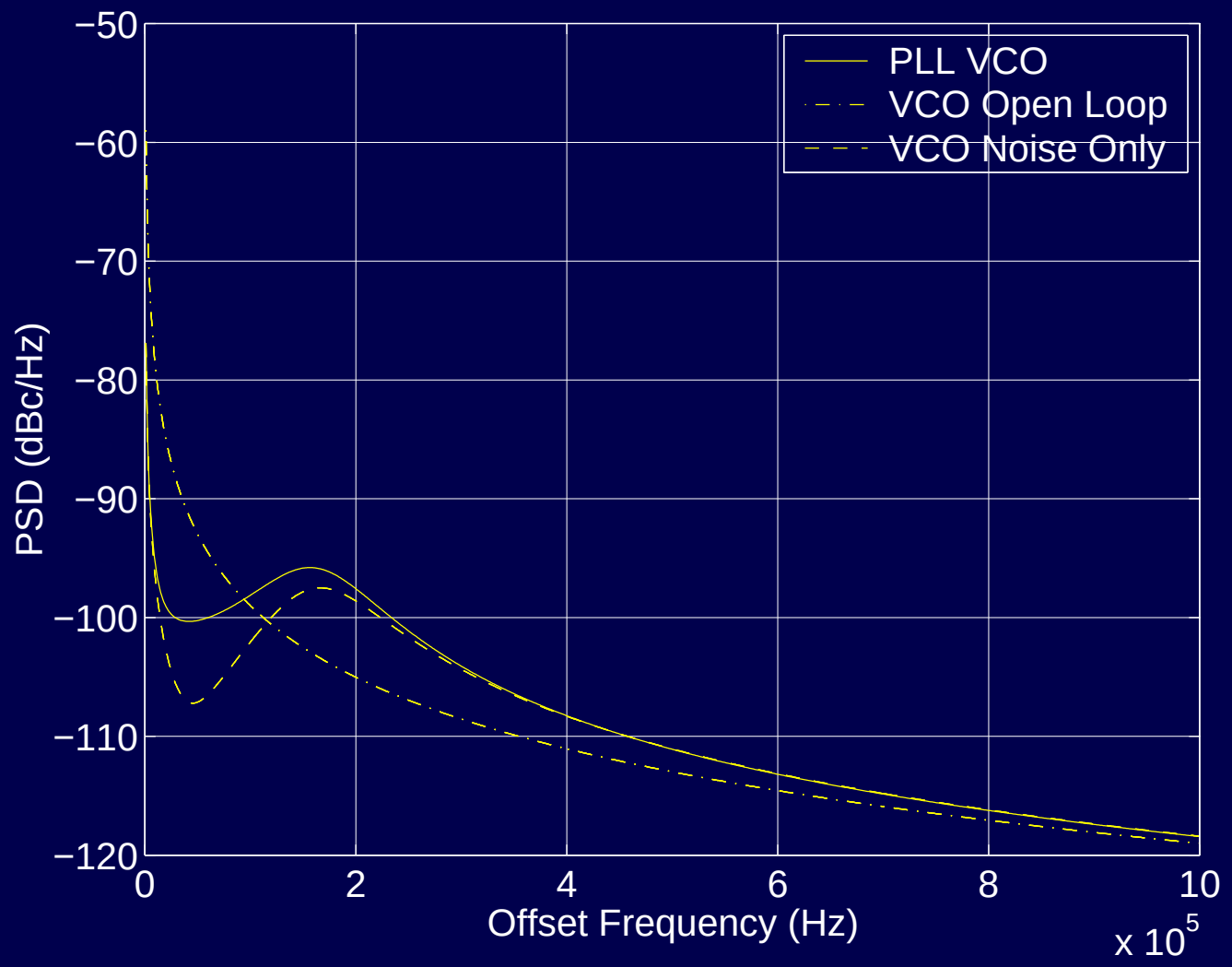
$$\mathbb{E}\left[\beta(t_1)\alpha_{in}(t_2)\right] = \sum_{i=1}^{n}\mu_i \exp(-\lambda_i \min(0, t_2 - t_1))$$

$$\mathbb{E}\left[\beta(t_1)\beta(t_2)\right] = \sum_{i=1}^{n}\nu_i \exp(-\lambda_i |t_1 - t_2|)$$

for appropriate μ_i and ν_i, λ_i eigenvalues of A, $n-1$ loop filter order

PLL Noise Analysis Results (contd.)

- PLL output $x_s(t+\alpha_{vco}(t))$ asymptotically wide-sense stationary
- spectrum

$$S_{x_s,x_s}(\omega) = \sum_{i=-\infty}^{\infty} \sum_{k_1,\ldots,k_n=0}^{\infty} 2X_i X_i^* \exp\left[-\omega_0^2 i^2 \sum_{l=1}^{n}(\mu_l+\nu_l)\right] \frac{\left[\prod_{l=1}^{n}\left[i^2\omega_0^2(\mu_l+\nu_l)\right]^{k_l}\right]\left(\frac{1}{2}\omega_0^2 i^2 c_{in} + \sum_{l=1}^{n} k_l\lambda_l\right)}{\left[\prod_{l=1}^{n} k_l!\right]\left[\left(\frac{1}{2}\omega_0^2 i^2 c_{in} + \sum_{l=1}^{n} k_l\lambda_l\right)^2 + (\omega+i\omega_0)^2\right]}$$

PLL Output Spectrum