NOISE IN OPTICAL SYSTEMS

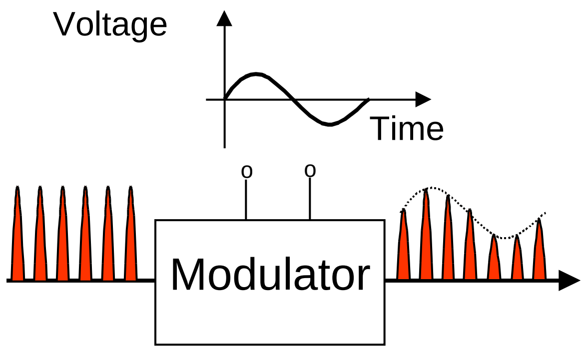
F. X. Kärtner

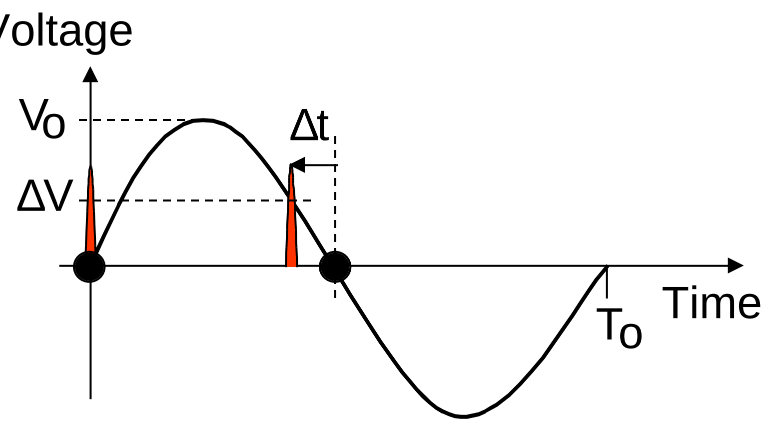
High-Frequency and Quantum Electronics Laboratory

University of Karlsruhe

Outline

High-Speed A/D-Conversion (100 GHz)





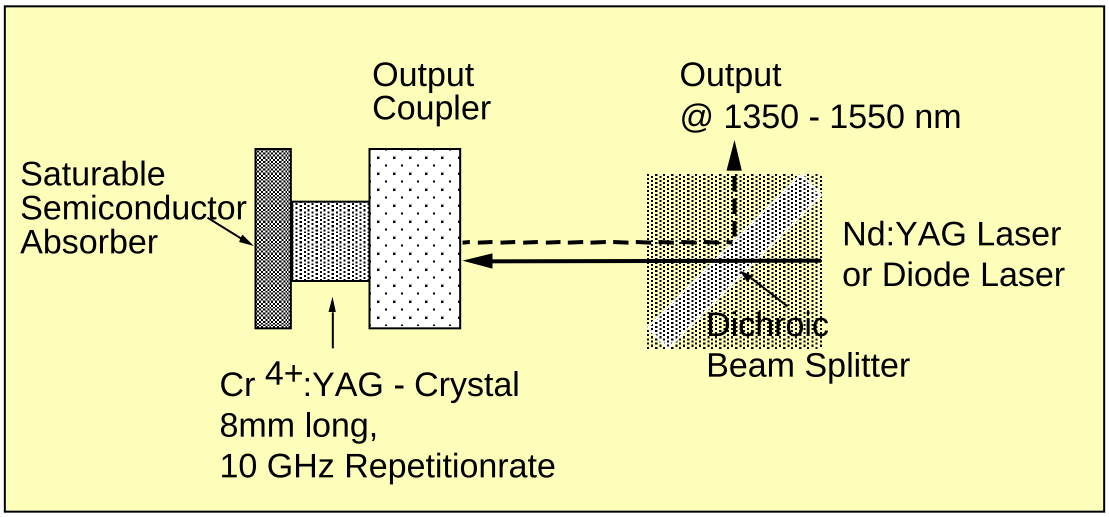
Timing-Jitter Δt: $\frac{\Delta V}{V_o} = 2\pi \frac{\Delta t}{T_o}$

$\frac{\Delta V}{V_o}$: 10 bit

$\frac{1}{T_o}$ =100 GHz

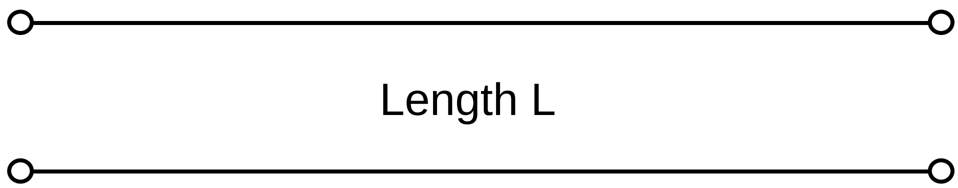
=> $\Delta t \sim 1$ fs

Mode-Locked Cr^{4+}: YAG Microchip-Laser



- Compact: Saturable Absorber,
  Dispersion Compensating Mirrors
- 10 GHz, 20 fs - 1 ps, @ 1350 - 1550 nm
- Very Small Timing-Jitter < 1 fs
- Cheap (< 10.000 $)

Classical and Quantum-Noise in Optical Systems
(Modes of the EM-Field)



$$A(z,t) = \frac{1}{\sqrt{L}}\ {}_{m}\ a_m e^{j(\beta_m z - \omega_m t)}, \quad \beta_m = \frac{2\pi}{L} m, \quad \omega_m = \omega(\beta_m)$$

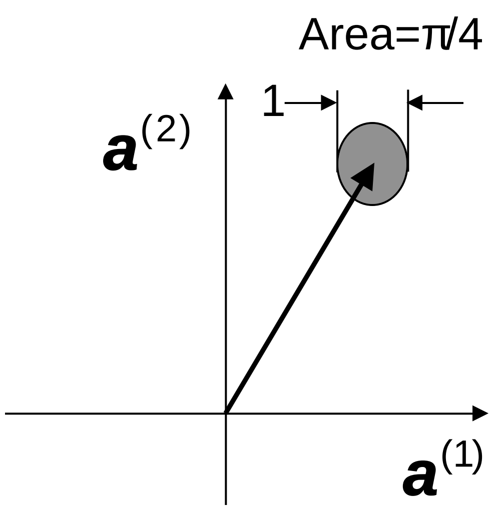
$$\hbar\omega_m a_m^* a_m : \text{Energy of m-th mode,} \qquad \text{QM}: a_m^*, a_m \ \blacklozenge\ \hat{a}_m^+, \hat{a}_m$$

Thermal Equilibrium

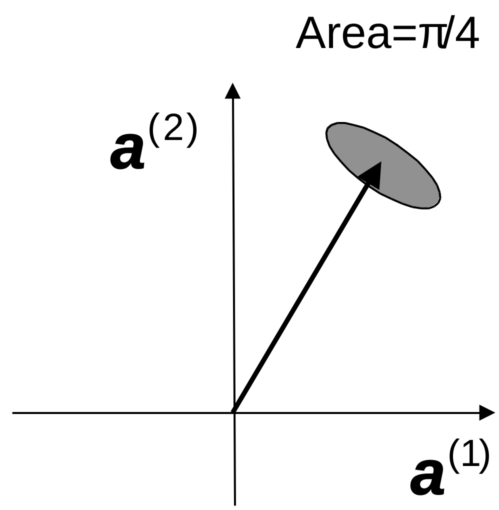
$$\langle \hat{a}_n^* \hat{a}_m \rangle = \delta_{n,m} \frac{1}{e^{\hbar\omega_m / kT} - 1} \overset{\omega_m \ \blacklozenge\ 0}{=} kT$$

States and Quadrature Fluctuations

$$\boldsymbol{a} = \boldsymbol{a}^{(1)} + j\boldsymbol{a}^{(2)}$$

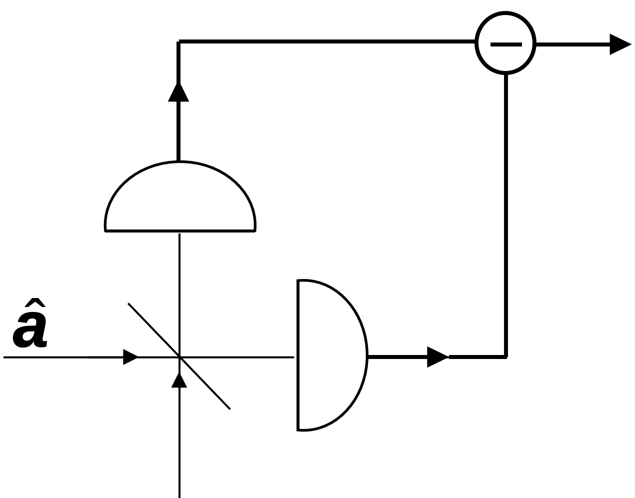


Coherent States
(Minium Uncertainty States)



Squeezed States

Balanced Homodyne-Detection



LO $\alpha = |\alpha| e^{j\varphi}$

$$|\alpha| >> |\langle a \rangle|$$

$$\hat{I} = \alpha^* \hat{a} + \alpha \hat{a}^+$$
$$= |\alpha| \left(\hat{a} e^{-j\varphi} + \hat{a}^+ e^{j\varphi} \right)$$

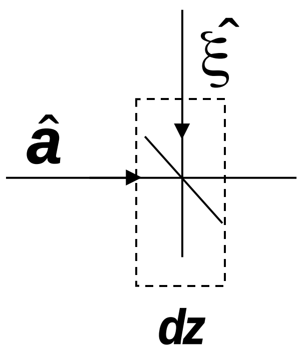
Continuum of modes

$$\hat{I} = \sum_m \left(\alpha_m^* \hat{a}_m + \alpha_m \hat{a}_m^+ \right)$$

$$\hat{I} = \int_{-\infty}^{\infty} \left(\alpha^*(t) \hat{a}(t) + \alpha(t) \hat{a}^+(t) \right) dt$$

Loss- and Amplifier-Noise

Loss:



$$\frac{d\hat{a}}{dz} = -\alpha\,\hat{a} + \hat{\xi}; \quad \left\langle \hat{\xi}^{+}(z')\hat{\xi}(z) \right\rangle = 2\alpha\delta(z - z')$$

Necessary noise for maintaining uncertainty circle

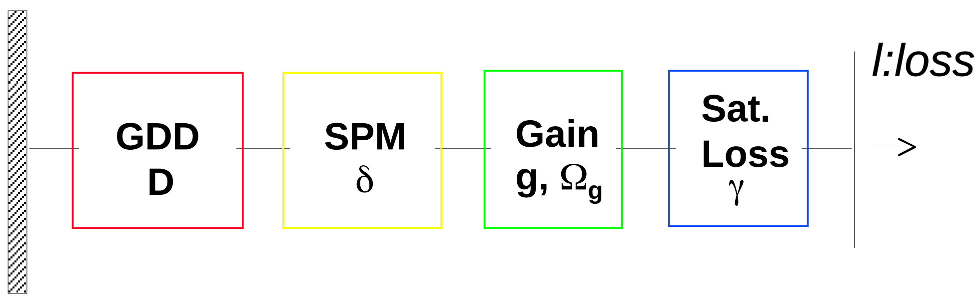
Amplifier:

$$\frac{d\hat{a}}{dz} = g\,\hat{a} + \hat{\xi}^{+}; \quad \left\langle \hat{\xi}^{+}(z')\hat{\xi}(z) \right\rangle = 2g\delta(z - z')$$

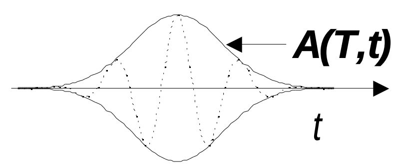
Spontaneous emission noise

Non-Ideal Amplifier: $\left\langle \hat{\xi}^{+}(z')\hat{\xi}(z) \right\rangle = 2g\Theta\delta(z - z'), \quad \Theta > 1$

Dynamics of Mode-Locked Lasers



T_R: cavity roundtrip time



small changes per roundtrip

$$T_R \frac{\partial}{\partial T} A(T,t) = \left[jD \frac{\partial^2}{\partial t^2} - j\delta\, |A|^2 \right] A + \left[g \left(1 + \frac{1}{\Omega_g^2} \frac{\partial^2}{\partial t^2} \right) - l + \gamma\, |A|^2 \right] A$$

Energy Conserving

Dissipative

Steady-State Solution

If $\frac{|D|}{\delta} = \frac{\frac{g}{\Omega_g^2}}{\gamma}$ pulses are solitonlike

$$A_s(T,t) = a_s\left(\frac{t-t_0}{\tau}\right)\exp\left(-j\Phi_0\frac{T}{T_R} + j\theta_0\right) \qquad a_s(t) = A_0\,\mathrm{sech}\left(\frac{t-t_0}{\tau}\right)$$

$$\text{Soliton Energy}: W_0 = 2A_0^2\tau$$

$$\text{Soliton Width}: \tau = \frac{4|D|}{\delta W_0}$$

$$\text{Phase Shift per Roundtrip}: \Phi_0 = \frac{|D|}{\tau^2} = \frac{1}{2}\delta A_0^2$$

$$\text{Area-Theorem}: A_0\tau = \sqrt{2\frac{|D|}{\delta}} \;\blacklozenge\; W_0 \quad A_0$$

The System with Noise

$$T_R \frac{\partial}{\partial T} A(T,t) = \left[jD\frac{\partial^2}{\partial t^2} - j\delta |A|^2 \right] A + \left[g\left(1+\frac{1}{\Omega_g^2}\frac{\partial^2}{\partial t^2}\right) - l + \gamma |A|^2 \right] A + S(T,t)$$

$$A(T,t) = \left(a_s(t-t_0) + \Delta a(t-t_0)\right)\exp\left[-j\Phi_0\frac{T}{T_R} + j\theta_0\right]$$

Amplifier Noise: $\left\langle S_q(T,t)^* S_q(T',t')\right\rangle = h\nu\ \Theta \frac{2g}{T_R}\delta(T-T')\delta(t-t')$

Gain Fluctuations: $S_{\Delta g}(T,t) = \frac{\Delta g(T)}{T_R}\left(1-\frac{1}{\Omega_g}\frac{\partial}{\partial t}\right)a_s(t)$

Cavity Length or Index Fluctuations:

$$S_{\Delta L}(T,t) = -\frac{\Delta L(T)}{T_R}\left[j\frac{\omega_0}{c}n + \frac{1}{v_g}\frac{d}{dt}\right] a_s(t)$$

Soliton-Perturbation Theory

$$\Delta a(T,t) = \Delta w(T) f_w(t) + \Delta\theta(T) f_\theta(t) + \Delta p(T) f_p(t) + \Delta t(T) f_t(t) + \Delta A_c(T,t)$$

Energy Phase Center-frequency Timing and Continuum

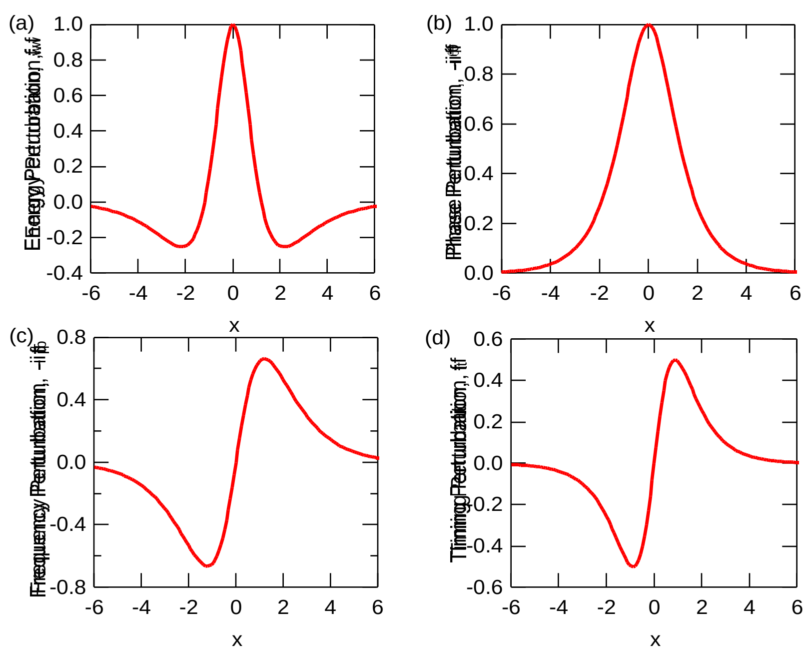
$$\text{with}: x = t/\tau$$

$$f_w(x) = \frac{1}{W_0}(1 - x\tanh x) a_s(x)$$

$$f_\theta(x) = j a_s(x)$$

$$f_p(x) = j\frac{2}{W_0} x\tau \tanh x a_s(x)$$

$$f_t(x) = \frac{1}{\tau}\tanh x a_s(x)$$



Linearized and Adjoint System

$$T_R \frac{\partial}{\partial T} \Delta a(T,t) = L\,\Delta a(T,t) + S(T,t)\exp\left(j\Phi_0 \frac{T}{T_R} - j\theta_0\right)$$

Linearized system is not hamiltonian,
it is pumped by the steady-state pulse

Scalar Product: $\langle f | g \rangle = \frac{1}{2}\int_{-\infty}^{+\infty} f(t)^{*} g(t)\,dt + c.c.$

Interpretation: Field *g* is homodyne detected by LO *f*

Adjoint System L^{+}: Biorthogonal Basis

$$\underline{f}_w(x) = 2a_s(x) \qquad \underline{f}_p(x) = j\frac{2}{W_0\tau}\tanh x\, a_s(x)$$

$$\underline{f}_\theta(x) = \frac{2j}{W_0}(1 - x\tanh x)\,a_s(x) \qquad \underline{f}_t(x) = \frac{2}{W_0}x\tau\tanh x\, a_s(x)$$

Basic Noise Processes

$$T_R \frac{f}{f T} \Delta w = -\frac{1}{\tau_w} \Delta w + T_R S_w(T) \qquad \frac{1}{\tau_w} = \frac{2}{T_R} \text{¾} \frac{f g}{f W_0} W_0 - \gamma A_0^2 \text{√}$$

$$T_R \frac{f}{f T} \Delta\theta = -2\Phi_0 \frac{\Delta w}{W_0} + T_R S_\theta(T)$$

$$T_R \frac{f}{f T} \Delta p = -\frac{1}{\tau_p} \Delta p + T_R S_p(T) \qquad \frac{1}{\tau_p} = \frac{4}{3} \frac{g}{T_R \Omega_g^2 \tau^2}$$

$$T_R \frac{f}{f T} \Delta t = -2|D|\Delta p - \frac{g}{\Omega_g} \frac{\Delta w}{W_0} + T_R S_x(T)$$

$$S_j(T) = \langle \underline{f}_j | S(T,t) \rangle, \quad j = w, \theta, p, t$$

Noise Sources

$$S_w(T) = 2\frac{\Delta g(T)}{T_R}W_0 + S_{w,g}(T)$$

$$S_\theta(T) = \omega_0 \;\; \frac{\Delta L(T)}{L} + \frac{\Delta n(T)}{n} \;\; + S_{\theta,g}(T)$$

$$S_p(T) = S_{p,g}(T)$$

$$S_t(T) = \frac{\Delta g(T)}{\Omega_g T_R} - \;\; \frac{\Delta L(T)}{L} + \frac{\Delta n(T)}{n} \;\; + S_{t,g}(T)$$

$$S_{j,g}(T) = \left\langle \underline{f}_j \middle| S_g(T,t) \right\rangle, \quad j = w, \theta, p, t$$

Amplitude- and Frequency Fluctuations

Amplitude- and frequency fluctuations are damped and remain bounded

Correlation Spectra | **Variances**

$$\left\langle \left| \Delta\tilde{\boldsymbol{w}}(\Omega) \right|^2 \right\rangle = \frac{\left\langle \left| \boldsymbol{S}_{\boldsymbol{w}}(\Omega) \right|^2 \right\rangle}{\Omega^2 + \tau_{\boldsymbol{w}}^{-2}} \qquad \left\langle \left| \Delta\boldsymbol{w} \right|^2 \right\rangle = \frac{\tau_{\boldsymbol{w}}}{2} \left| \hat{\boldsymbol{S}}_{\boldsymbol{w}}(0) \right|^2$$

$$\left\langle \left| \Delta\tilde{\boldsymbol{p}}(\Omega) \right|^2 \right\rangle = \frac{\left\langle \left| \hat{\boldsymbol{S}}_{\boldsymbol{p},\boldsymbol{g}}(\Omega) \right|^2 \right\rangle}{\Omega^2 + \tau_{\boldsymbol{p}}^{-2}} \qquad \left\langle \left| \Delta\boldsymbol{p} \right|^2 \right\rangle = \frac{\tau_{\boldsymbol{p}}}{2} \left\langle \left| \hat{\boldsymbol{S}}_{\boldsymbol{p},\boldsymbol{g}}(0) \right|^2 \right\rangle$$

Phase- and Timing Fluctuations

Phase- and timing fluctuations are unbounded diffusion processes

$$\left\langle \left| \Delta\tilde{\theta}(\Omega) \right|^2 \right\rangle = \left(\frac{2\Phi_0}{T_R} \right)^2 \frac{\left\langle \left| \hat{S}_w(\Omega) \right|^2 \right\rangle}{\Omega^2 \left(\Omega^2 + \tau_w^{-2} \right)} + \frac{\left\langle \left| \hat{S}_\theta(\Omega) \right|^2 \right\rangle}{\Omega^2}$$

$$\left\langle \left| \Delta\tilde{t}(\Omega) \right|^2 \right\rangle = \underbrace{\left(\frac{2D}{T_R} \right)^2 \frac{\left\langle \left| \hat{S}_p(\Omega) \right|^2 \right\rangle}{\Omega^2 \left(\Omega^2 + \tau_p^{-2} \right)}} + \frac{\left\langle \left| \hat{S}_t(\Omega) \right|^2 \right\rangle}{\Omega^2}$$

Gordon-Haus Jitter

Timing Fluctuations

$$
\begin{aligned}
\left\langle \left| \Delta t(T+T_0) - \Delta t(T_0) \right|^2 \right\rangle = {} & \frac{2\pi^2 g\Theta}{3} \frac{\tau^2}{(W_0/h\nu)} \frac{T}{T_R} + \\
& + \frac{8\pi^2 \Phi_0^2 \Theta}{(W_0/h\nu)} (\Omega_g \tau)^2 \frac{\tau_p^2}{T_R^2} \left[\frac{T}{\tau_p} - \left(1 - \exp\left(-\frac{T}{\tau_p} \right) \right) \right] +
\end{aligned}
$$

Quantum Noise

$$
\begin{aligned}
& + \frac{\langle \Delta L^2 \rangle}{L^2} \tau_L^2 \left[\frac{T}{\tau_L} - \left(1 - \exp\left(-\frac{T}{\tau_L} \right) \right) \right] + \\
& + \frac{\langle \Delta n^2 \rangle}{n^2} \tau_n^2 \left[\frac{T}{\tau_n} - \left(1 - \exp\left(-\frac{T}{\tau_n} \right) \right) \right] + \\
& + \frac{\langle \Delta g^2 \rangle}{g^2} \frac{g^2}{\Omega_g^2} \frac{\tau_g^2}{T_R^2} \left[\frac{T}{\tau_g} - \left(1 - \exp\left(-\frac{T}{\tau_g} \right) \right) \right]
\end{aligned}
$$

Classical Noise

Long-Term Timing Fluctuations T >> τ_p, τ_L, τ_n, τ_g

$$\left\langle \left| \Delta t(T+T_0) - \Delta t(T_0) \right|^2 \right\rangle = \frac{2\pi^2 g\Theta}{3} \frac{\tau^2}{(W_0/h\nu)} +$$

$$+ \frac{8\pi^2 \Phi_0^2 \Theta}{(W_0/h\nu)} (\Omega_g \tau)^2 \frac{\tau_p T}{T_R^2} +$$

$$+ \frac{\langle \Delta L^2 \rangle}{L^2} \tau_L T +$$

$$+ \frac{\langle \Delta n^2 \rangle}{n^2} \tau_n T +$$

$$+ \frac{\langle \Delta g^2 \rangle}{g^2} \frac{g^2}{\Omega_g^2} \frac{\tau_g T}{T_R^2}$$

? Quantum Noise ?

? Classical Noise ?

Semicondutor versus Solid-State Lasers

	$W_0/h\nu$	g	Ω_g THz	τ fs	$\Omega_g\tau$	τ_p/T_R	$\Delta n/n$	τ_n ns	$\Delta g/g$	τ_g ns	Θ	$\sqrt{\langle \lvert\Delta t\rvert^2 \rangle}$
Semicon-ductor Laser	10^7	0.2	40	300	10	375	10^{-3}	1	10^{-3}	1	10	450 fs
Solid-State Laser	10^{10}	0.01	200	10	1	75	0	0	10^{-3}	1000	2	1 fs

Other parameters are: $T=T_R=100$ ps, $\Phi_o=1$

Dominant sources for timing jitter:

Semiconductor -Laser: Gordon-Haus-Jitter + Index-Fluctuations

Solid-State Laser: Gain-Fluctuations

Conclusions

- Classical and quantum noise in modes of the EM-Field
- Spontaneous emission noise of amplifiers
- Dynamics of modelocked lasers (solitonlike pulses)
- Amplitude-, phase-, frequency- and timing-fluctuations
- Solid-State Lasers: no index fluctuations; possibly small Gordon-Haus Jitter; very short pulses; superior timing jitter in comparison to semiconductor lasers

References:

H. A. Haus and A. Mecozzi: „Noise of modelocked lasers," IEEE JQE-**29**, 983 (1993).

J. P. Gordon and H. A. Haus: „Random walk of coherently amplified solitons in optical fiber transmission," Opt. Lett. **11**, 665 (1986).

H. A. Haus, M. Margalit, and C. X. Yu: „Quantum noise of a mode-locked laser," JOSA B**17**, 1240 (2000).

D. E. Spence, et. al.: „Nearly quantum-limited timing jitter in a self-mode-locked Ti:sapphire laser," Opt. Lett. 19, 481 (1994).