UNIVERSITY OF CALGARY

Carrier Phase Measurements Characteristics and Utilization Overview

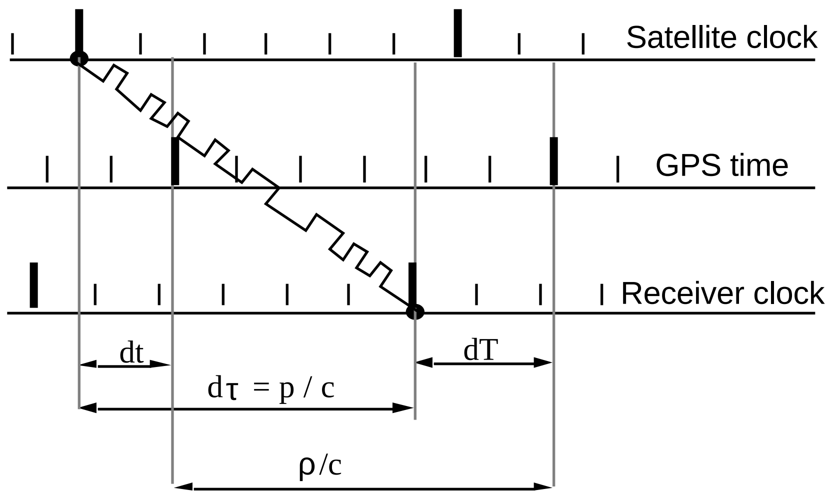
G. Lachapelle
Department of Geomatics Engineering
University of Calgary

www.ensu.ucalgary.ca/lachap/lachap.html
www.ensu.ucalgary.ca/GPSRes/GPSResIndex.html

University of Minnesota IMA Workshop on Mathematical Challenges in GPS

August 16-18, 2000

PSEUDORANGE OBSERVABLE



- Observation Equation

$p = \rho + d\rho + c\,(dt - dT) + d_{ion} + d_{trop} + \varepsilon(p)$

$\rho =$ $\|$ **r** - **R** $\|$, where **R** (x, y, z) is the (unknown) position vector of the observation point and **r**, that of the satellite observation point

$d\rho$ … orbital error

dt, dT... satellite & receiver clock errors

d_{ion}, d_{trop}... ionospheric & tropospheric delays

$\varepsilon(p)$... $f\{\varepsilon(p_{rx}),\ \varepsilon(p_{mult})\}$

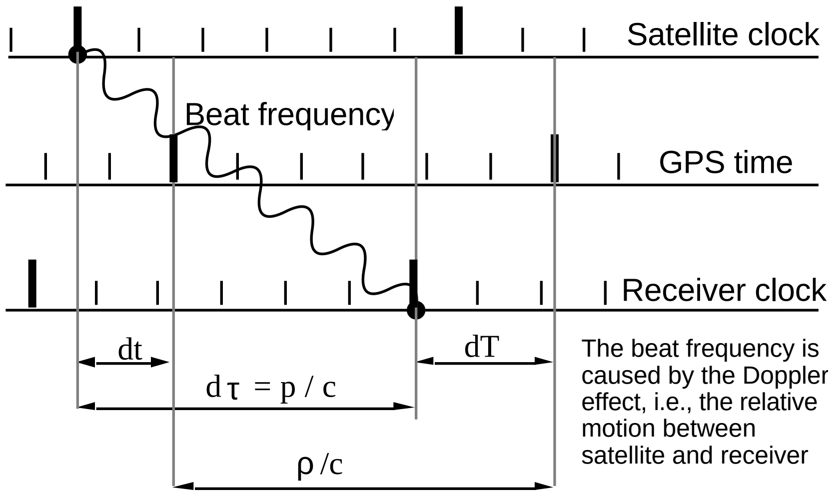
$\varepsilon(p_{rx})$ ... receiver noise (Gaussian)

$\varepsilon(p_{rx[c/a]}) \approx 10 - 300$ cm

$\varepsilon(p_{rx[P]}) \approx 10 - 30$ cm

$\varepsilon(p_{mult}) \leq 1$ chip (non-Gaussian)

CARRIER PHASE OBSERVABLE



- Observation equation (in metres)

$$\Phi = \rho + d\rho + c(dt - dT) + \lambda N - d_{ion} + d_{trop} + \varepsilon_{\Phi}$$

where

ρ geometric range (i.e. $\| \mathbf{r} - \mathbf{R} \|$)

$d\rho$… orbital error

dt… satellite clock error

dT receiver clock error

N cycle ambiguity (integer number)

d_{ion} ionospheric delay

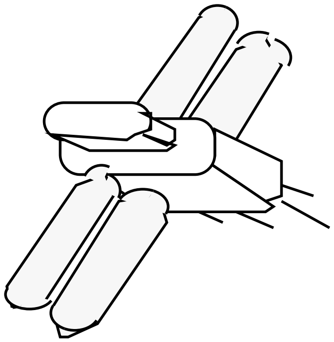
d_{trop} tropospheric delay

ε_{Φ} noise ($\varepsilon_{\Phi_{rx}} + \varepsilon_{\Phi_{multipath}}$)

$\varepsilon_{\Phi_{rx}} \approx 1 - 5$ mm

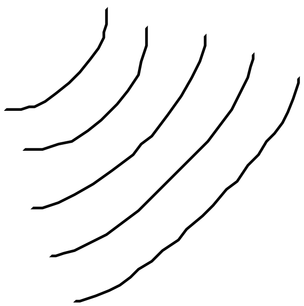
$\varepsilon_{\Phi_{multipath}} \leq 0.25\lambda$

GPS ERROR SOURCES

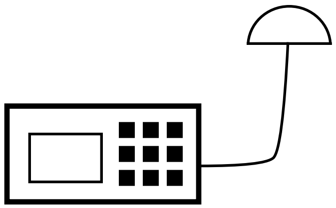


ERROR MAGNITUDE

- Satellite errors (1σ):
  Orbit & clock:< 5 m



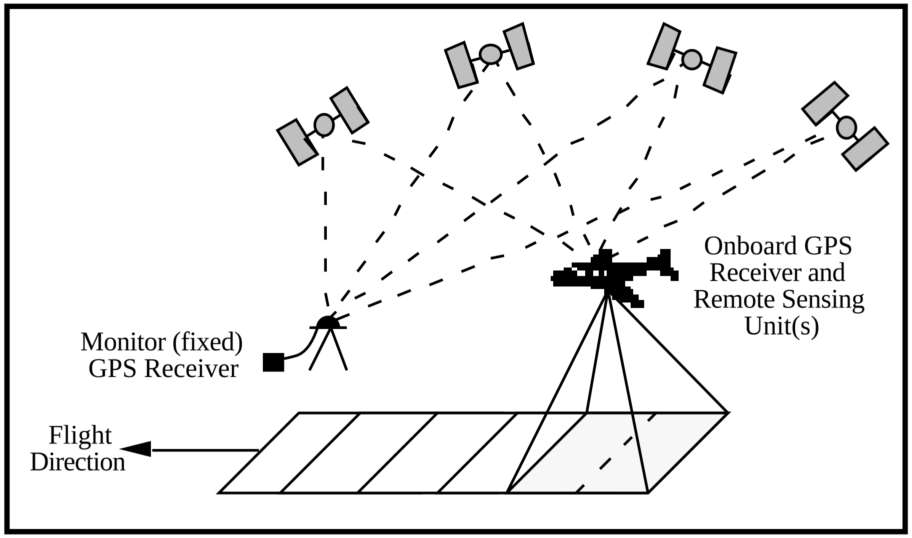
- Propagation errors:
  Ionosphere: < 10 m
  Troposphere: 0.2 - 1.0 m



- C/A code receiver errors

  Code Multipath: < 5 m
  Code Noise: < 1 m
  Carrier Multipath: < 50 mm
  Carrier Noise: < 3 mm

SINGLE REFERENCE STATION DGPS CONCEPT



- **Advantages**

(i) Reduction/elimination of following errors

- Orbital (reduced)
- Ionosphere and troposphere (reduced)
- Satellite and receiver clock errors (eliminated)

(ii) Better quality control

- **Remaining errors**

- receiver noise (amplified)
- multipath (amplified)
- differential troposphere
- differential ionosphere (for single frequency users)

WHY DIFFERENTIAL GPS?

- to decrease spatially correlated errors, namely
  - orbital errors
  - atmospheric errors

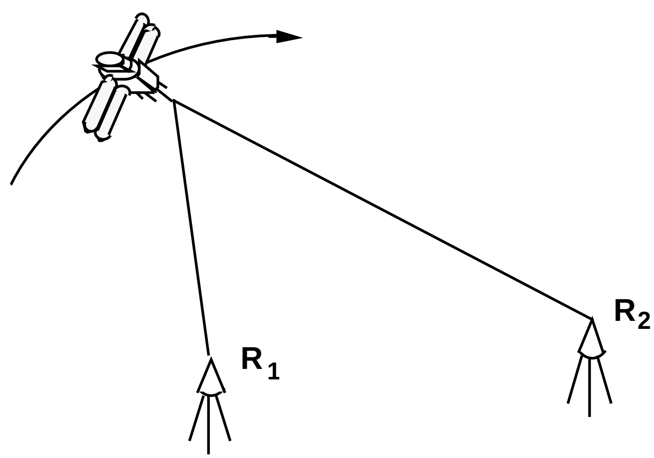
- to eliminate satellite (differencing) and receiver clock errors (Δ differencing)

- site dependent errors (noise and multipath) are not reduced but amplified

- DGPS is required to fully exploit the high accuracy (cm-level) of carrier phase measurements, even in the absence of Selective Availability

- DGPS can also improve reliability, depending on system specifications

BETWEEN RECEIVER SINGLE DIFFERENCE (Δ)



- Concept: Subtract pseudorange at reference station (R_1) from that at remote (R_2):

$$\Delta = (\bullet)_{rx_2} - (\bullet)_{rx_1}$$

$$\Delta p = \Delta\rho + \Delta d\rho - c\Delta dT + \Delta d_{ion} + \Delta d_{trop} + \varepsilon_{\Delta p}$$

- Reduces orbital and atmospheric errors
- Eliminates satellite clock error, dt
- Does not reduce $\varepsilon_{\Delta p}$

- **Method used for real-time applications**:
- Filtered dρ's and dρ/dt's are transmitted from R_1 to R_2 at regular intervals:
- RTCM SC104 Specifications: 50 - 100 bps data rate
- Positioning accuracy: 0.5 - 5 m

CARRIER PHASE SMOOTHING (1/2)

MOTIVATION

- Merge 'absolute' pseudorange capability and 'relative' carrier phase capability - pseudorange is not ambiguous but noisy, carrier phase is ambiguous but accurate
- Provides an alternative to pure pseudorange observations and is used in virtually all rx's firmware

METHODOLOGY

- Recursive filter to progressively increase weight on Φ while decreasing weight on P
- For example, smoothed pseudorange $\tilde{\mathbf{P}}_k$ at time k:

$$\tilde{\mathbf{P}}_k = W_{P_k} P_k + W_{\Phi_k} \{ \tilde{\mathbf{P}}_{k-1} + (\Phi_k - \Phi_{k-1}) \}$$

$\tilde{\mathbf{P}}_k$ ↓	$W_{P_k} P_k$ ↓	$\tilde{\mathbf{P}}_{k-1}$ ↓	$(\Phi_k - \Phi_{k-1})$ ↓
computed smoothed pseudorange	measured pseudorange	previously smoothed pseudorange	range difference from measured carrier phase

- W_{P_k} and W_{Φ_k} are the weights on the measured pseudorange and carrier phase components

CARRIER PHASE SMOOTHING (2/2)

$W_{P_k} = W_{P_{k-1}} - 0.01 \quad \{e.g., 0.01 \leq W_{P_k} \leq 1.00\}$

$W_{\Phi_k} = W_{\Phi_{k-1}} + 0.01 \quad \{e.g., 0.00 \leq W_{\Phi_k} \leq 0.99\}$

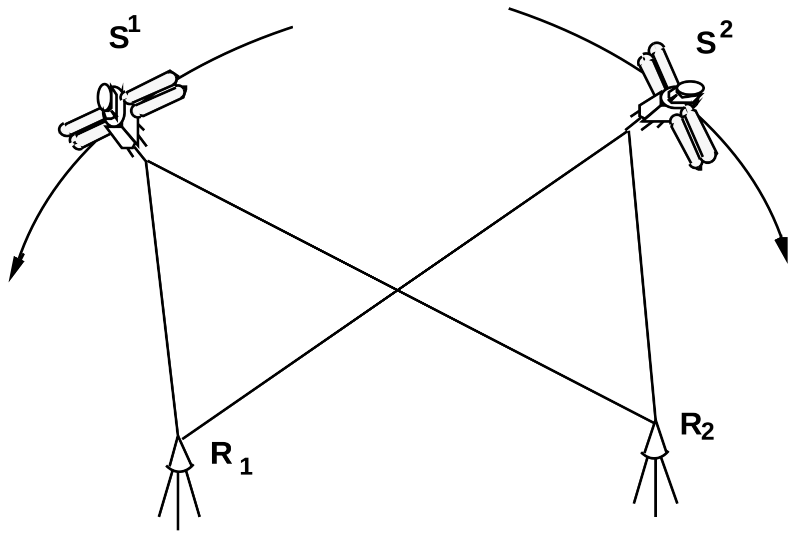
- At initialization (t_1)

  $\tilde{\mathbf{P}}_1 = P_1 \quad \{W_{P_{t_1}} = 1.0; W_{\Phi_{t_1}} = 1.0 - W_{P_{t_1}} = 0.0\}$

- Use parallel filters to deal with code/carrier divergence and make the procedure more robust

HATCH, R. (1982) The Synergism of GPS Code and Carrier Measurements. Proceedings of 3rd International Geodetic Symposium on Satellite Doppler Positioning, DMA/NGS, pp. 1213-1232, Washington, D.C.

SATELLITE/RECEIVER DOUBLE DIFFERENCE ($\Delta\nabla$)



$\Delta\nabla = \{(\bullet)_{sat_2} - (\bullet)_{sat_1}\}_{rx_2} - \{(\bullet)_{sat_2} - (\bullet)_{sat_1}\}_{rx_1}$

$\Delta\nabla p = \Delta\nabla\rho + \Delta\nabla d\rho + \Delta\nabla d_{ion} + \Delta\nabla d_{trop} + \Delta\nabla\varepsilon(p)$

$\Delta\nabla\Phi = \Delta\nabla\rho + \Delta\nabla d\rho + \lambda\Delta\nabla N - \Delta\nabla d_{ion} + \Delta\nabla d_{trop} + \Delta\nabla\varepsilon(\Phi)$

- Assumes that all observations are taken at same time
- Reduces orbital and atmospheric errors
- Eliminates satellite and receiver clock errors dt and dT
- Does not reduce ε(p)'s or ε(Φ)'s
- Ambiguity term $\Delta\nabla N$ can be held to an integer value for short baselines/monitor-remote separations:

$$\Delta\nabla\Phi = \Delta\nabla\rho + \lambda\Delta\nabla N + \Delta\nabla\varepsilon(\Phi)$$

OTF AMBIGUITY RESOLUTION (1/2)

- OTF: On-The-Fly, i.e., in kinematic mode
- Resolution of (Δ → double difference) integer ambiguities using a search technique while remote receiver is moving
- Real-time or post-mission batch algorithm
- Assumes that no irrecoverable cycle slips occur during the resolution period
- Numerous methods have been developed since early 80s, e.g., Ambiguity Function Method, least-squares search, FASF, FARA, lambda, etc.
  - methods explicitly/implicitly assume a known stochastic behavior for unmodelled errors (e.g., noise, multipath, differential atmospheric effects)

OTF AMBIGUITY RESOLUTION (#2/2)

- **The time to resolution is a function of:**
  - Use of L_1 vs L_1 - L_2 (widelane) observable ($\lambda L1$= 19 cm, λL_1-L_2 = 86 cm)
  - distance between reference and remote receivers
  - number and geometry of satellites
  - differential atmospheric conditions
  - multipath conditions, code and carrier phase noise
  - ambiguity search method used
  - level of (statistical) reliability required