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Statistical Reliability Measures for GPS

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Reliability Theory

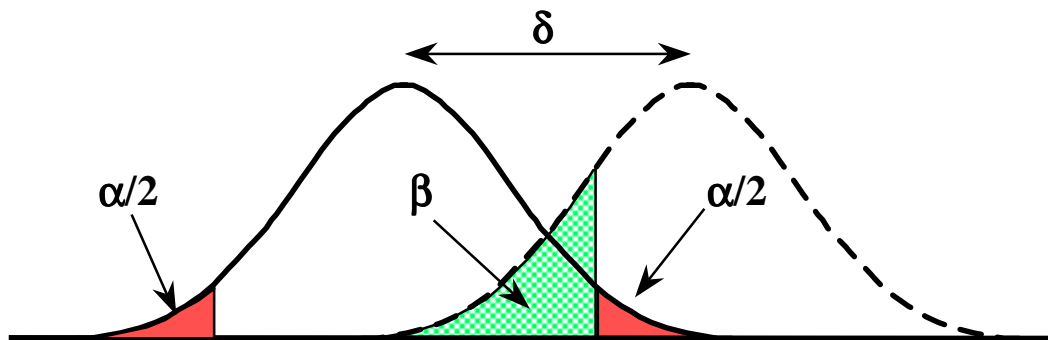
Overview

- *Reliability refers to the controllability of observations, that is, the ability to detect blunders and to estimate the effects that undetected blunders may have on a solution.*
- **Redundancy Number of an Observation**
 - Measure of Absorption of a Blunder
- **Internal Reliability**
 - Capability of a system to detect a blunder
- **External Reliability**
 - Impact of an undetected blunder on the parameters

Reliability Theory

Type I and II Errors

- Type I Error (α)
 - Probability of Rejecting a good observation
- Type II Error (β)
 - Probability of Accepting a blunder



α	β	δ
5.0%	20%	2.80
2.5%	20%	3.10
5.0%	10%	3.24
2.5%	10%	3.52
0.1%	20%	4.12
0.1%	10%	4.57

Reliability Theory

Redundancy Number

- Residuals: $\hat{r} = -C_{\hat{r}} C_1^{-1} w = -Rw$
- Trace of $(C_{\hat{r}} C_1^{-1}) = n - u$ redundancy
- $v_i = \frac{\sigma_{\hat{r}_i}^2}{\sigma_{l_i}^2}$ is observation i 's redundancy number
- Covariance Matrix: $C_{\hat{r}} = C_1 - AC_{\hat{x}}A^T$
- Thus from Covariance Matrix $0 \leq v_i \leq 1$
- It is desirable that v_i be close to 1 for all observables

Reliability Theory

External Reliability

- Calculate the Marginally Detectable Blunder (MDB) for each observation:

$$|\nabla_i| = \frac{\delta \sigma_{l_i}}{\sqrt{V_i}}$$

- Calculate the impact of each MDB on the parameters.
 - Assume only 1 Blunder Occurs at any time
 - Calculate the effect that each MDB could have on the parameters:

$$\Delta X = -\left(A^T * C_1^{-1} * A\right)^{-1} * A^T * C_1^{-1} * \nabla$$

- For each blunder determine the Horizontal Error:

$$\text{Horizontal Error} = \sqrt{\Delta\phi^2 + \Delta\lambda^2}$$

- The MDB that produces the Maximum Horizontal Position Error (HPE) represents the External Reliability.

Reliability Theory

Least Squares Blunder Detection

- Residual Testing:

$$\hat{r}_i^* = \left| \frac{\hat{r}_i}{\sqrt{C_{rii}}} \right| < n_{1-\frac{\alpha}{2}}$$

- If σ_0 unknown, the student distribution must be used.
- If a Blunder is Detected, Perform Sub-Set testing:
 - Reject one (1) observation at a time and test the resulting subset of residuals.
 - If only one (1) sub-set passes, the blunder has been isolated
 - Otherwise the blunder has been detected, but cannot be isolated.

Reliability Theory

Least Squares Multiple Blunder Detection

- Assume that blunders are present on satellite's "i" and "j". The k^{th} satellite's normalized residual must be $< \delta$.

$$\frac{\hat{\mathbf{r}}_k}{\sqrt{\mathbf{C}_{\hat{\mathbf{r}}_k}}} = \frac{|\mathbf{R}_{ki} * \nabla_i + \mathbf{R}_{kj} * \nabla_j|}{\sqrt{\mathbf{C}_{\hat{\mathbf{r}}_k}}} \leq \delta$$

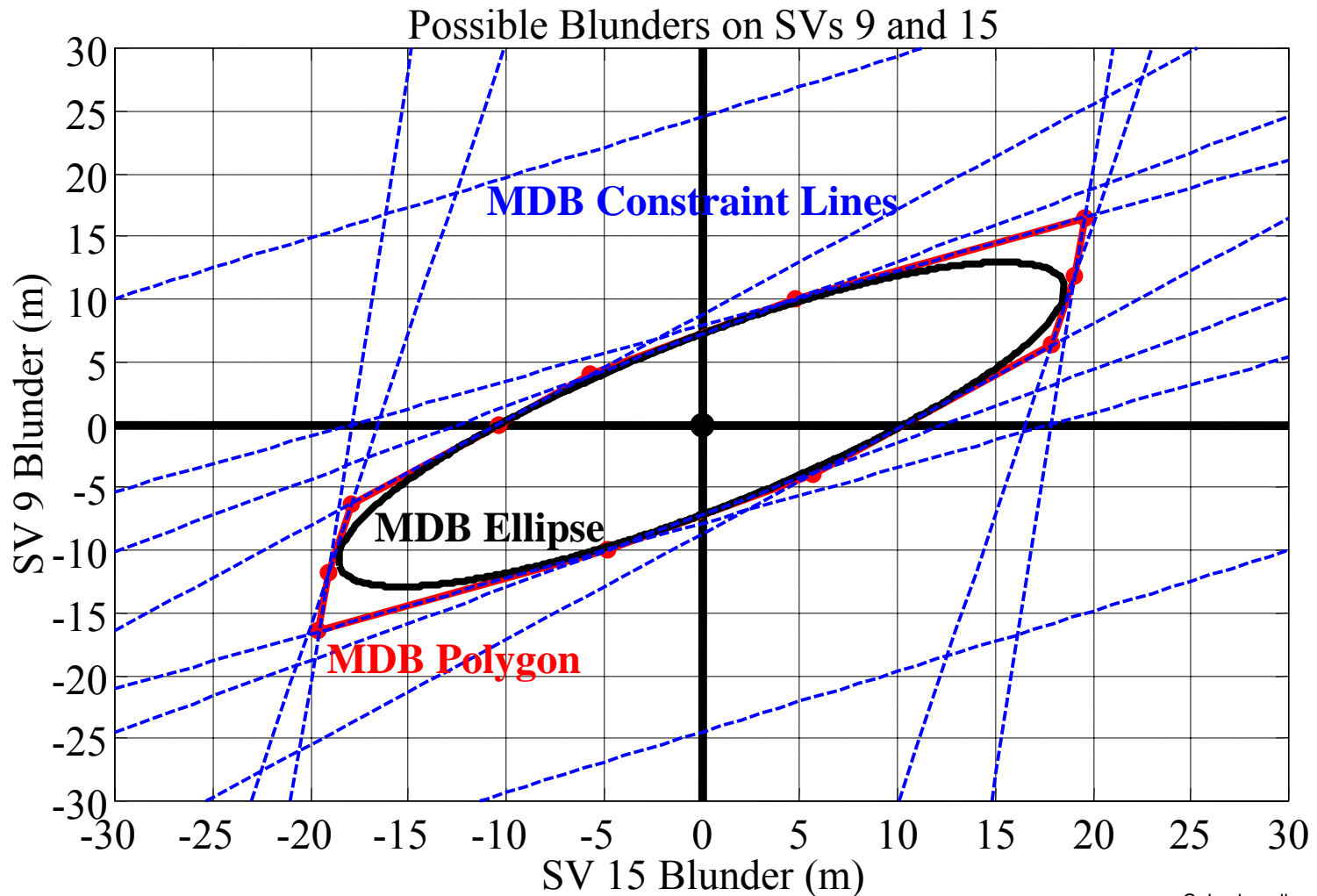
- With n observations there will be $2n$ of these constraints on the blunders, which define a MDB polygon in "i" and "j" blunder space. Substituting into the SSR results in the MDB ellipse.

$$\hat{\mathbf{r}}^T * \mathbf{C}_1^{-1} * \hat{\mathbf{r}} \leq \delta^2$$

$$\sum_{k=1}^{k=n} \left(\mathbf{R}_{ki}^2 * \nabla_i^2 + 2 * \mathbf{R}_{ki} * \mathbf{R}_{kj} * \nabla_i * \nabla_j + \mathbf{R}_{kj}^2 * \nabla_j^2 \right) * \mathbf{C}_{1_{kk}}^{-1} \leq \delta^2$$

Reliability Theory

Least Squares Multiple Blunder Detection Example



Kalman Filtering

Introduction

- Use all previous data to detect failures in the current epoch.

- Model:
$$\mathbf{x}_k = \Phi_k * \mathbf{x}_{k-1} + \mathbf{w}_k, \mathbf{w}_k \sim n(0, \mathbf{Q}_k)$$

$$\mathbf{z}_k = \mathbf{H}_k * \mathbf{x}_k + \mathbf{e}_k, \mathbf{e}_k \sim n(0, \mathbf{R}_k)$$

- Based on our vehicle, we select a Dynamics Model and use it to estimate our parameters.
 - Constant Velocity (P and V)
 - Constant Acceleration (P, V, and A)
 - Time Correlation (ie Gauss Markov) (P, V, and A)

Kalman Filtering

Propagation and Updating

- Use the Dynamics Model to propagate the parameters to the next epoch.

$$\hat{\mathbf{x}}_k^- = \Phi_k * \hat{\mathbf{x}}_{k-1}^+ \quad \mathbf{P}_k^- = \Phi_k * \mathbf{P}_{k-1}^+ * \Phi_k^T + \mathbf{Q}_d$$

- Update our Parameters, using the current measurements and the propagated parameters.

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (z_k - \mathbf{H}_k * \hat{\mathbf{x}}_k^-) \quad \mathbf{P}_k^+ = \mathbf{P}_k^- - \mathbf{K}_k * \mathbf{H}_k * \mathbf{P}_k^-$$

- Use the innovation sequence to detect blunders, similar to Least Squares.

Kalman Filtering

Innovation Sequence Testing

- Test the Normalized Sum Square of the Innovations

$$\text{Test} = \mathbf{i}^T * \mathbf{C}_i^{-1} * \mathbf{i} \sim \chi^2(m, 0), \text{ where } \mathbf{i} = \mathbf{z}_k - \mathbf{H}_k * \hat{\mathbf{x}}_k^-$$

- Assume only one Blunder Occurs at any time and calculate the MDB.

$$\lambda_o = \nabla_k^2 * (\mathbf{C}_i^{-1})_{kk} \Rightarrow \nabla_k = \sqrt{\frac{\lambda_o}{(\mathbf{C}_i^{-1})_{kk}}}$$

- This gives the same MDB as Least Squares if we include a priori information on the Parameters.
- Calculate the impact of each MDB on the parameters and generate the HPE similar to Least Squares.

Kalman Filtering

Spectral Densities

- The filter is only as good as the model you give it.
(Garbage in Garbage Out)
- In the simulations to follow, the Dynamics Model and Spectral Densities were extrapolated from an actual Canadian Coast Guard Survey Launch.

First Order Gauss Markov Process

Direction	σ^2 (10^{-3})	Time Constant
North & East	300	10 s
Up	10	1 s

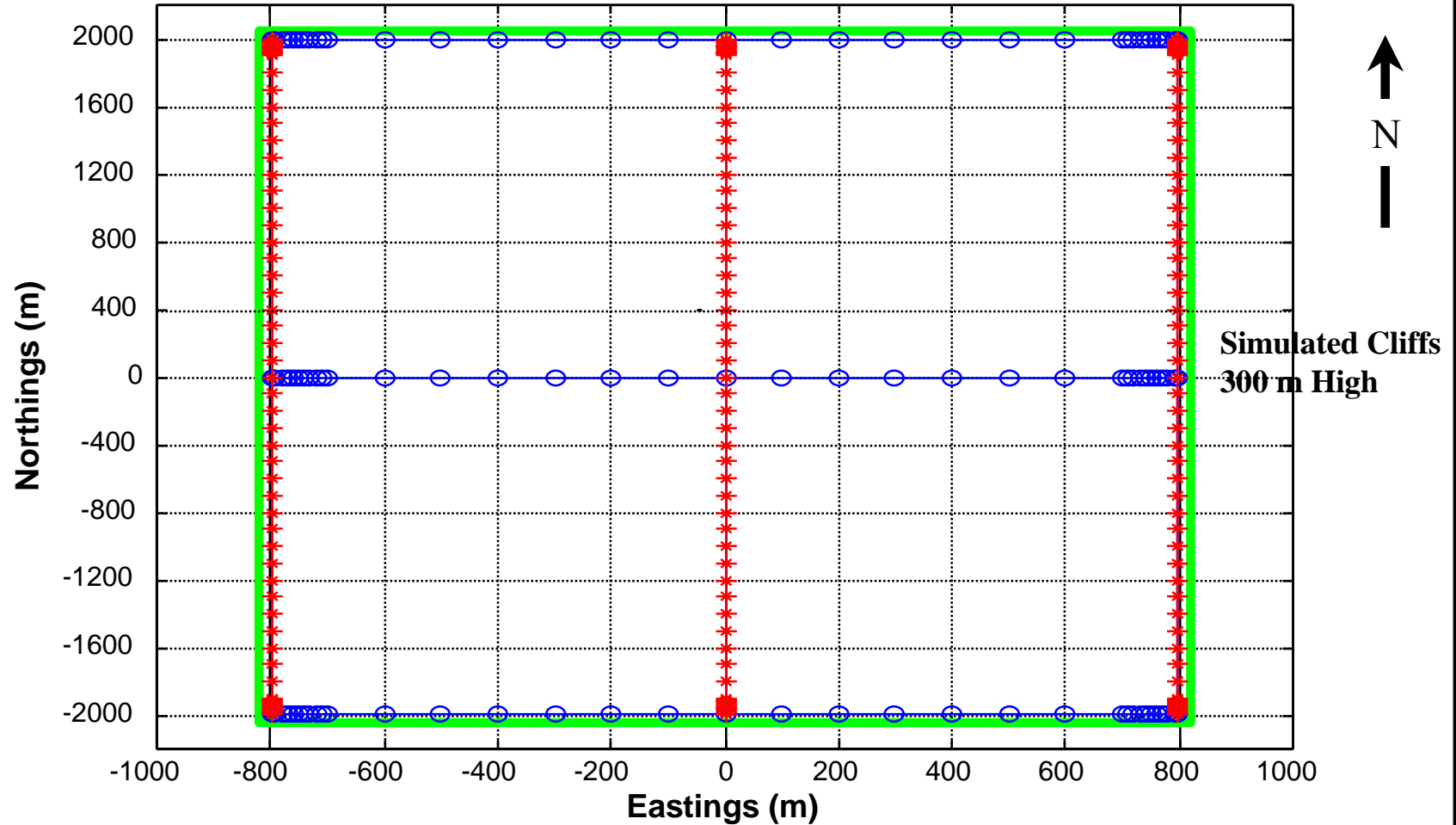
Simulation Description

Test Parameters

- Kalman Filter:
 - Survey Launch
- Constellations:
 - DGPS (1 m²)
 - DGPS+DGEO (1 m²)
 - DGPS+DGLO (1 m²)
 - DGPS+DGEO+DGLO (1 m²)
- Constraints:
 - Height Constraint (4 m²)
 - Clock Constraint (1 m²)
- Reliability Parameters:
 - $\alpha = 0.1\%$, $\beta = 10\%$, $\delta = 4.57$
- Simulation Data:
 - Date: July 25, 1997
 - Time: 24 Hours
 - Location: IOS Victoria, BC
48° N 123° W
 - 25 GPS SV
 - 15 GLONASS SV
 - 6 Geostationary SV

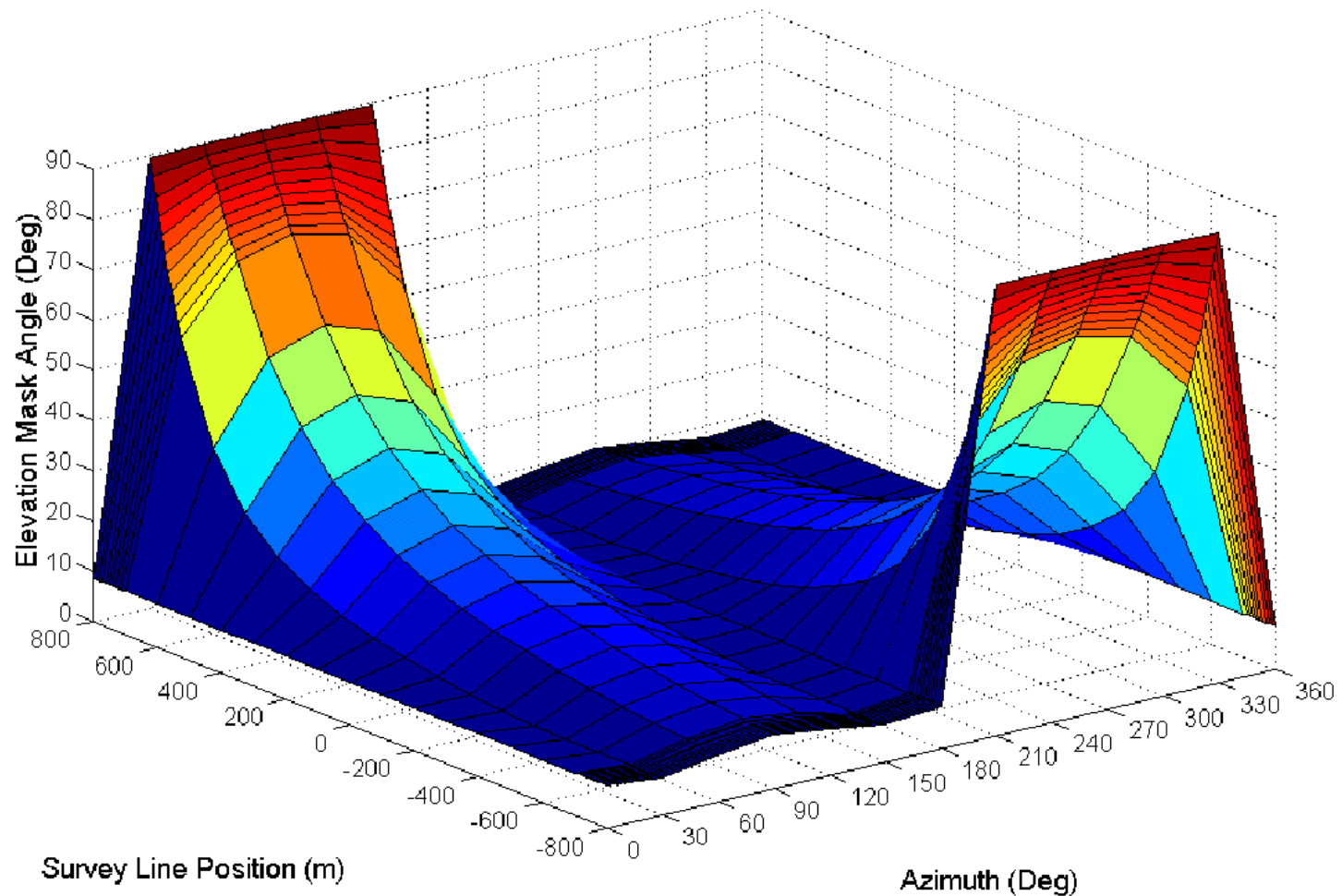
Simulation Description

Trajectory and Terrain



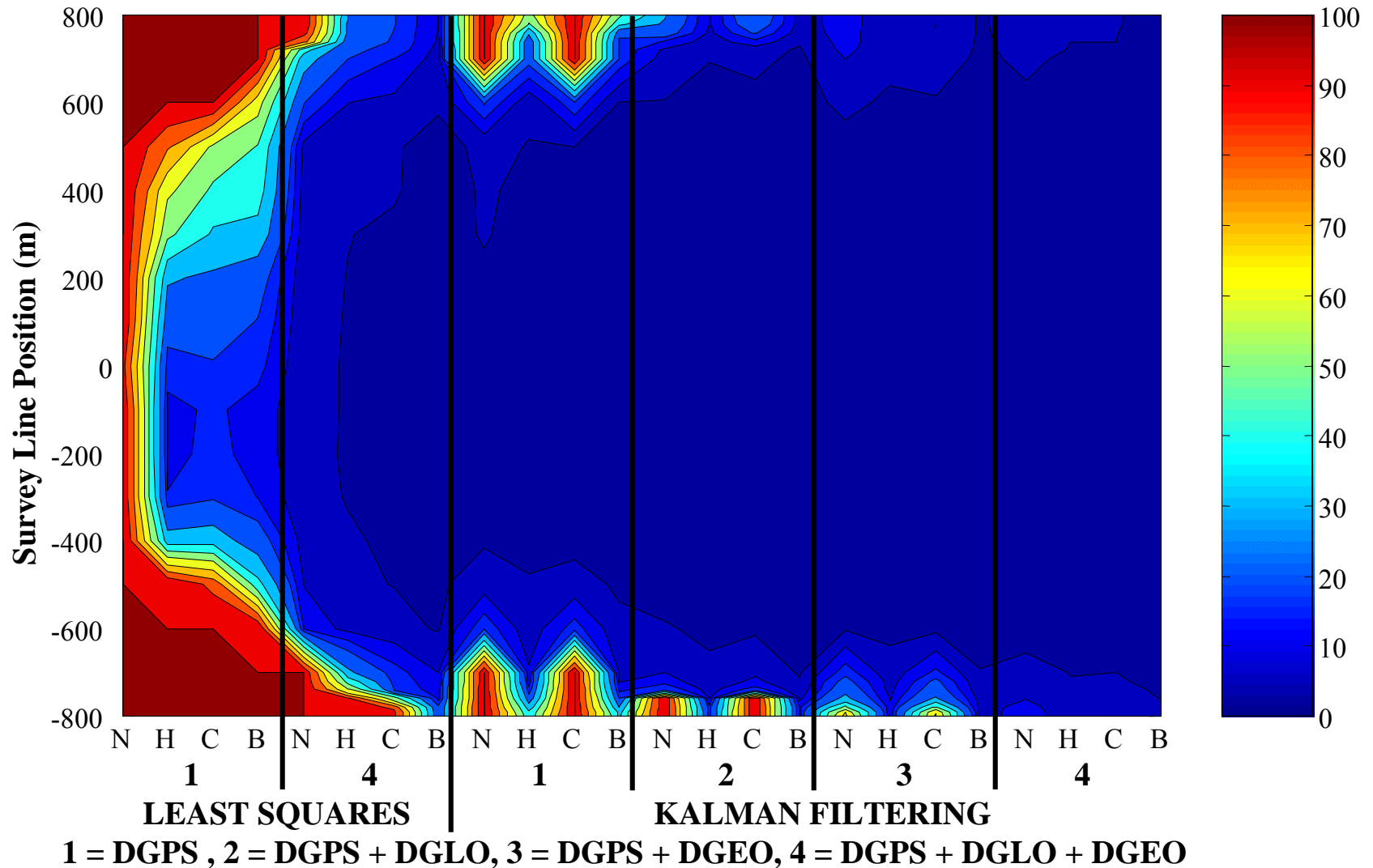
Simulation Description

East/West Line in the Middle - Mask Profile



Simulation Results (24 Hours)

East/West Line - HPE 95% - DGPS & KF



Double Blunder Simulations

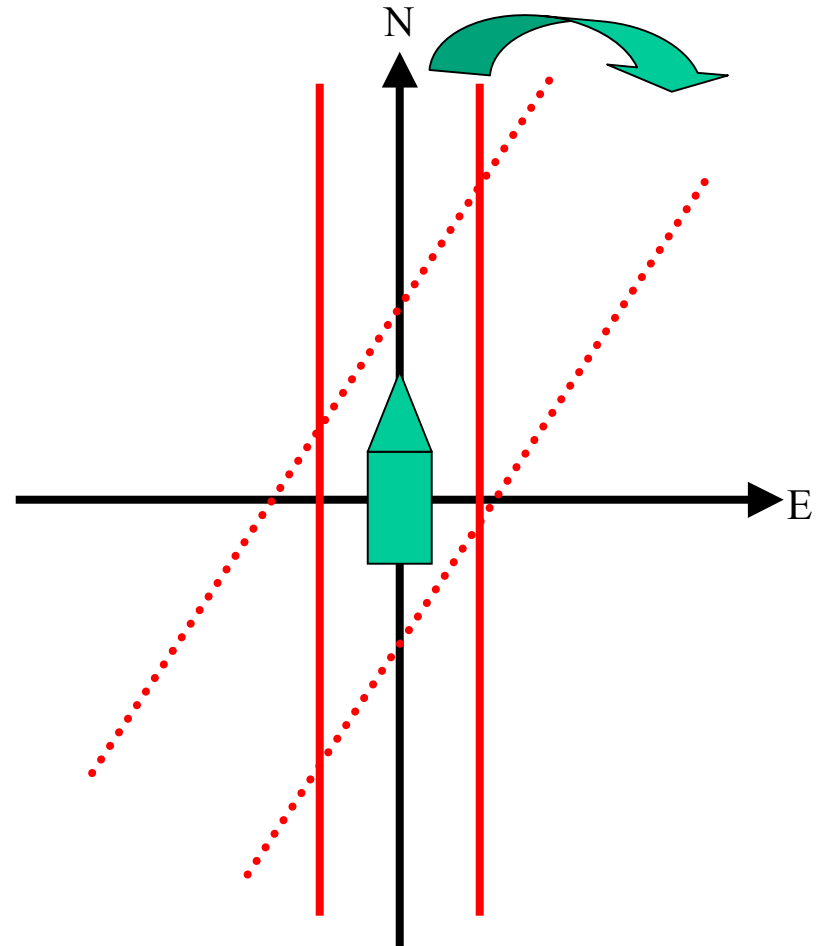
Test Parameters

- Constellations:
 - DGPS (Obs m^2)
 - DGPS+DGLO (Obs m^2)
 - DGPS+DGEO (Obs m^2)
 - DGPS+DGEO+DGLO (Obs m^2)
- Constraints:
 - None
 - Height Constraint (4 m^2)
 - Clock Constraint (1 m^2)
 - Height+Clock (4/1 m^2)
- Observation Variance
 - Narrow Correlator 1 m^2
 - Wide Correlator 9 m^2
- Mask Profile
 - Channel Rotated 180°
- Reliability Parameters:
 - $\alpha = 0.1\%$, $\beta = 10\%$, $\delta_o = 4.57$
 - Single and Double Blunders
- Simulation Data:
 - Date: March 22, 2000
 - Time: 24 Hours
 - Location: 50° N 114° W
 - 27 GPS SVs
 - 8 GLONASS SVs
 - 6 GEO SVs

Simulation Description

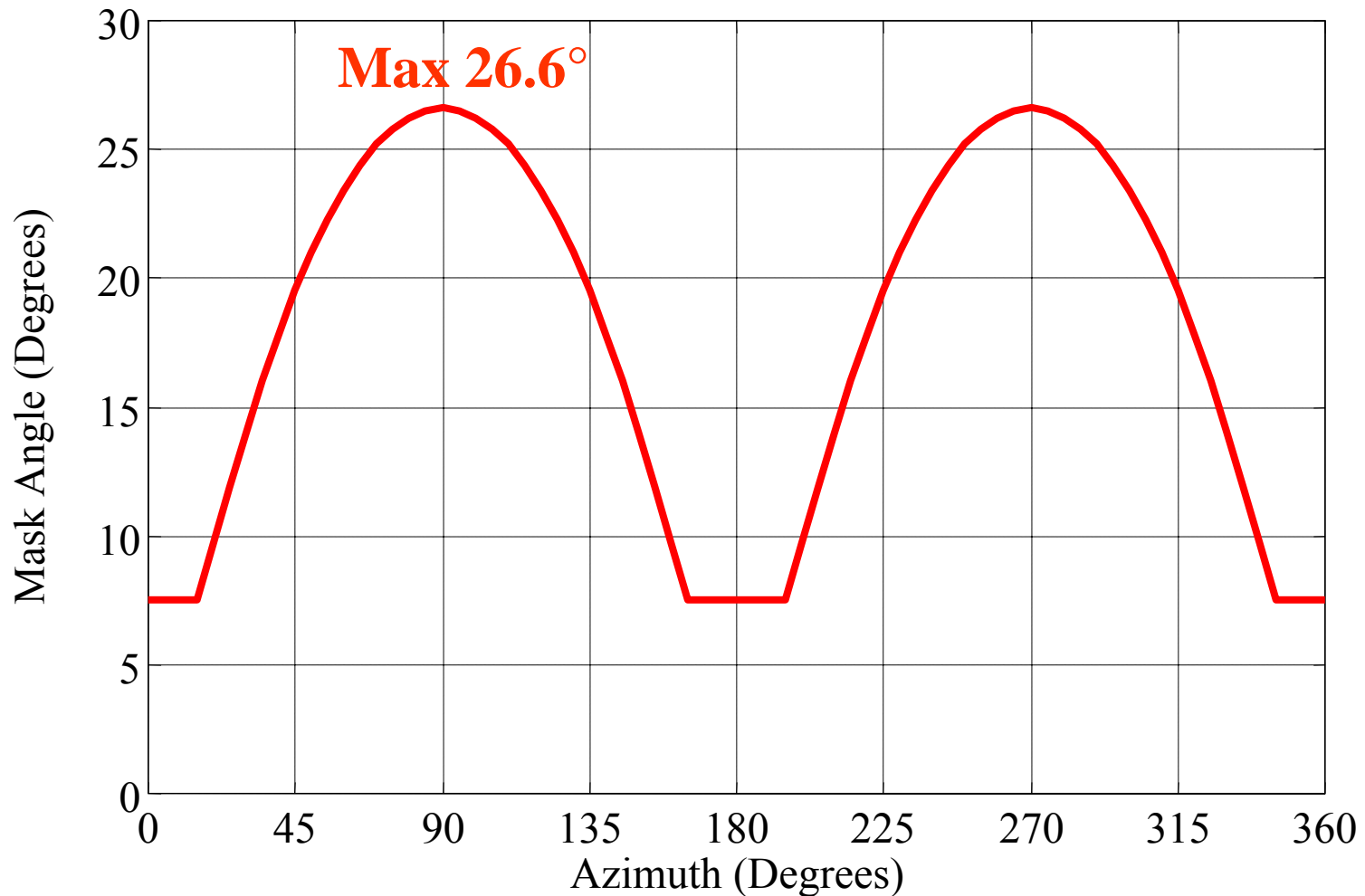
Constricted Channels

- Channel rotated 180° in 30° increments
- Reliability Analysis
 - Single Blunder
 - Double Blunder



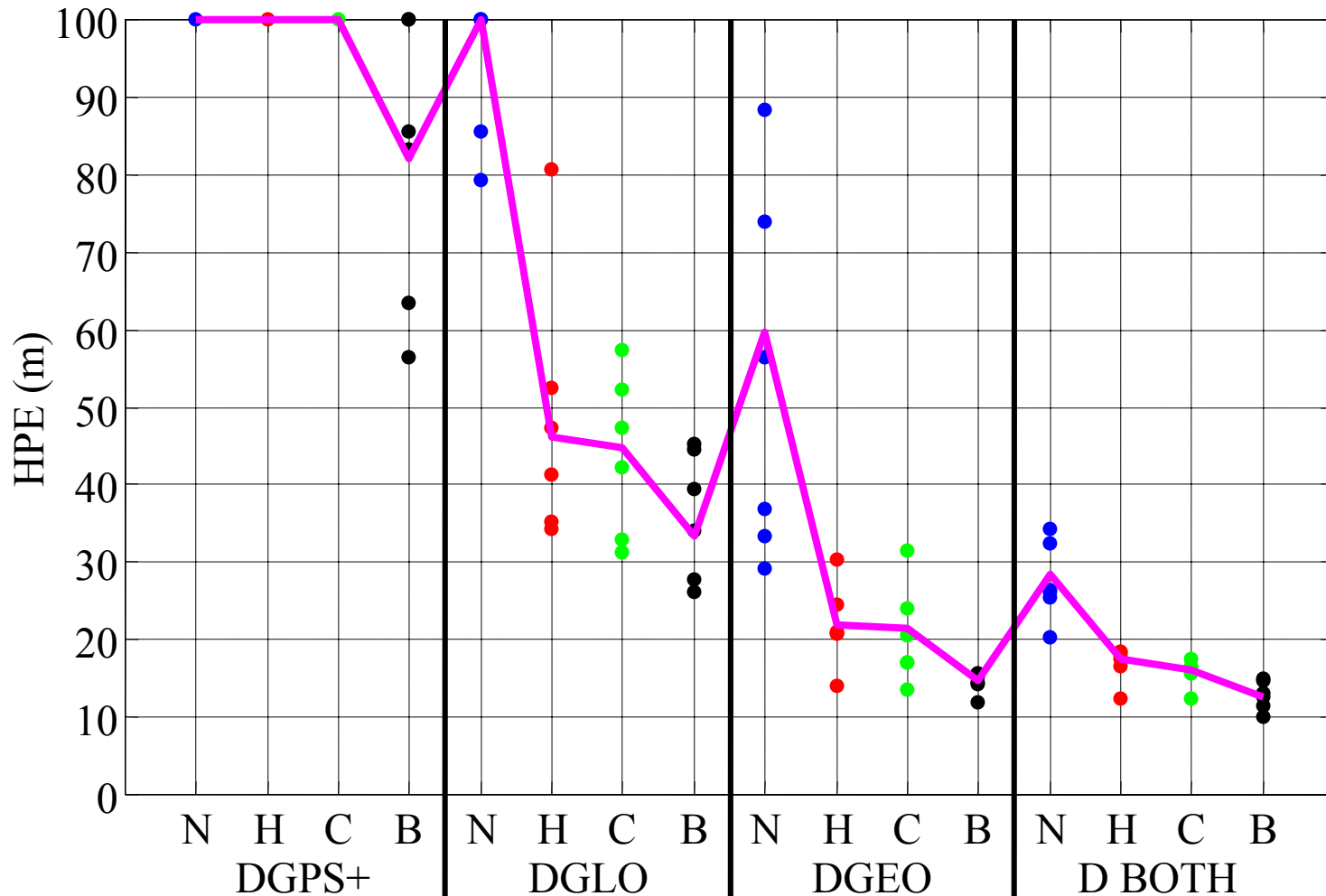
Simulation Description

Masking Profiles for the Constricted Channel



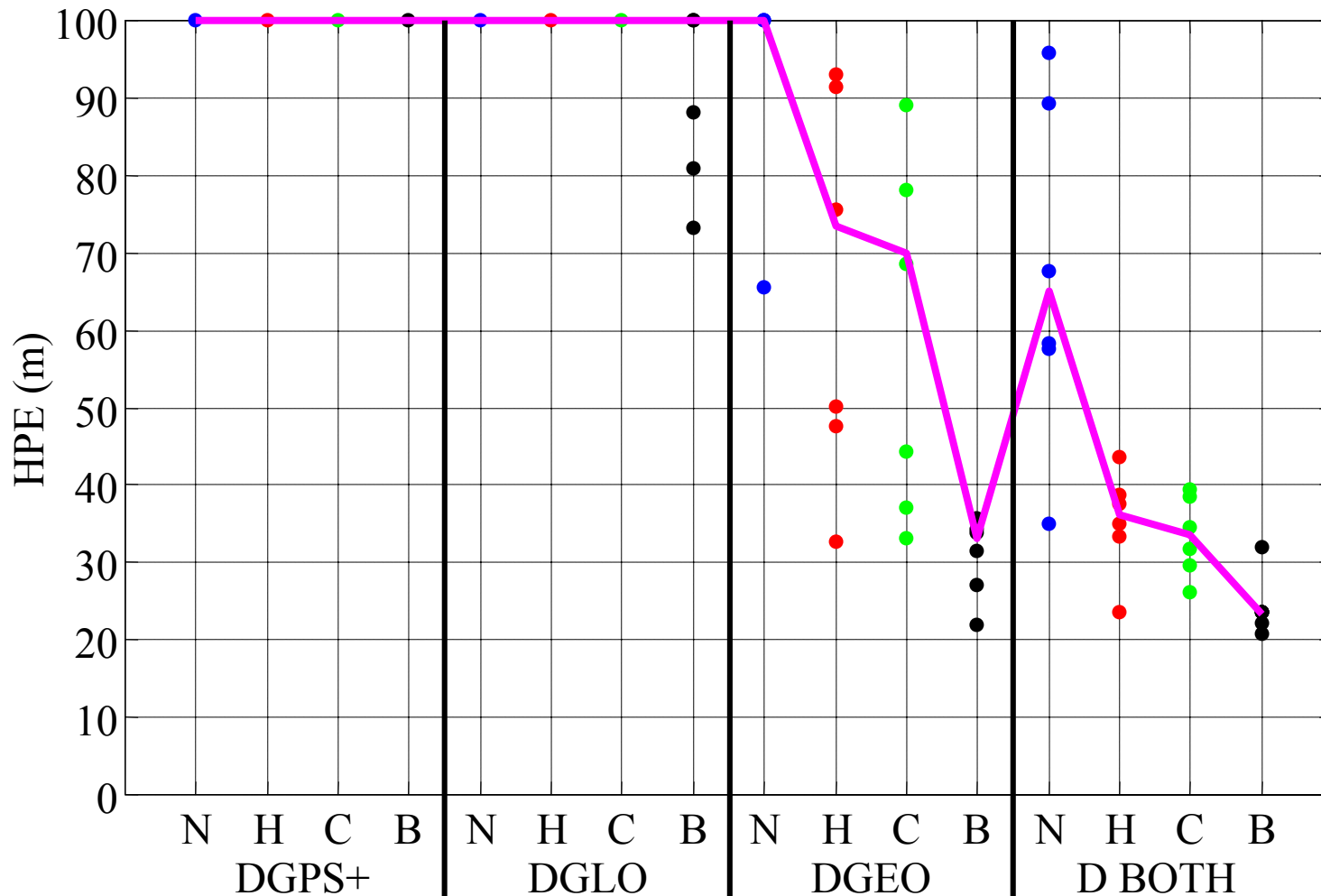
Simulation Results

95% HPE - Single Blunder - Wide Correlator



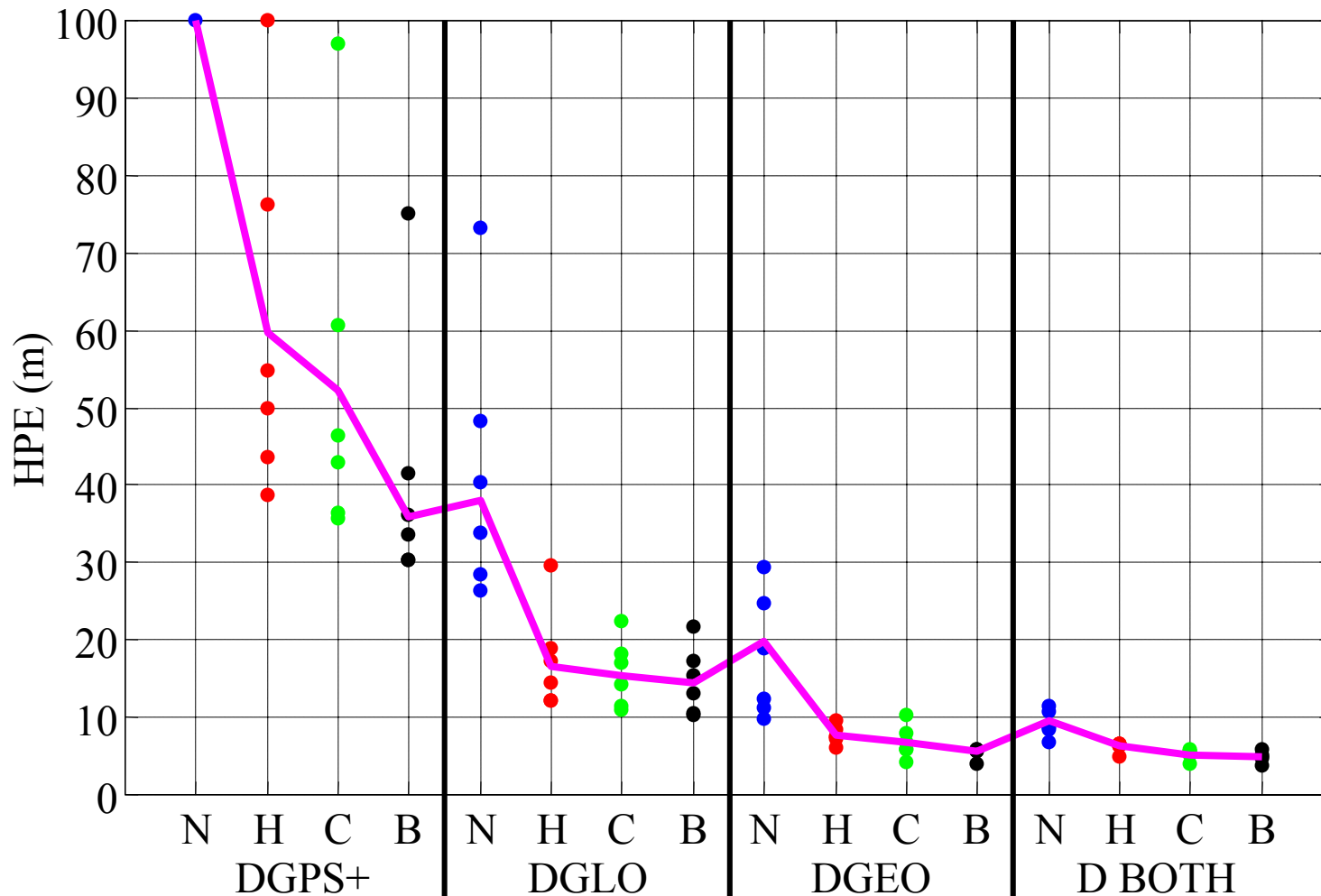
Simulation Results

95% HPE - Double Blunder - Wide Correlator



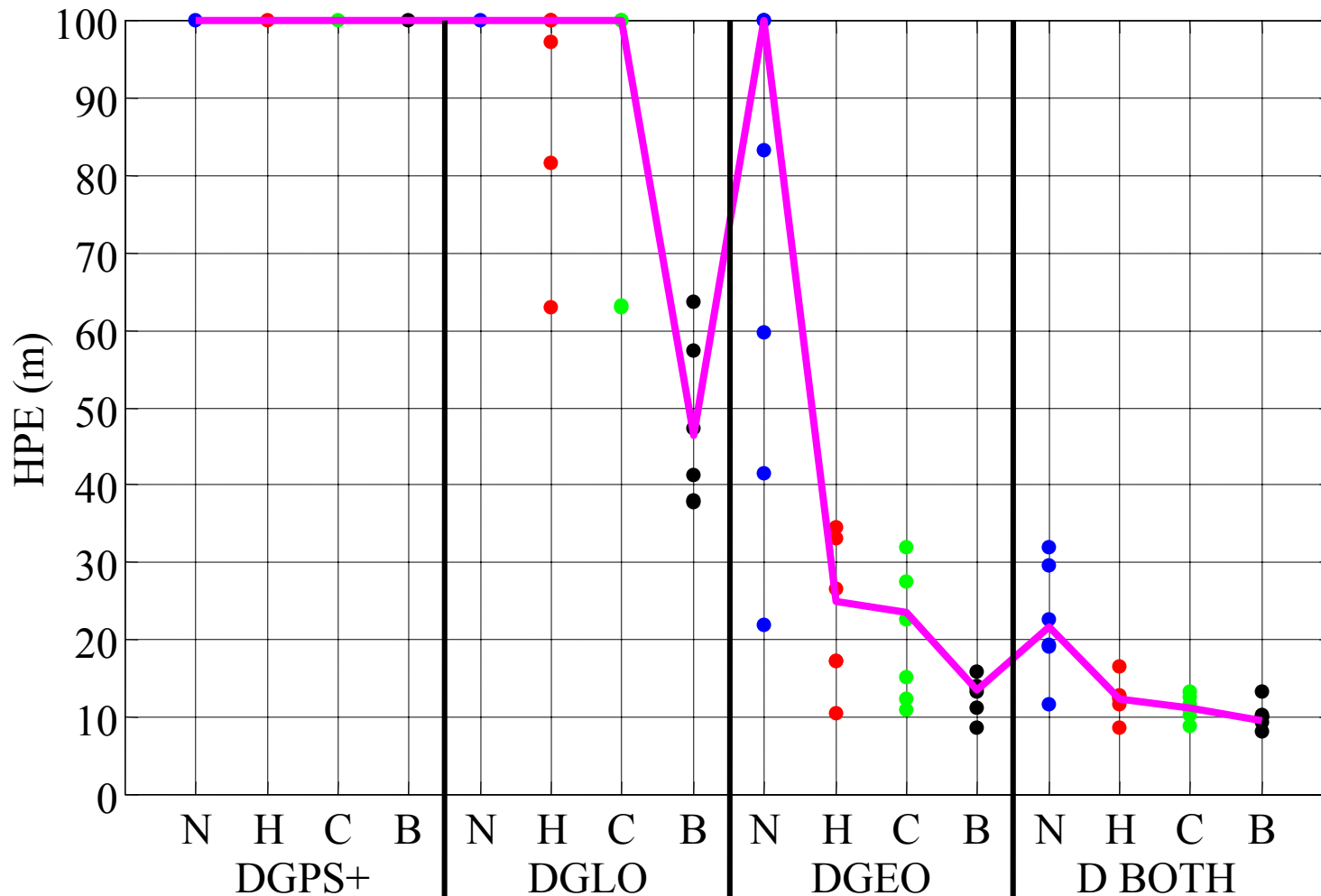
Simulation Results

95% HPE - Single Blunder - Narrow Correlator



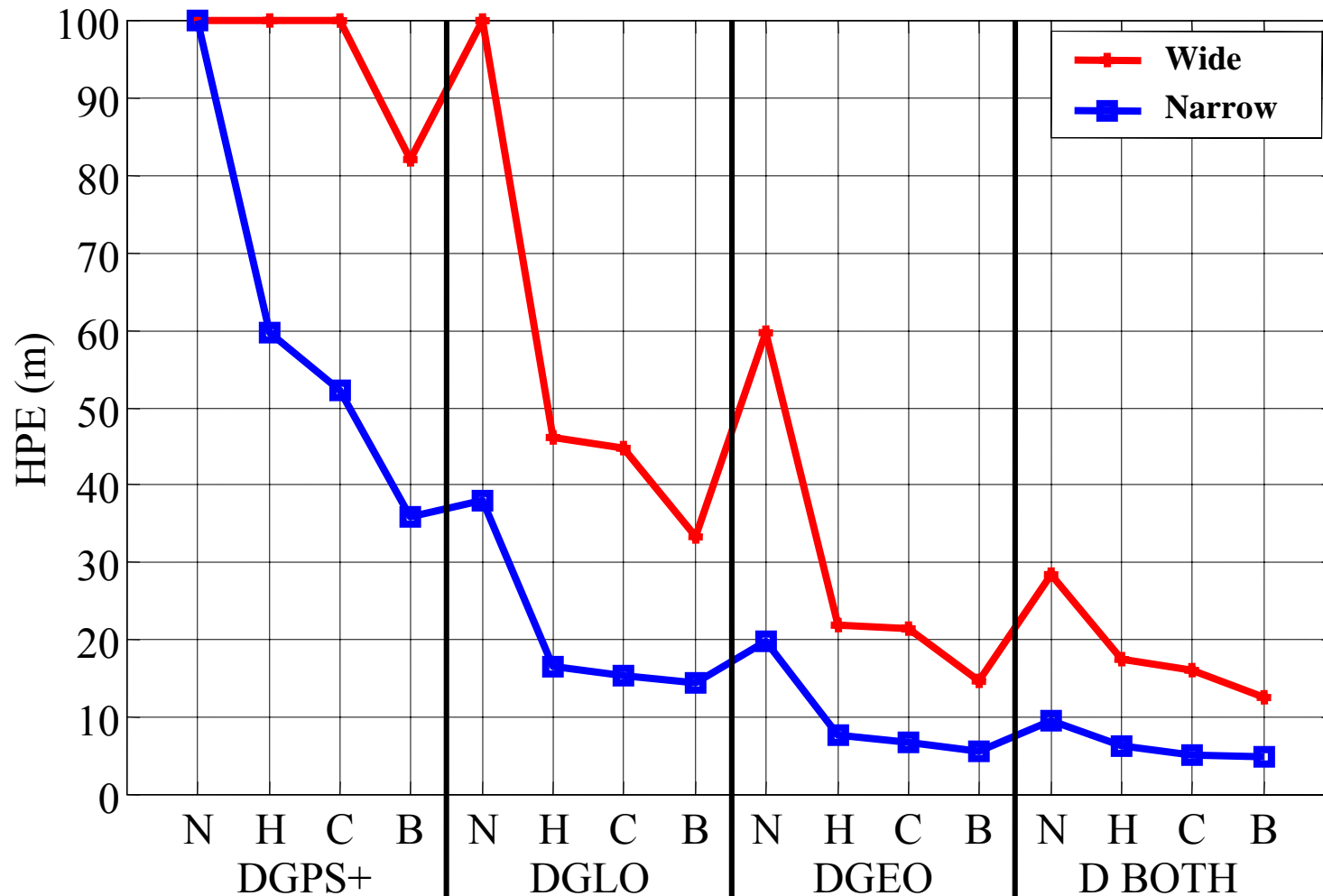
Simulation Results

95% HPE - Double Blunder - Narrow Correlator



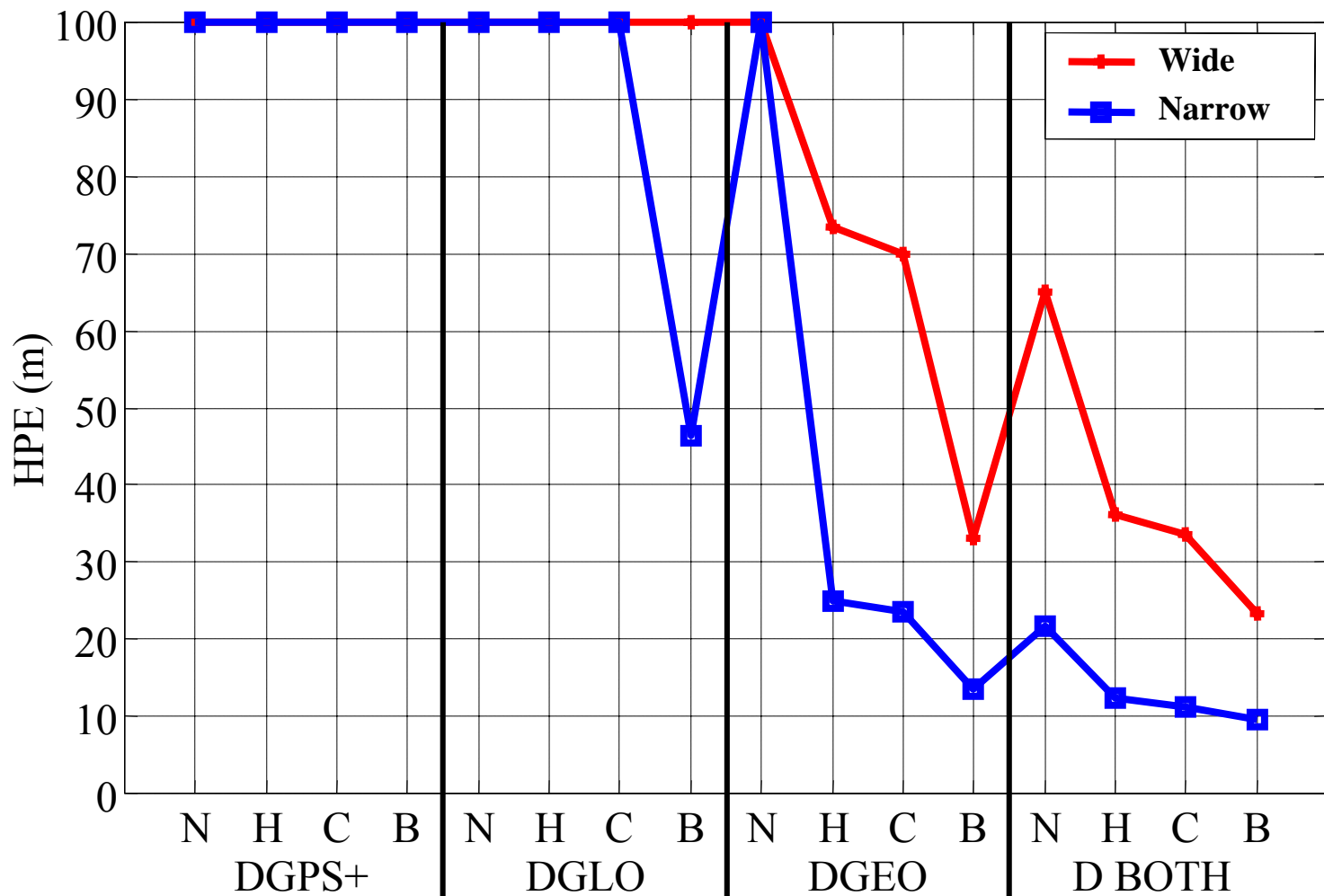
Simulation Results

95% HPE - Single Blunder



Simulation Results

95% HPE - Double Blunder



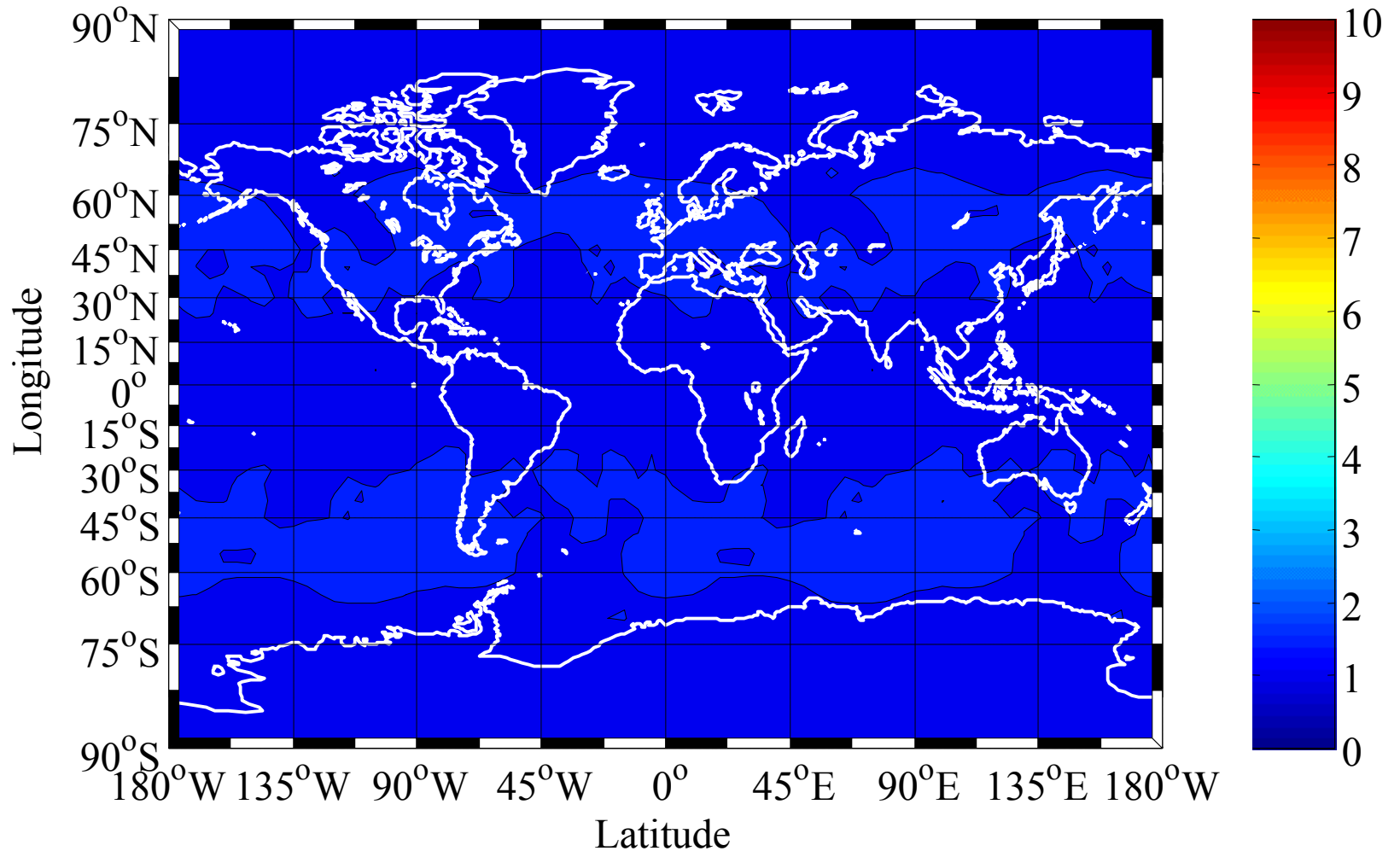
GPS + Galileo Simulations

Test Parameters

- Constellations:
 - DGPS
 - DGPS + Galileo1 (MEO+GEO)
 - DGPS + Galileo2 (MEO)
- Constraints:
 - None
 - Height (4 m²)
 - Clock (1 m²)
 - Height + Clock (4 / 1 m²)
- Masking Environment
 - Isotropic Mask Angles 0° - 40°
- Observation Variance:
 - 1 m²
- Reliability Parameters:
 - $\alpha = 0.1\%$, $\beta = 10\%$, $\delta = 4.57$
 - Least Squares epoch by epoch reliability checking
- Simulation Data:
 - Date: June 6, 2000 24 Hours
 - 28 GPS SV
 - 2 Galileo Configurations
- Locations:
 - World (1780 Points)

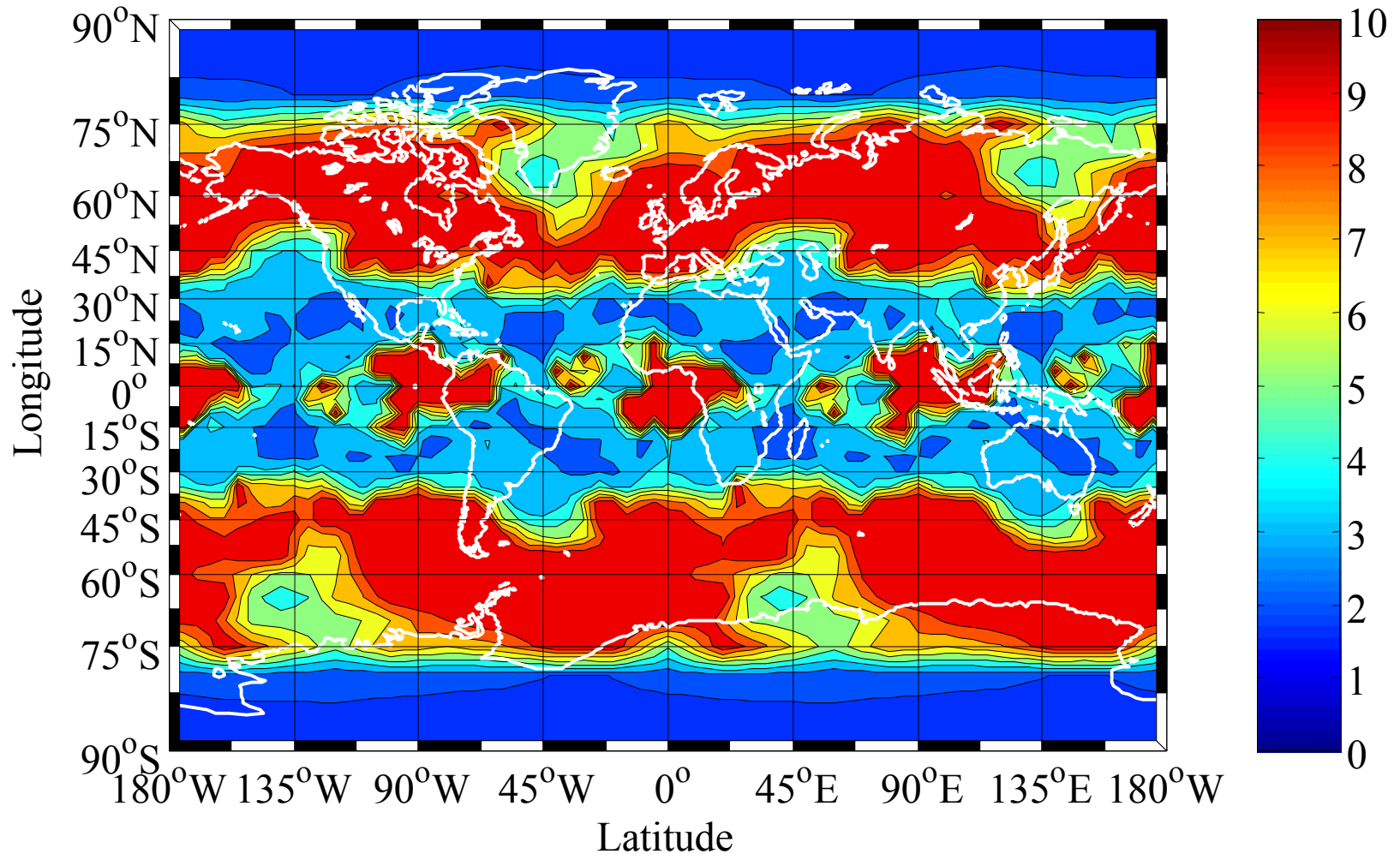
Simulation Results (24 Hours)

HDOP 95%, DGPS, 10° Mask Angle



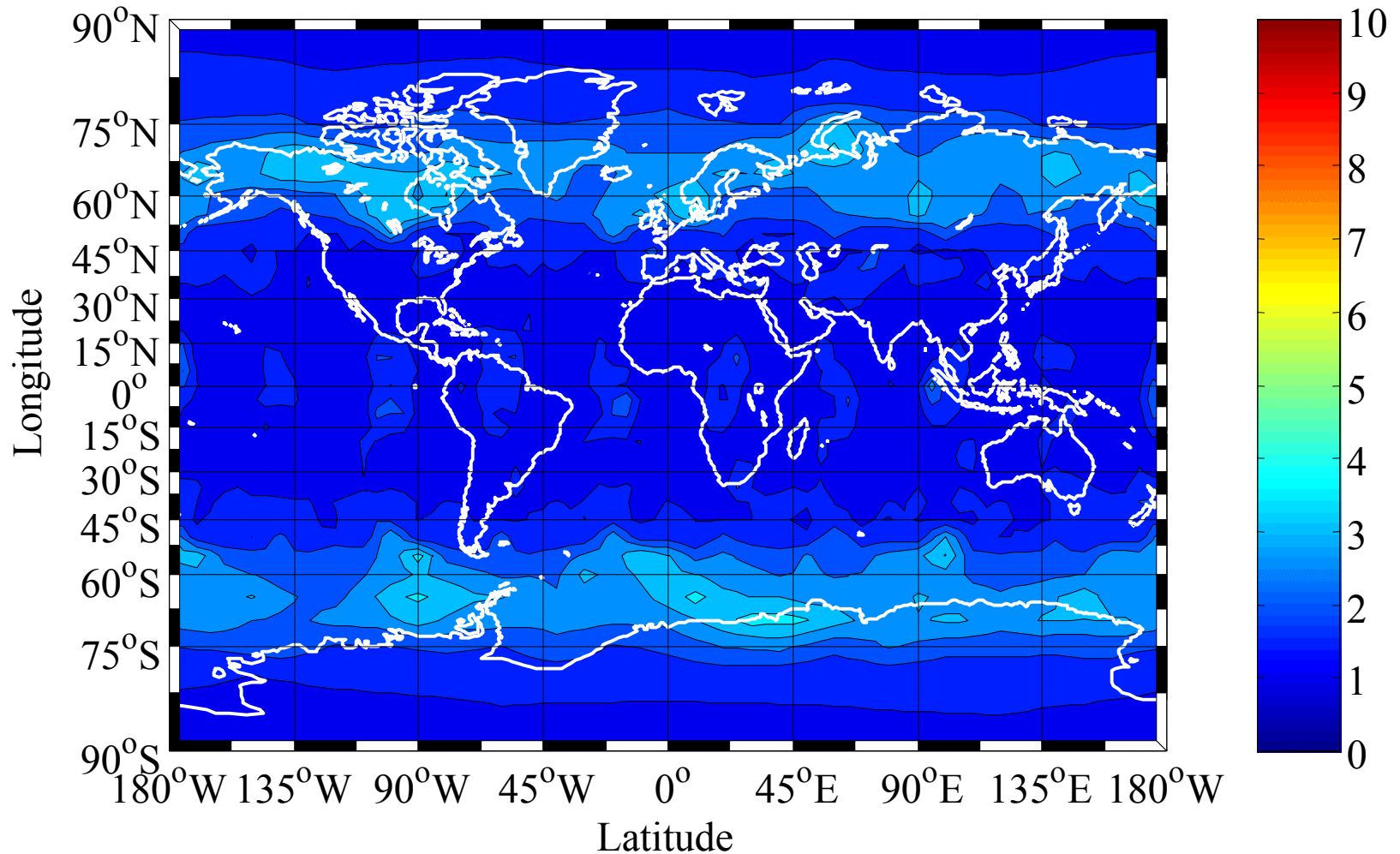
Simulation Results (24 Hours)

HDOP 95%, DGPS + H, 30° Mask Angle



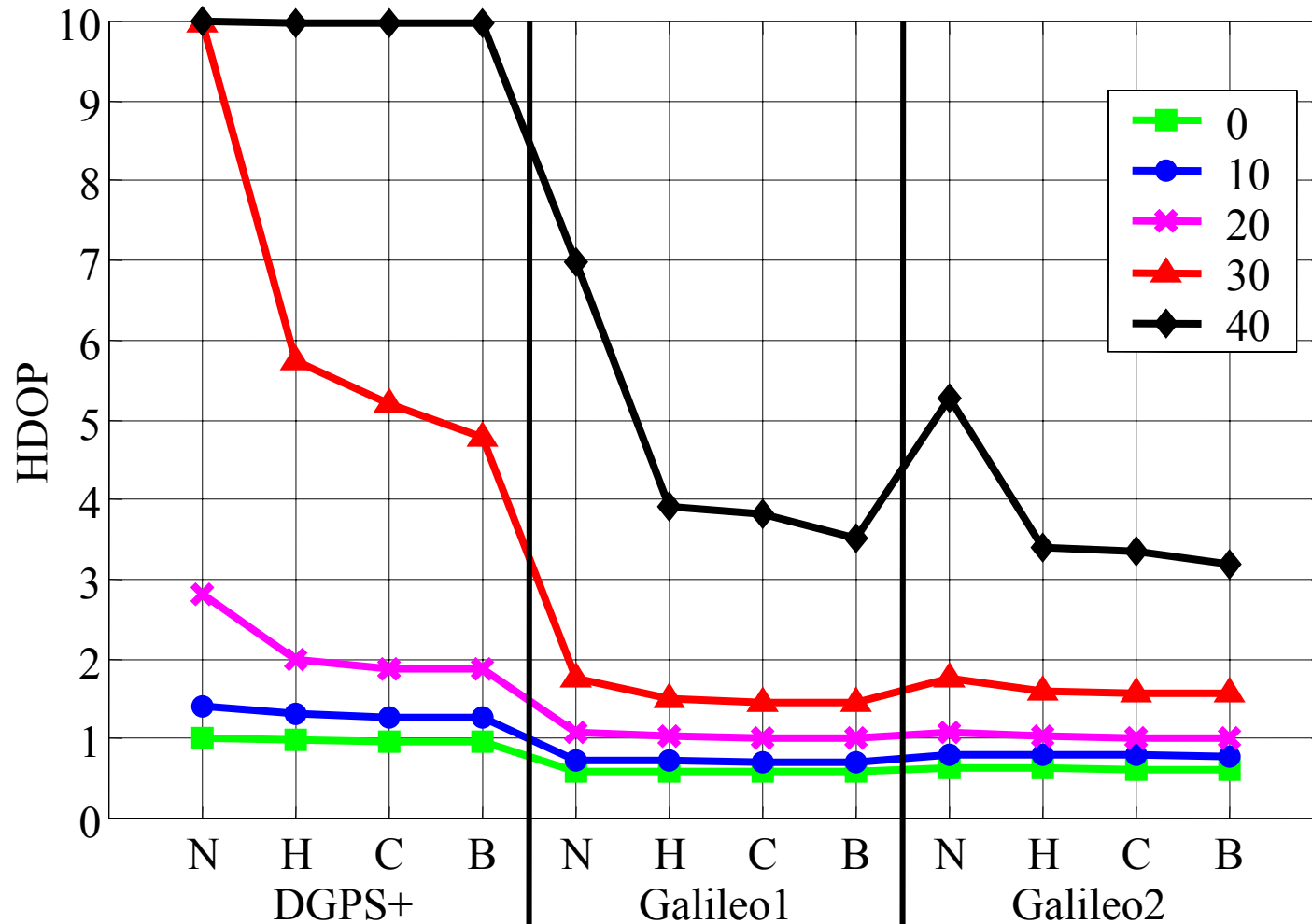
Simulation Results (24 Hours)

HDOP 95%, DGPS + Galileo1, 30° Mask Angle



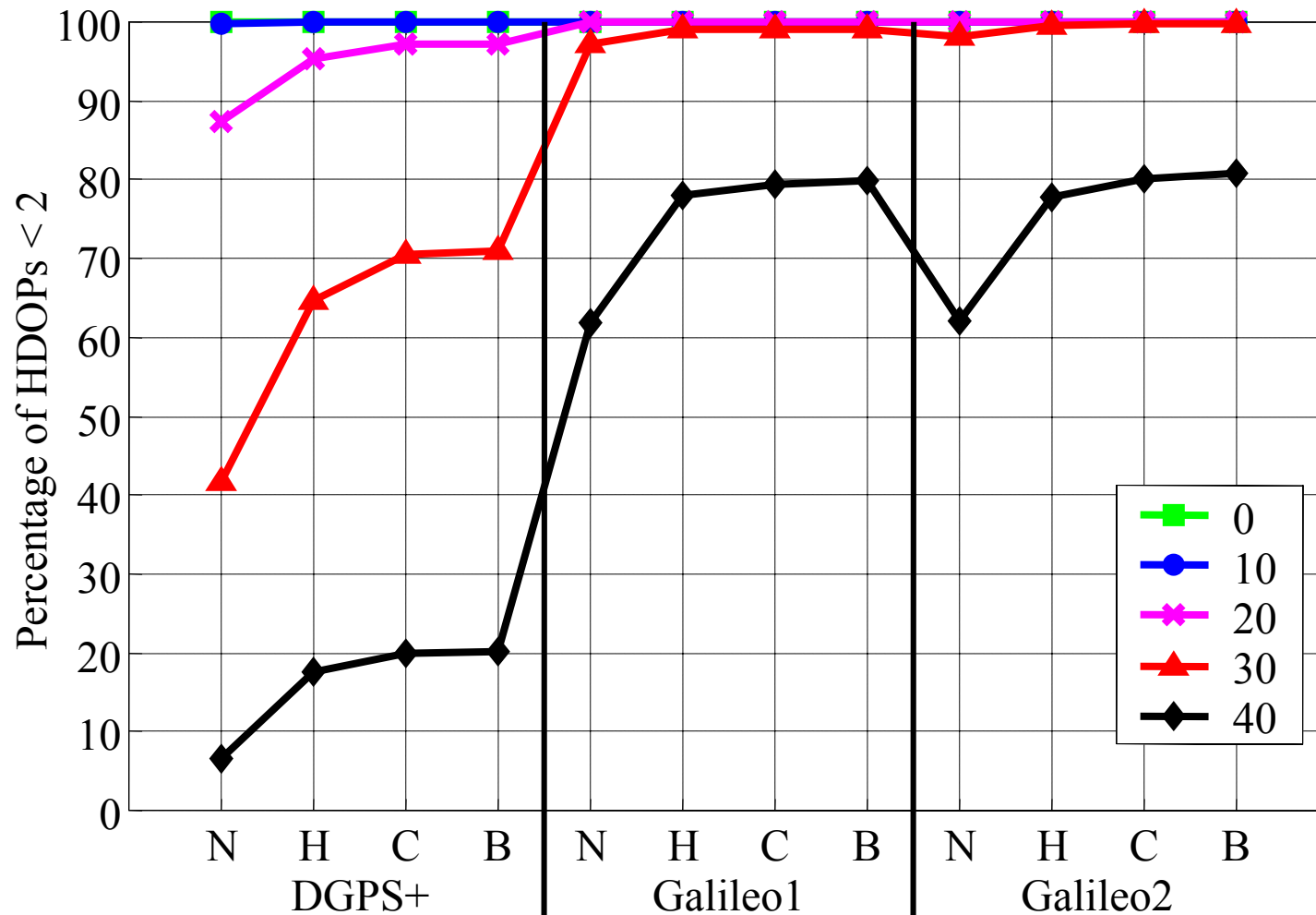
Simulation Results (24 Hours)

HDOP 95% for the World (Isotropic Mask)



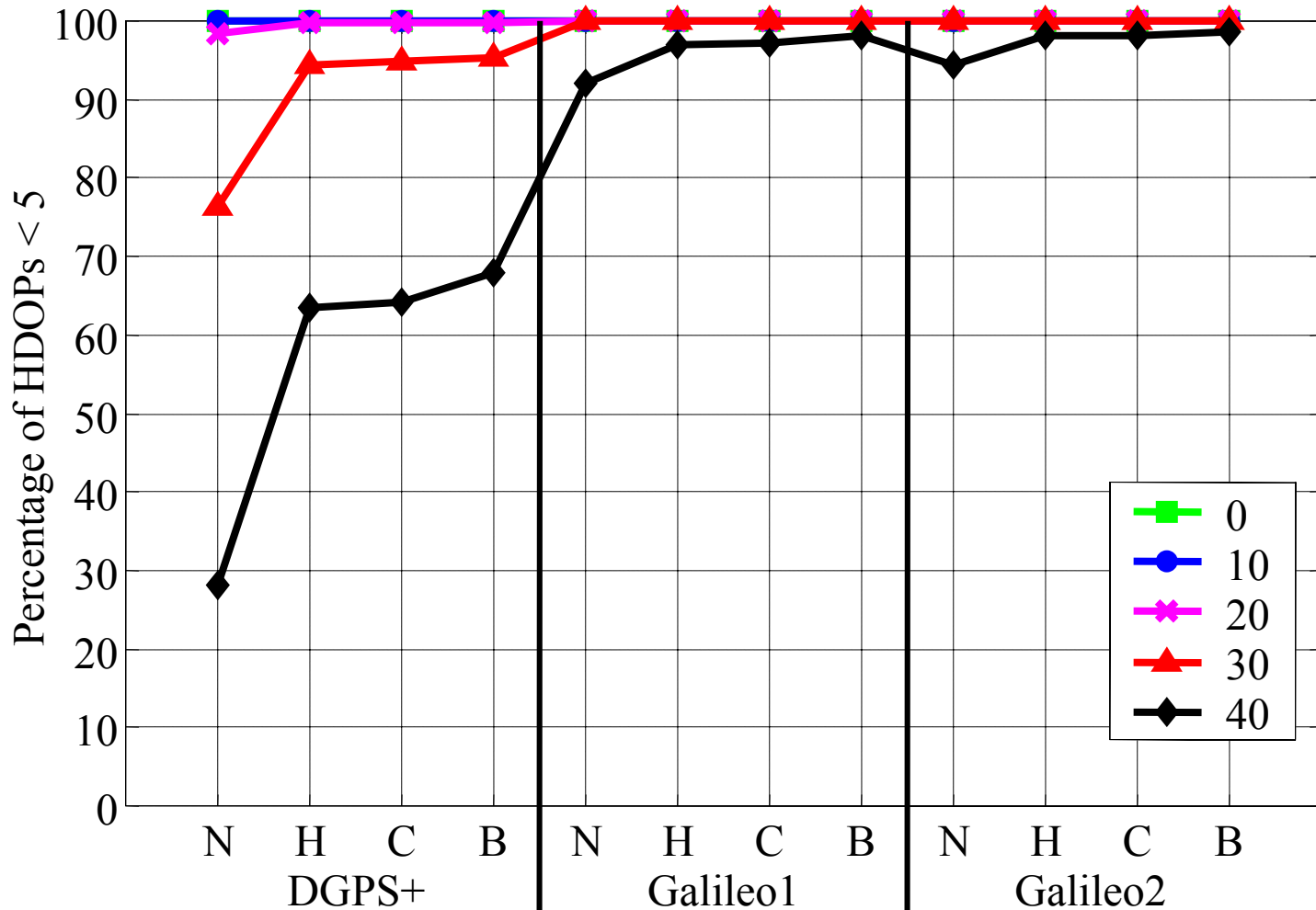
Simulation Results (24 Hours)

% of HDOPs < 2 for the World (Isotropic Mask)



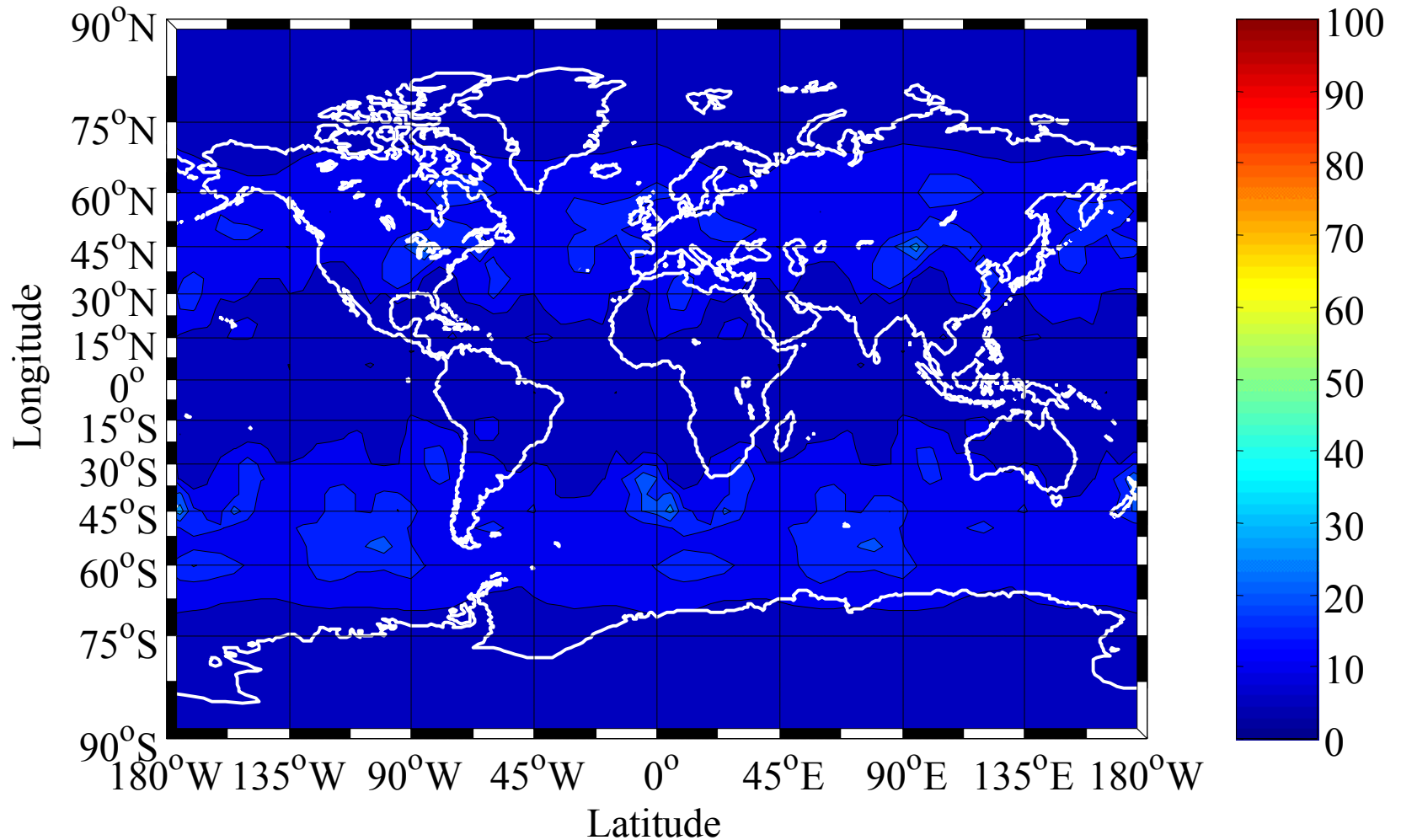
Simulation Results (24 Hours)

% of HDOPs < 5 for the World (Isotropic Mask)



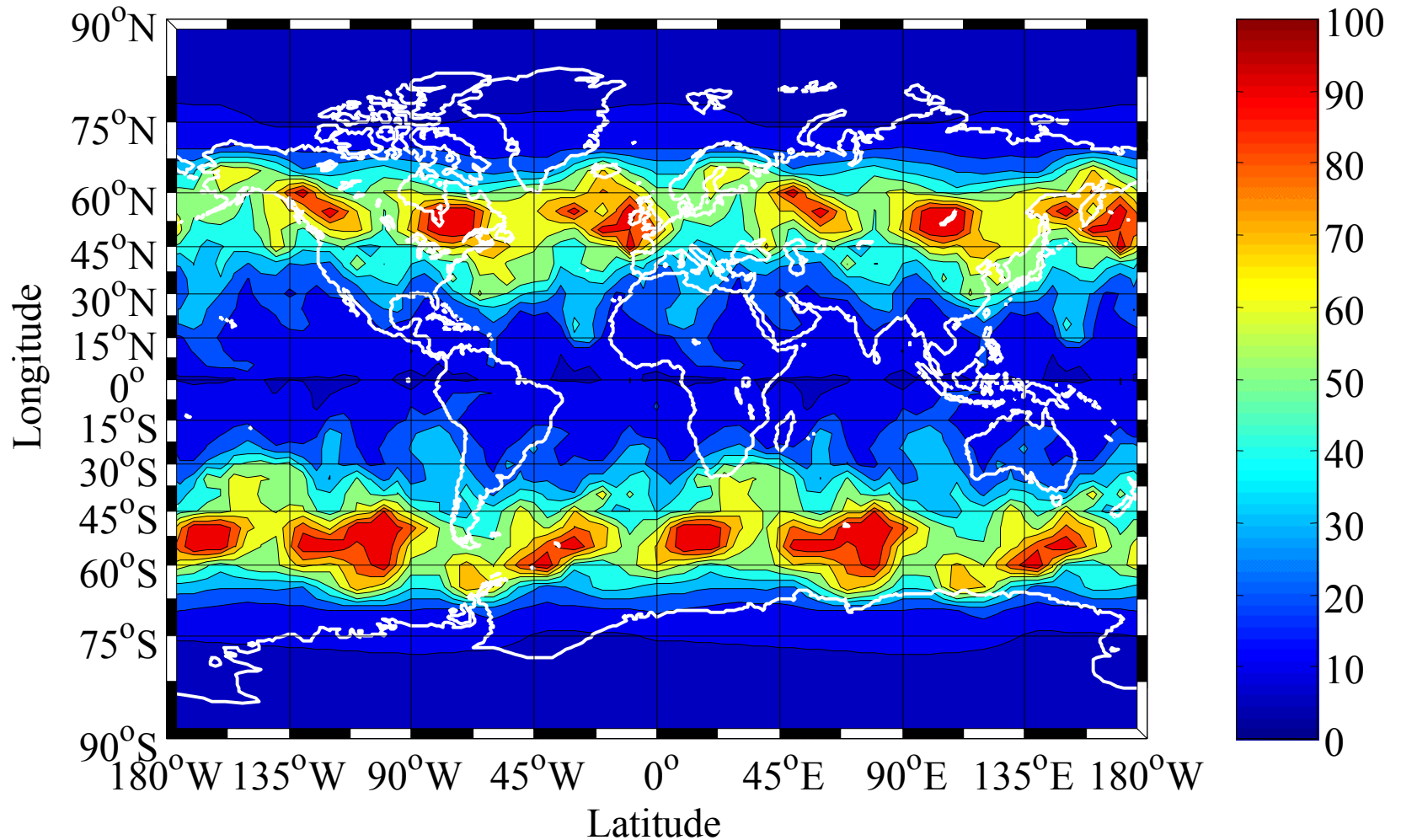
Simulation Results (24 Hours)

HPE 95%, DGPS, 10° Mask Angle



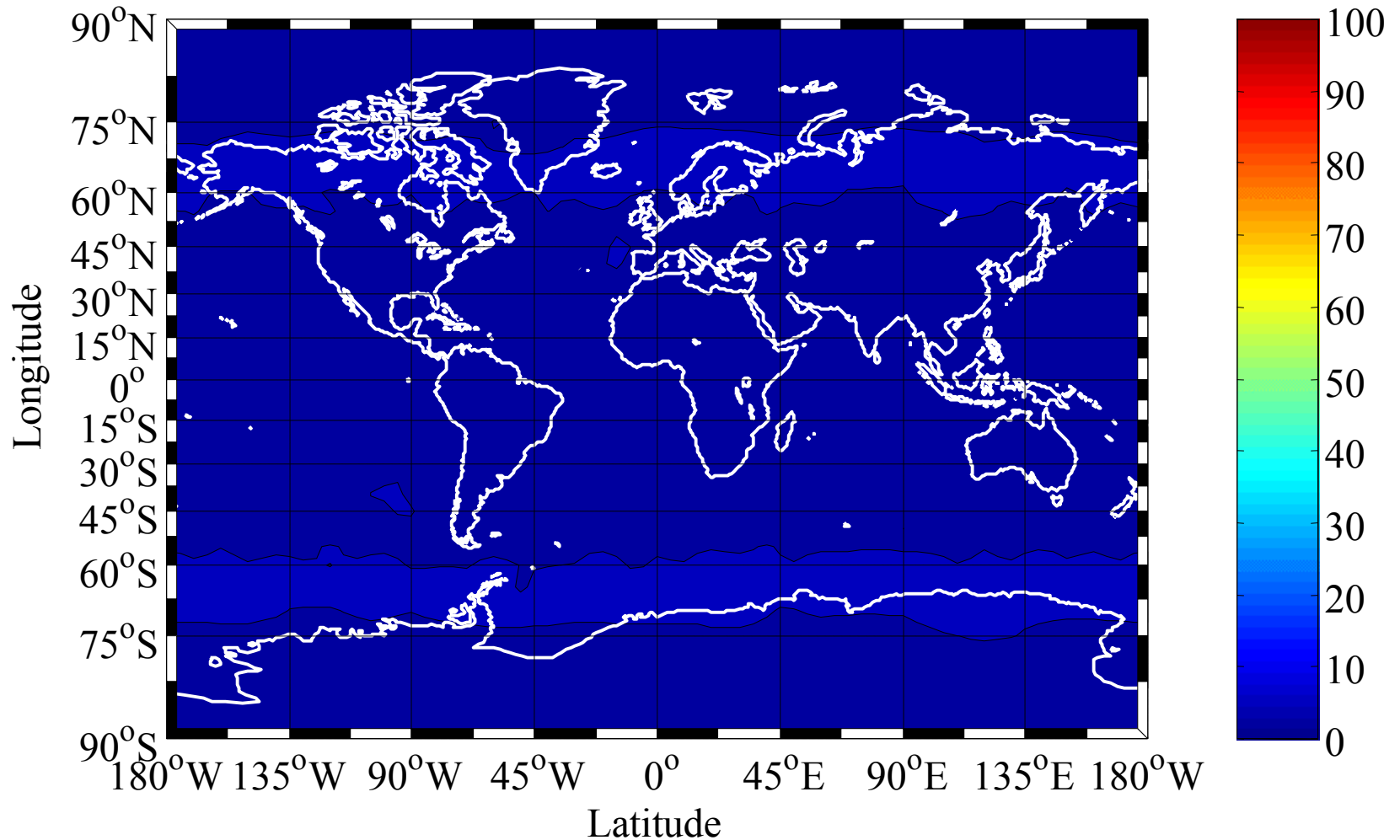
Simulation Results (24 Hours)

HPE 95%, DGPS + H, 20° Mask Angle



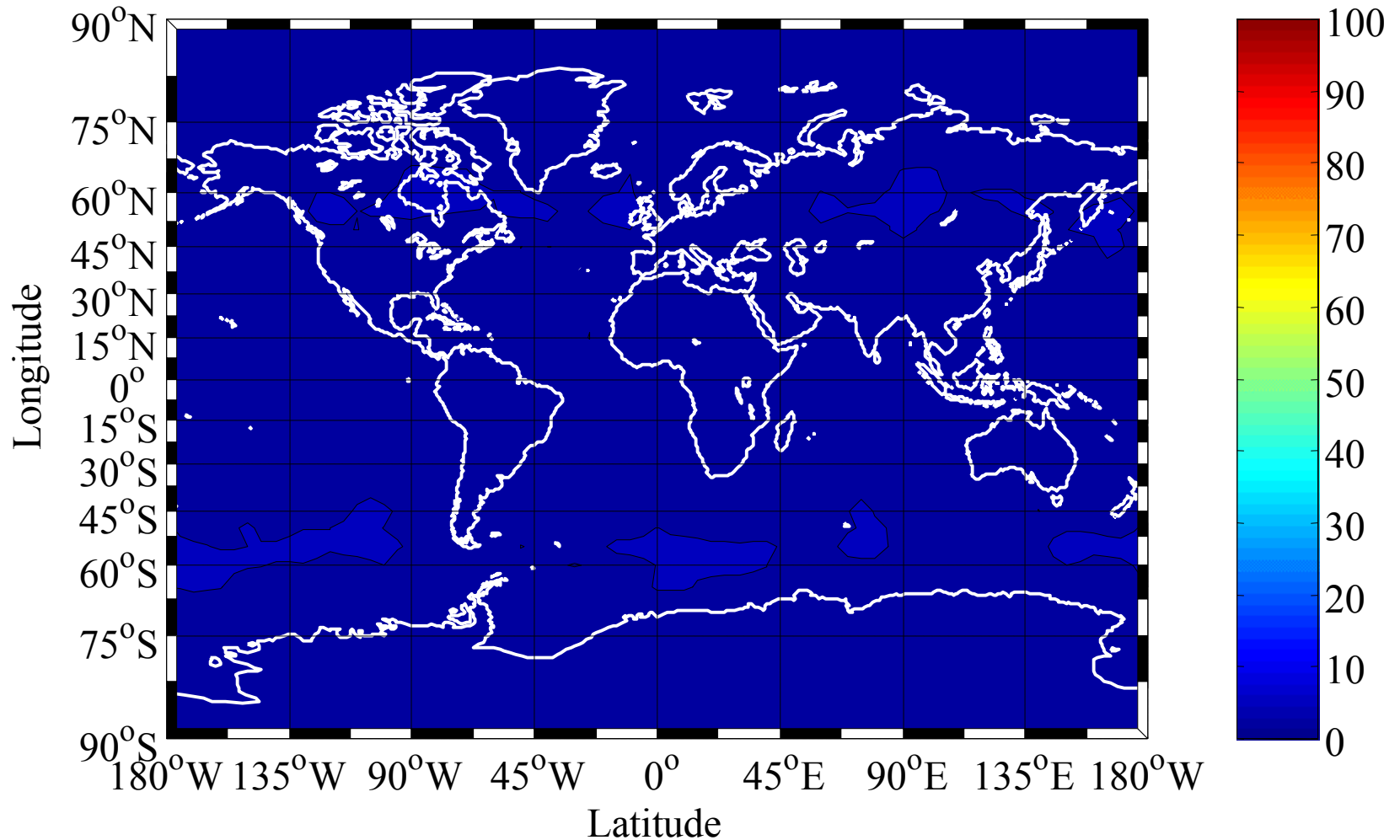
Simulation Results (24 Hours)

HPE 95%, DGPS + Galileo1, 20° Mask Angle



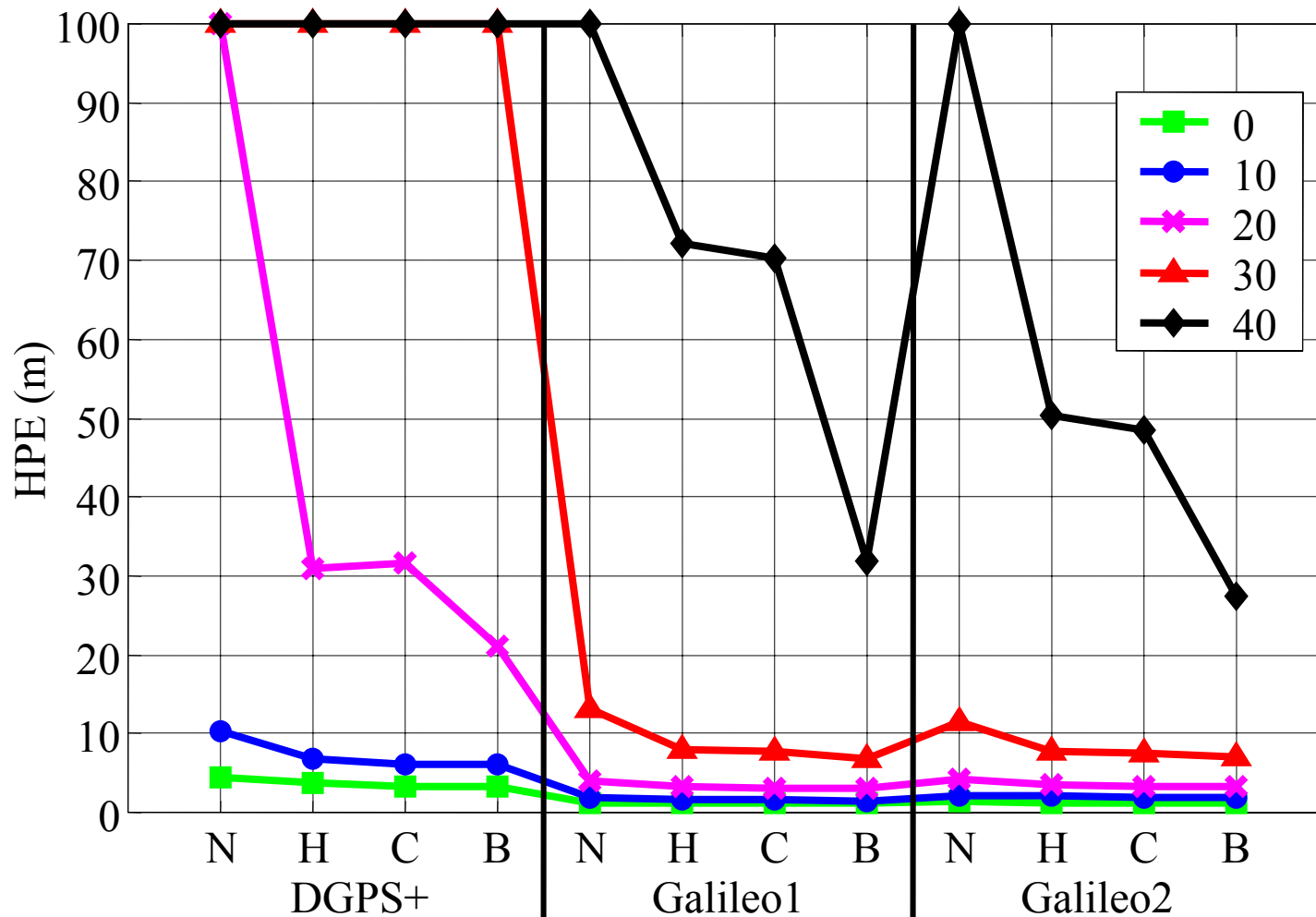
Simulation Results (24 Hours)

HPE 95%, DGPS + Galileo2, 20° Mask Angle



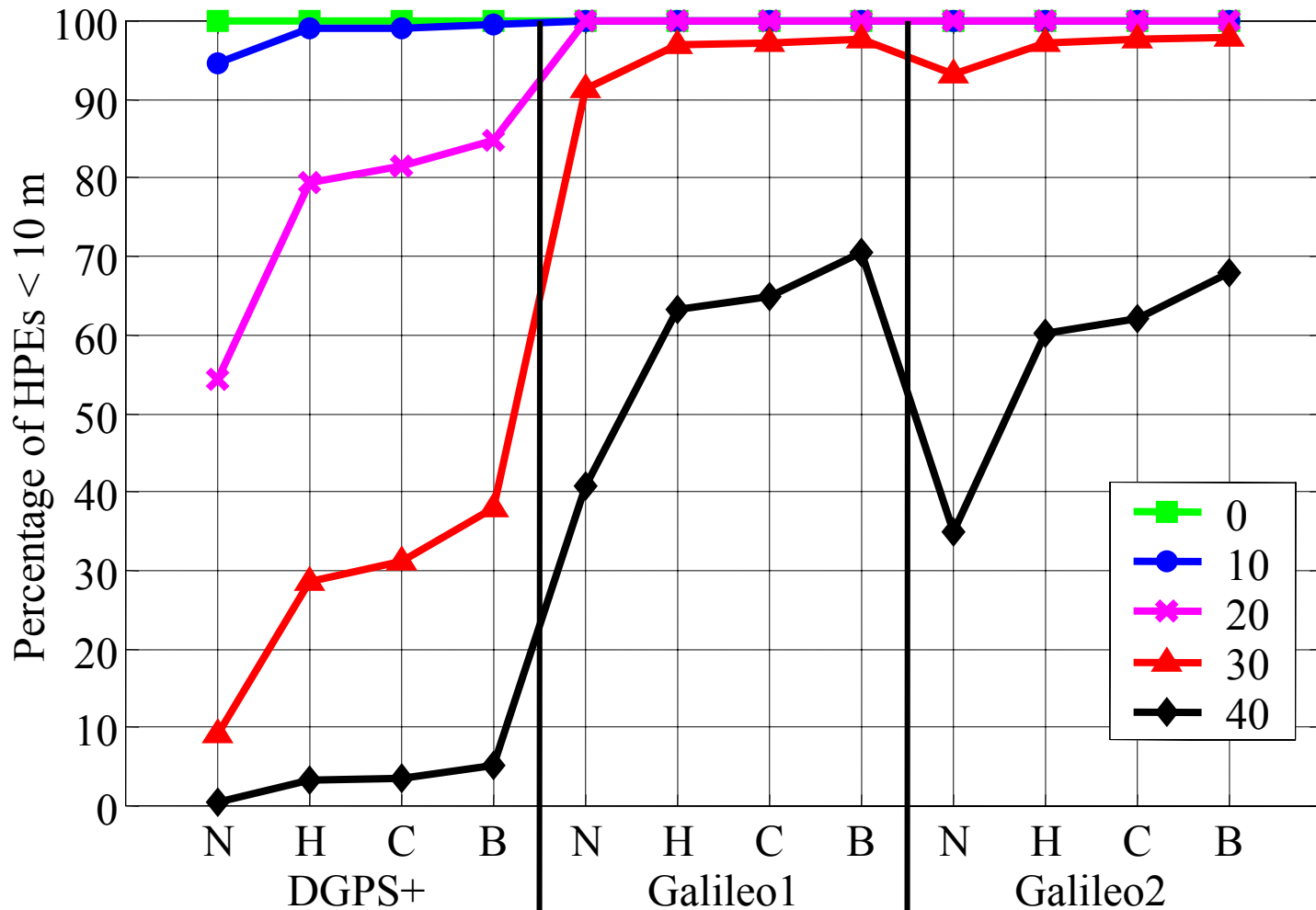
Simulation Results (24 Hours)

HPE 95% for the World (Isotropic Mask)



Simulation Results (24 Hours)

% of HPEs < 10 m for the World (Isotropic Mask)



User Receiver

Reliability Evaluation

- Purpose
 - To demonstrate that many DGPS User Receivers do not use a reliability algorithm and to illustrate the inherent dangers that this causes
- Procedure
 - A GSS DGPS Signal Simulator was used to test three receivers under various multipath conditions
 - Multipath ramps were added to SV #8 and each receiver was analyzed to determine if it could detect or otherwise mitigate the multipath error

GPS Signal Simulation Description

Test Receivers

- Receiver “A” - DGPS Survey Receiver
 - 12 Channel Dual Frequency with Raw Data Output
 - High Performance Correlator
- Receiver “B” - Integrated DGPS Sensor
 - 12 Channel L1 Only Receiver with Raw Data Output
 - High Performance Correlator, Integrated Radiobeacon
- Receiver “C” - Wide Correlator DGPS Sensor
 - 12 Channel L1 Only Receiver with Raw Data Output
 - Wide Correlator

GPS Signal Simulation Description

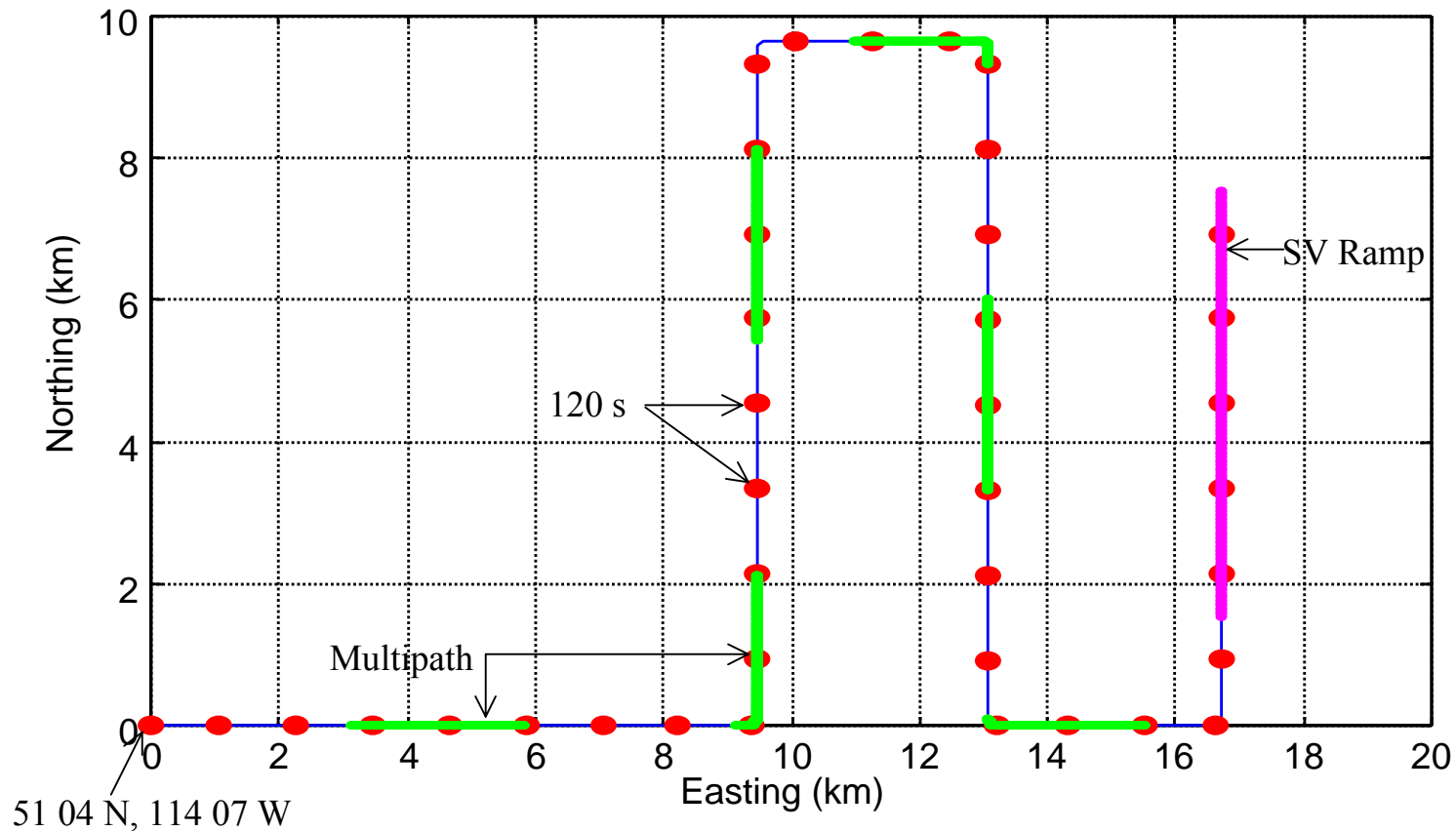
Test Parameters

- Test Date and Duration - Jan 1, 1999 from 1:30 - 2:45
- Data rate - 1 Hz
- No SA/AS
- No satellite clock or ephemeris errors
- Dynamic Ship Motion with moderate Sea States
- Tropospheric and Ionospheric Models On
- User Receiver Multipath ramping errors and a Satellite ramping clock error added to SV #8
- DGPS Corrections from a CCG DGPS Station

GPS Signal Simulation Description

Test Trajectory

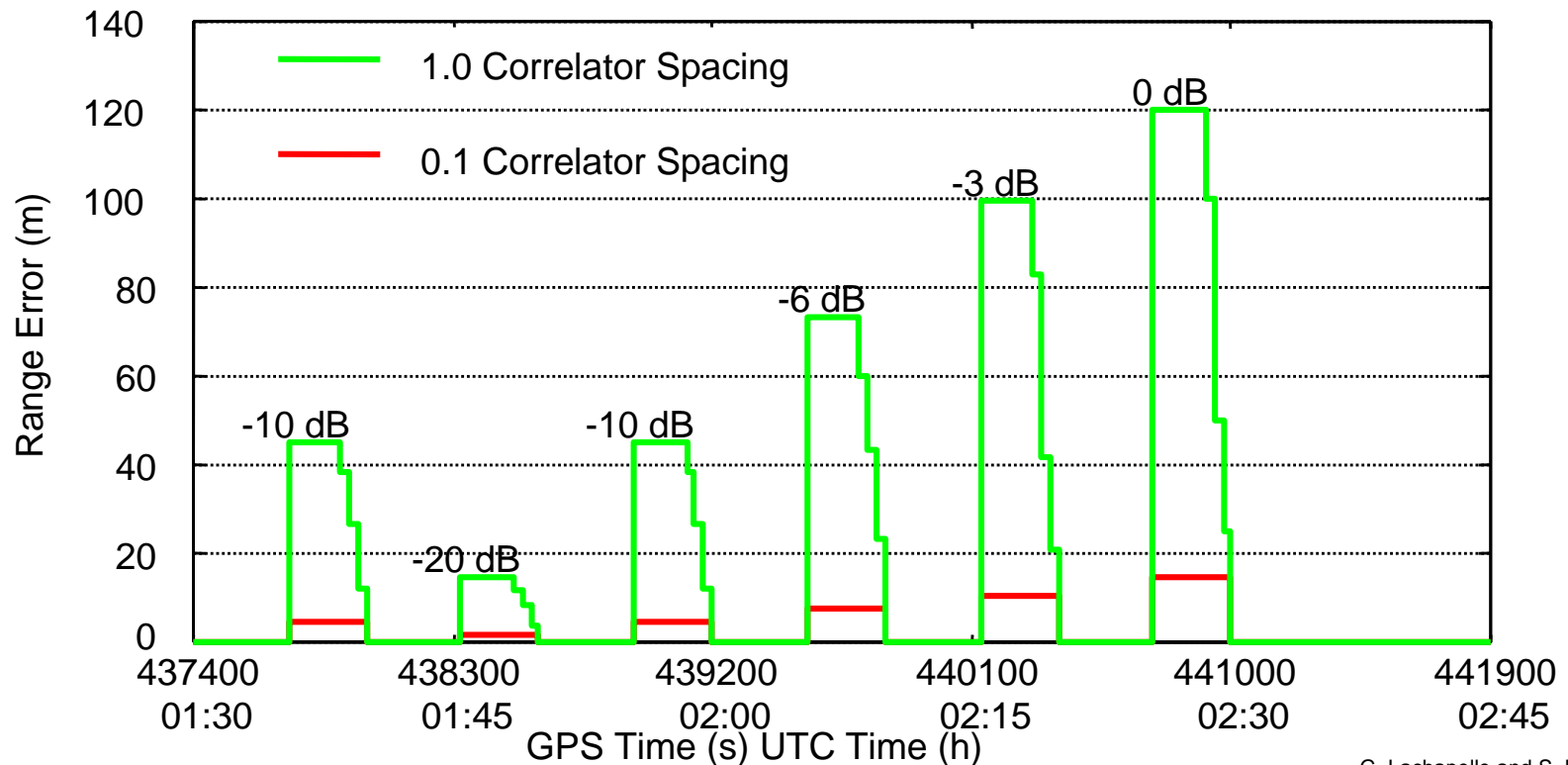
- Maximum speed of 36 km/h, Sea state varied 0 to 3.



GPS Signal Simulation Description

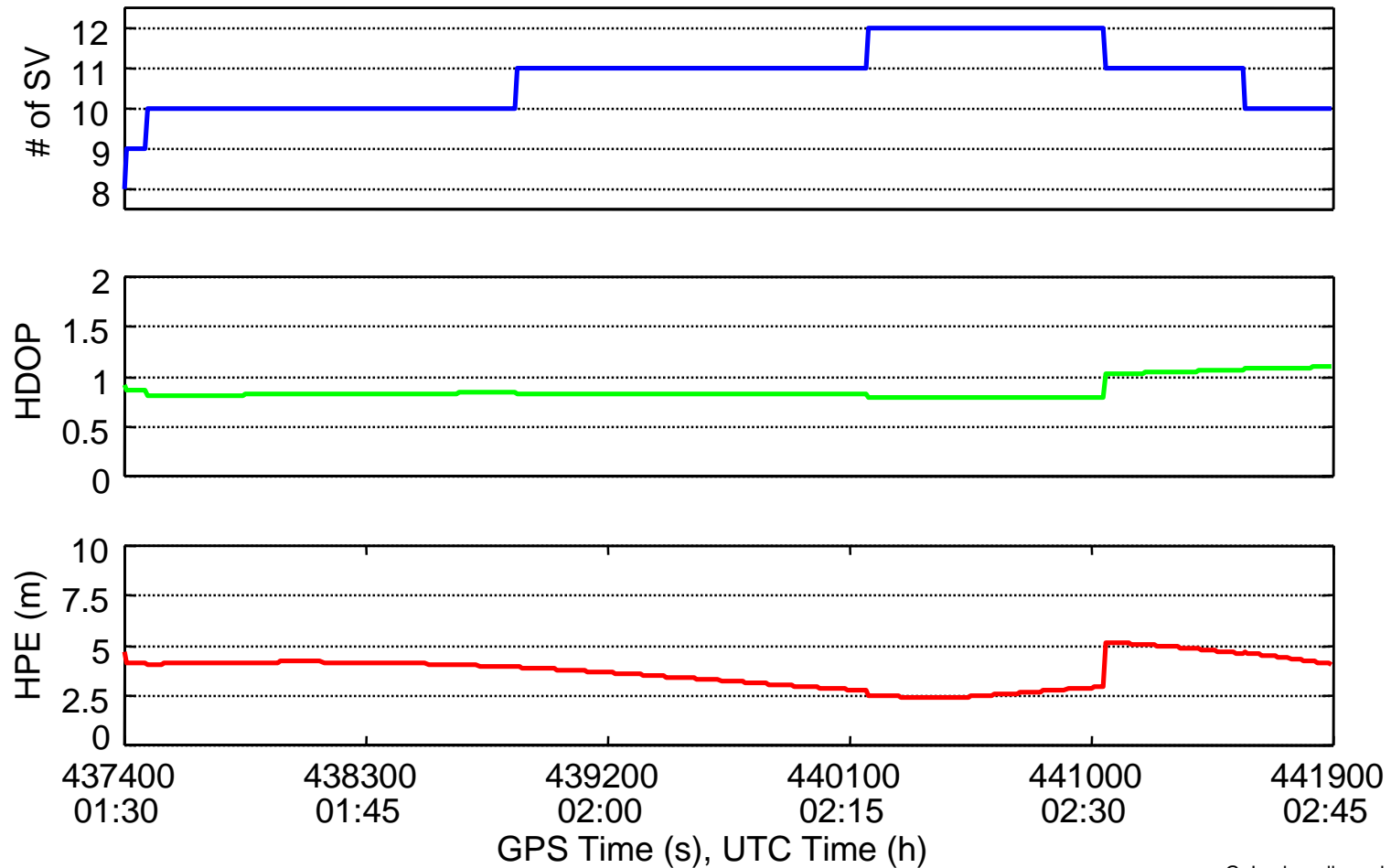
Multipath Errors Added to SV #8

- Six Multipath ramps were added to SV #8. The Resulting Range Errors are plotted against time, assuming an infinite bandwidth.



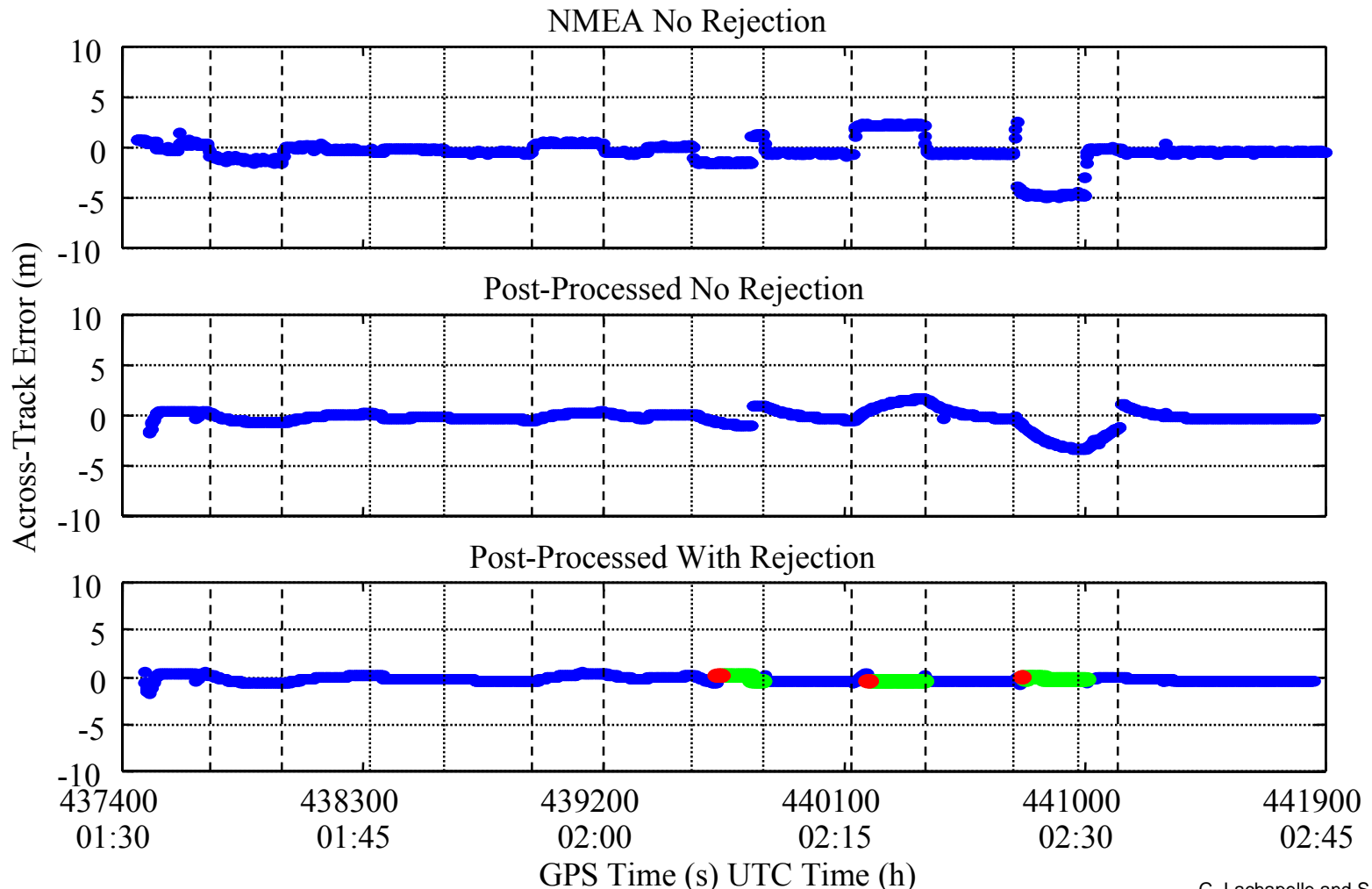
GPS Signal Simulation Description

of SVs, HDOP, and HPE



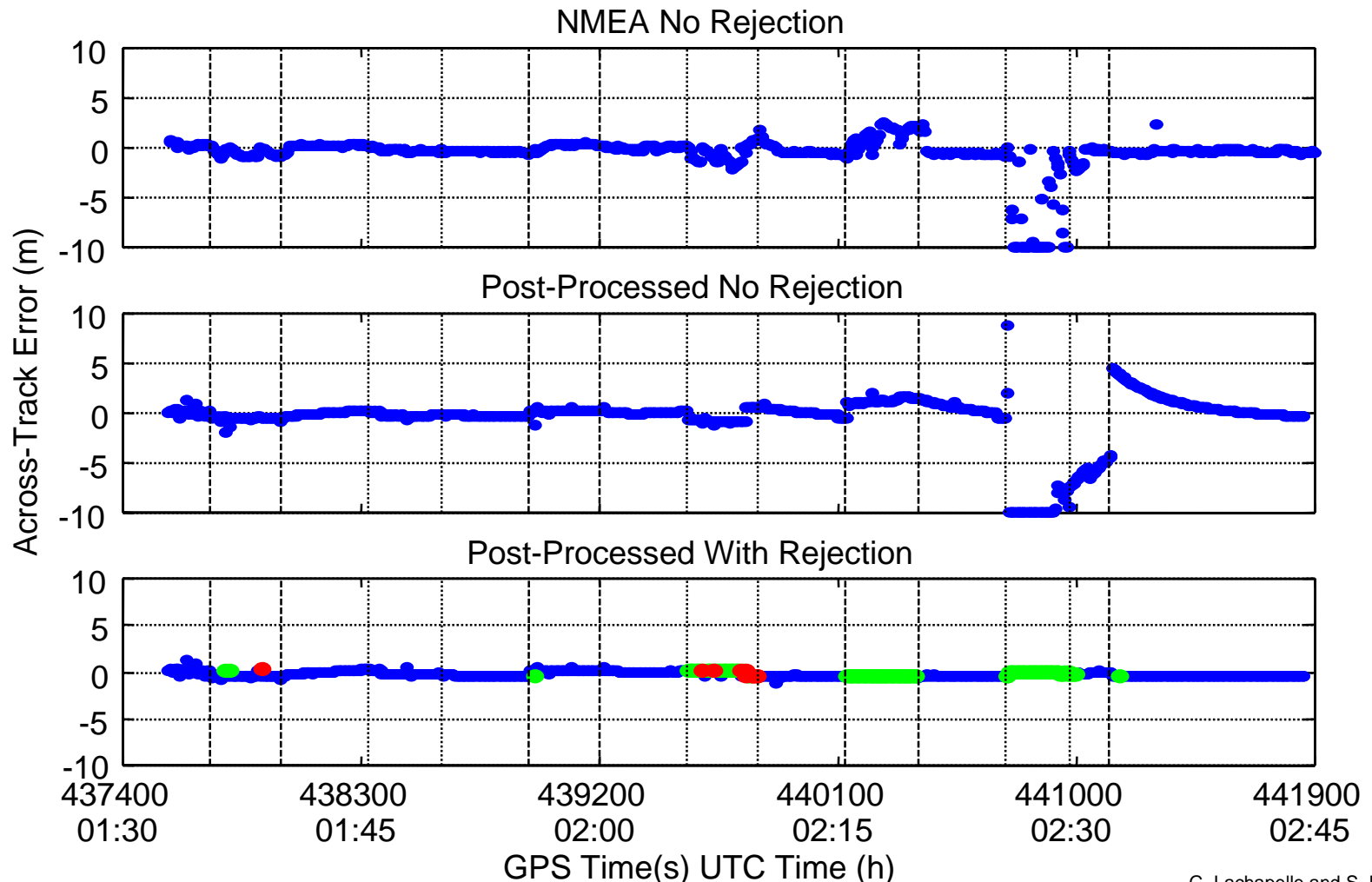
GPS Signal Simulation Results

Receiver "A" - DGPS Survey Receiver



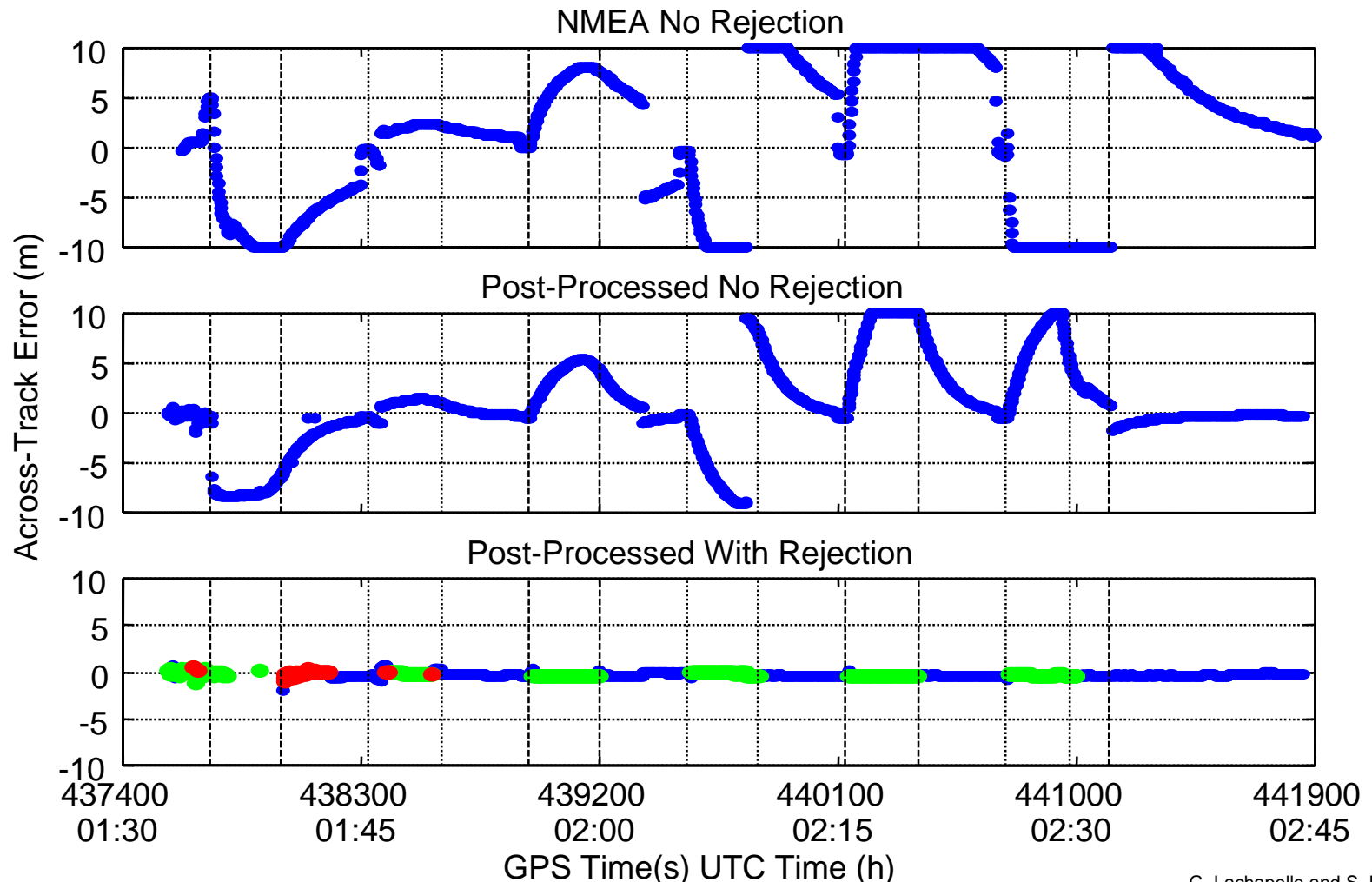
GPS Signal Simulation Results

Receiver "B" - Integrated DGPS Sensor



GPS Signal Simulation Results

Receiver "C" - Wide Correlator DGPS Sensor



GPS Signal Simulation Results

Summary

Max and RMS Errors (m) during the Multipath Ramps

RX	NMEA No Rejection		Post-Processing No Rejection		Post-Processing With Rejection	
	Max	RMS	Max	RMS	Max	RMS
“A”	5.0	2.2	3.4	1.1	0.8	0.4
“B”	46.9	7.9	63.4	9.0	1.0	0.3
“C”	26.7	13.1	14.1	7.1	1.1	0.4

Conclusions

- Statistical reliability theory is powerful to analyze systems in simulation mode.
- Statistical reliability should be implemented in receivers, to increase integrity.
- Areas of Research
 - Actual probability of single and multiple blunder occurrence.
 - Augmentation of GPS with other systems to improve reliability.
 - Improved system and receiver performance to reduce and detect blunders (multipath and ionosphere).

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