

**IMA HOT TOPICS WORKSHOP: Mathematical Challenges in
Global Positioning
Systems (GPS)**

University of Minnesota, 16 - 19 August 2000

**SPEEDING UP FILTER CONVERGENCE IN HIGH
PRECISION, VERY LARGE AREA
KINEMATIC NAVIGATION**

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**References: O.L. Colombo, Proc. ION GPS-91
(Albuquerque); O.L. Colombo and A.G. Evans, Proc. ION
GPS-98 (Nashville); P. Teunissen and A. Kleusberg (Eds.)
"GPS for Geodesy" (Springer, 1998); O.L. Colombo et al.,
Proc. ION GPS-2000 (Salt Lake City)**

LONG-RANGE KINEMATIC GPS:

LONG BASELINES (0 TO > 2000 km)

USE: LARGE AREA REMOTE-SENSING WITH PRECISE GEOGRAPHIC DATA REGISTRATION

MAIN DATA TYPE: THE L_c COMBINATIONS OF L1, L2 (carrier phase) AND P1, P2 (p-range)

FORM: (THIS TALK) DOUBLE DIFFERENCES

EXTRA UNKNOWN FOR:

Tropo. Refraction Correction Errors (one residual z-delay per receiver)

GPS Orbit Errors (6 initial state + random x, y, z forces per satellite)

L_c Biases ("Floated Ambiguities"; one per observation)

NAVIGATION SOLUTION:

Kalman Filtering (Real-Time)

K-Filtering + Smoothing (Post-Processing)

(50 - 200 Error States)

PROS: HIGH PRECISION ONCE FILTER CONVERGES (~5 CM, 3-D SIGMA)

CAN BE IMPLEMENTED IN PENTIUM II PC'S, OR EQUIVALENT

CON : FILTER USUALLY TAKES MORE THAN HALF AN HOUR TO CONVERGE (ALSO A POST-PROCESSING PROBLEM WITH SHORT/INTERRUPTED DATA TAKES)

CONVERGENCE SPEED DEPENDS ON APPROACH:

**STATIC: TRAJECTORY WITH CONSTANT X, Y, Z
STRONGEST DYNAMIC CONSTRAINT AND FASTEST
CONVERGENCE**

**LEO ORBIT: DYNAMICS USUALLY WELL KNOWN WITH
FORCES MODELED TO $< 10^{-6}$ G.
FAST CONVERGENCE (DYNAMIC, SEMI-DYNAMIC POD)**

**KINEMATIC GPS: NO DYNAMIC CONSTRAINTS ON
TRAJECTORY, SO VERY FLEXIBLE, BUT OF SLOW
CONVERGENCE (IF AMBIGUITIES CANNOT BE FIXED)**

CONSTRAINING LONG-RANGE KINEMATIC TO SPEED UP CONVERGENCE

**FIXED AMBIGUITY/ L_c BIAS (WITH IONOSPHERIC
CORRECTION)**

GEOMETRIC CONSTRAINTS:

KNOWN SITES OCCUPIED

VERTICAL* OR HORIZONTAL POSITION CONSTRAINTS**

MEAN SEA HEIGHT CONSTRAINT

FOR SURFACE CRAFT (BUOYS, BOATS, SHIPS)

MEAN SEA HEIGHT VARIES SLOWLY AND IN MANY PLACES IT AND THE TIDES ARE WELL KNOWN FROM SAT. ALTIMETRY, RECENT GPS DETERMINATION, ETC.

IN A PROCEDURE THAT ALLOWS SOLVING FOR A MEAN POSITION VECTOR X ON AN INTERVAL LONG ENOUGH TO SMOOTH OUT WAVES (e.g. DATA COMPRESSION):

PSEUDO-OBSERVATION:

$H = Xv+n$ (H = Mean Height, v = Mean Vertical Vector,
n = Mean Data Noise + Residual Wave Action)

ERROR STATE DYNAMICS:

$H(i) = H(i-1)+w$ (w = Radom Walk System Noise)

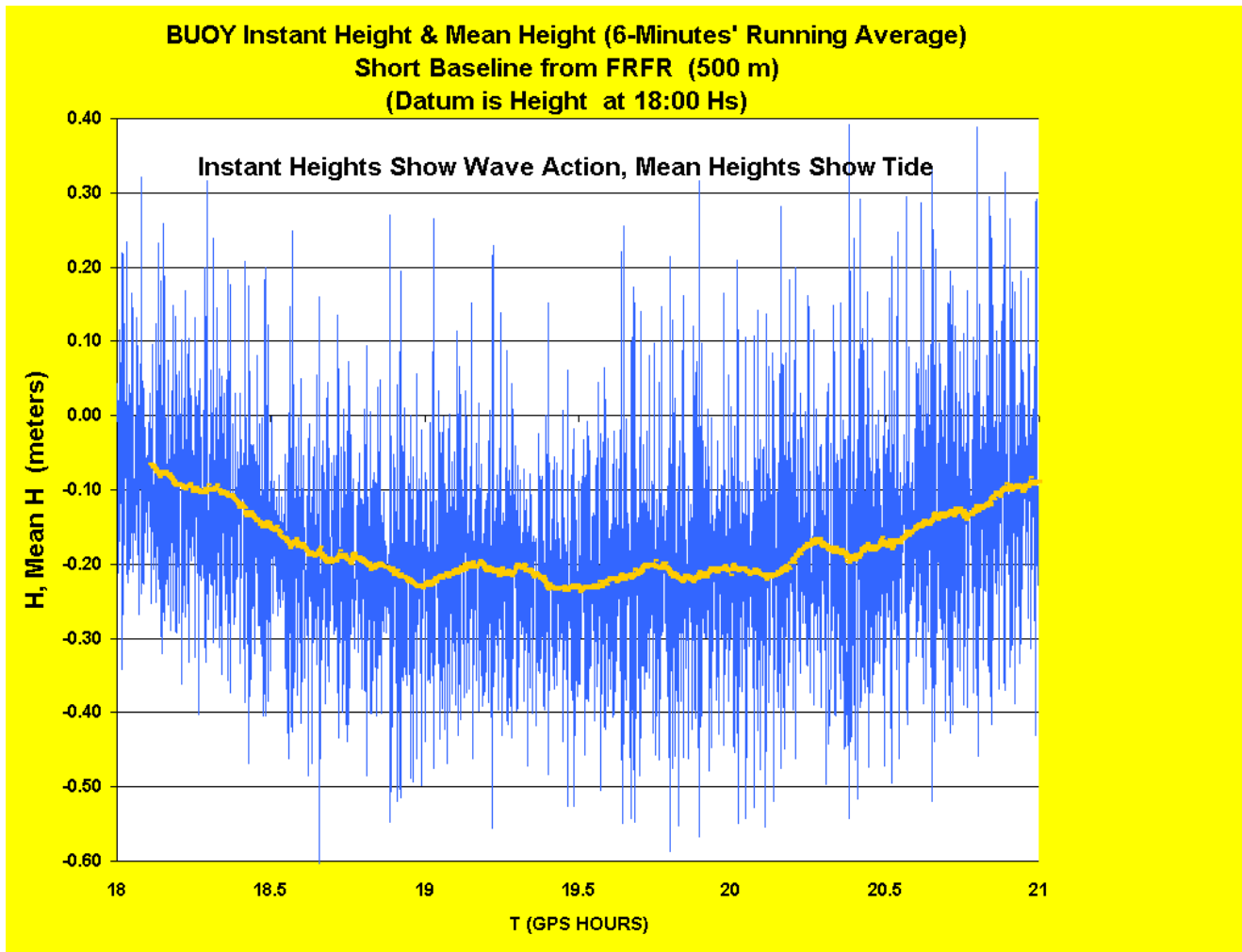
ASSIGNING A PRIORI SIGMAS TO H(0), w, AND MAKING

$$\sigma_n = \sigma_m + W T w (2^{3/2} \pi T a)^{-1}$$

(σ_n = Sigma n, σ_m = Sigma Mean Data Noise, W = Typical Wave Height (Peak), Ta = Averaging Interval)

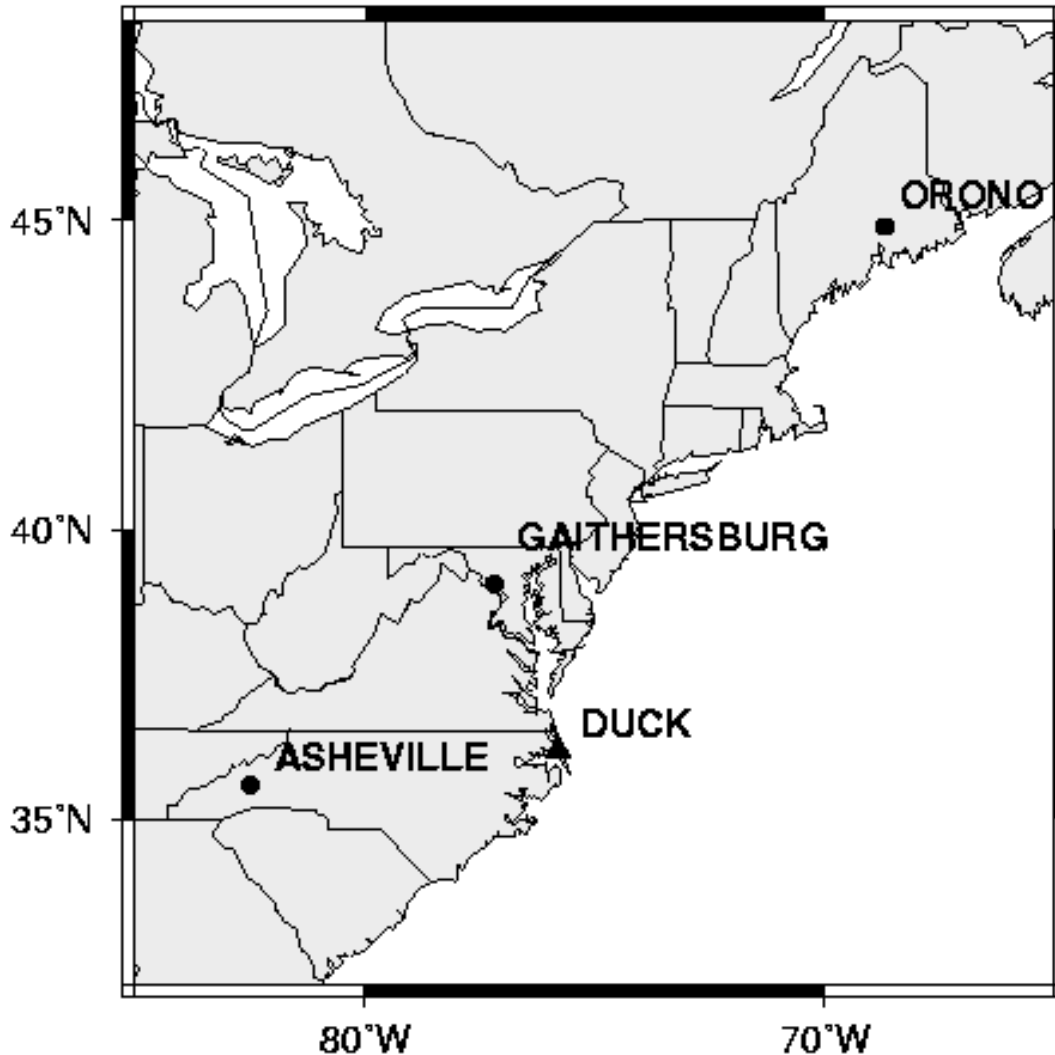
THESE EXPRESSIONS DEFINE A DYNAMIC CONSTRAINT ON THE MEAN VERTICAL POSITION

DIFFERENTIAL, KINEMATIC GPS WITH CARRIER PHASE has been used in the past to position buoys relative to nearby coastal stations. A running average of the observed instantaneous buoy height, with a window of 5 or 6 minutes duration, largely eliminates the short-term fluctuations due to ordinary waves (with periods of 5 to 30 seconds), revealing more gradual changes in sea level, such as those caused by tsunamis. The accuracy is a few centimeters, so one can detect tsunamis of 10 cm or more in height.



Such accuracy is possible because of the short baselines involved (less than 1 km). To detect such things as tsunamis in real time well in advance of their arrival in coastal areas, the buoys must be placed in the high seas, at much greater distances (where a tsunami may rise only one foot at its crest). What follows is a description and results from a test carried out by Dr. A. Evans from NSWCD, on 29 October 1999, at Duck, North Carolina.

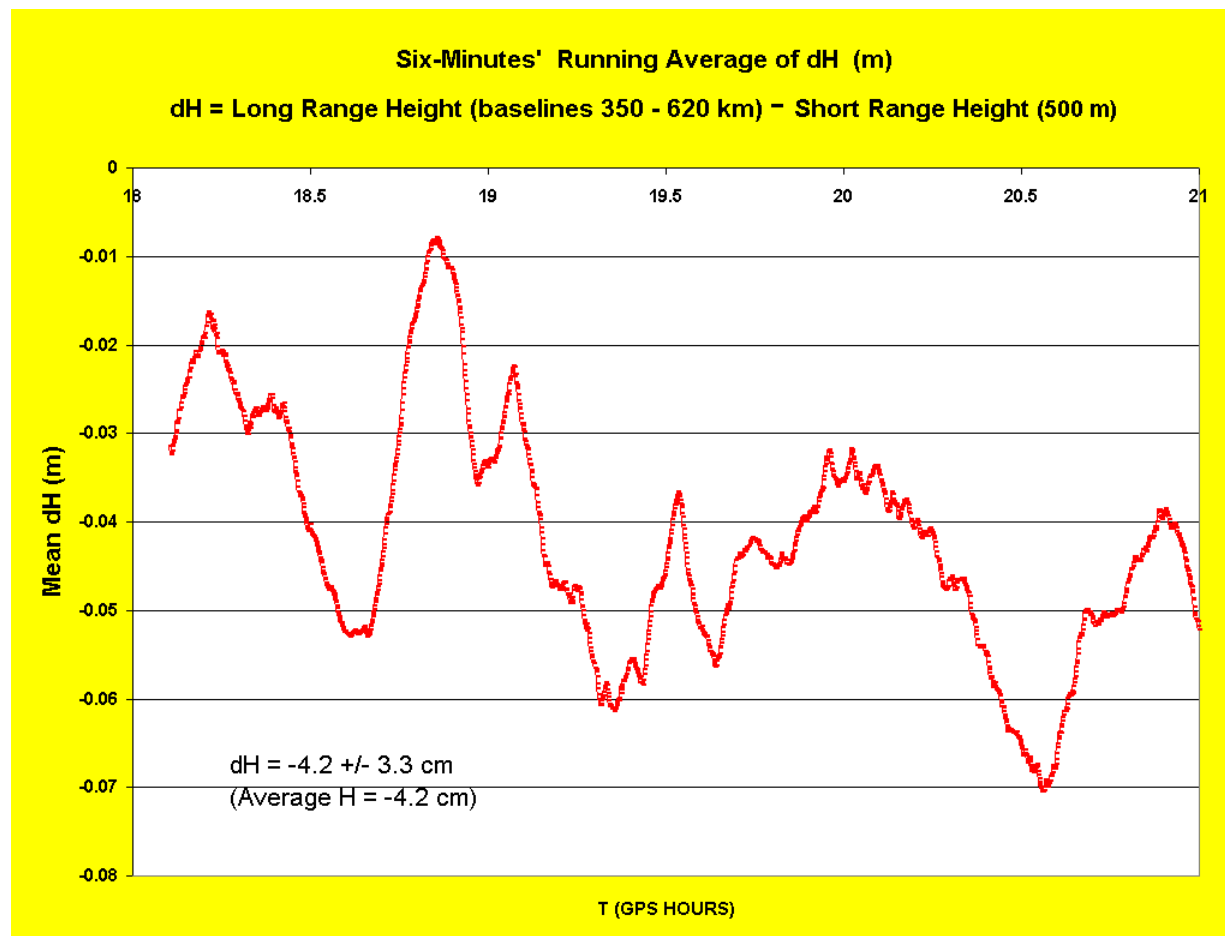
**LOCATION OF THE DISTANT C.O.R.S. SITES
(GAITHERSBURG "GAIT", ASHEVILLE "ASHV", ORONO
"ORON"), IN RELATION TO THE SITE AT DUCK.**



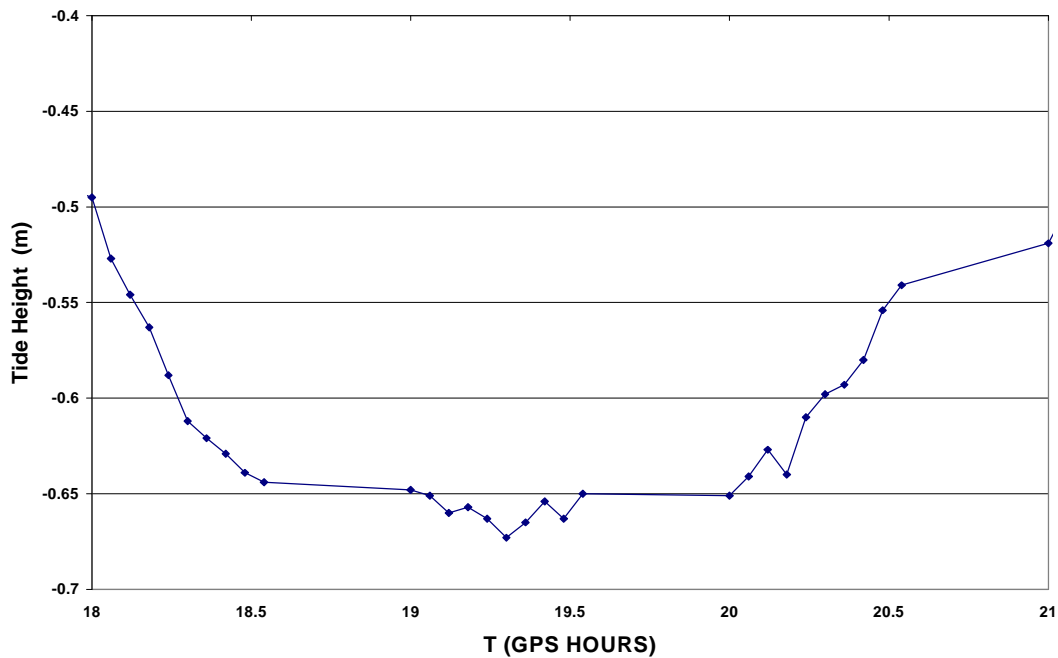
These distant sites were used as base stations in a long-range kinematic solution for the buoy, which was then compared to a short-range solution relative to a receiver on top of a building near the pier's landing. The differences in short and long-range positions were regarded as possible errors in the long-range results.

The plot of the difference between 6-minute mean sea heights, with both a short baseline (500 m), and with two long ones (352 km and 617 km), shows a constant offset of 4.2 cm (irrelevant to tsunami detection), plus a fluctuation of +/- 3.3 cm. So if a tsunami of 10 cm or more can be detected with the short-baseline kinematic solution, it should also be detected with the long-baseline solution.

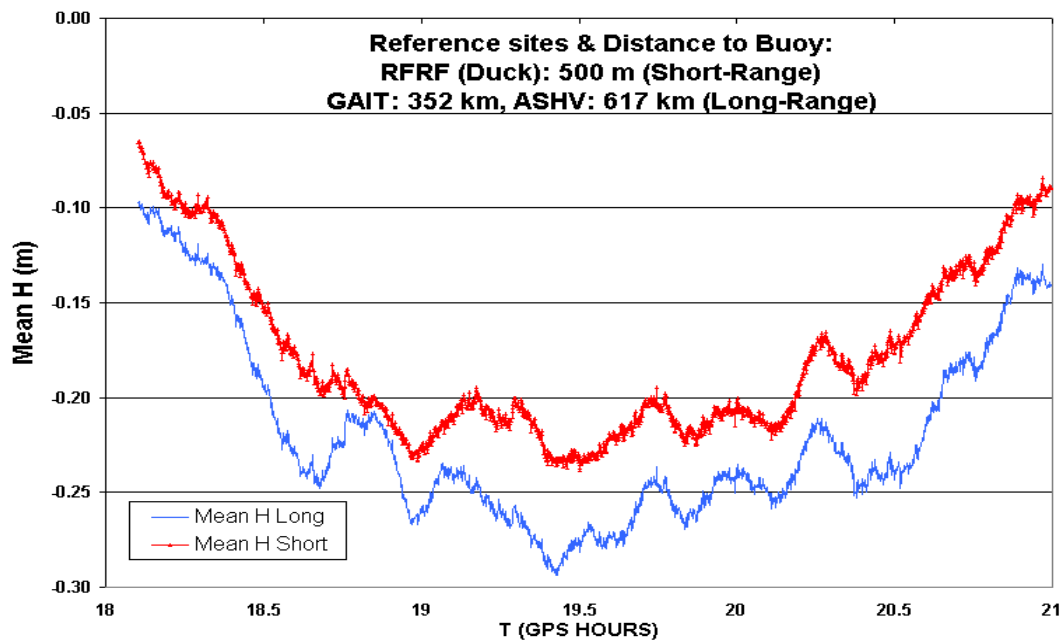
(The differences with the observed tide reflect, in part, errors in the earth-tide correction, and ocean loading.)



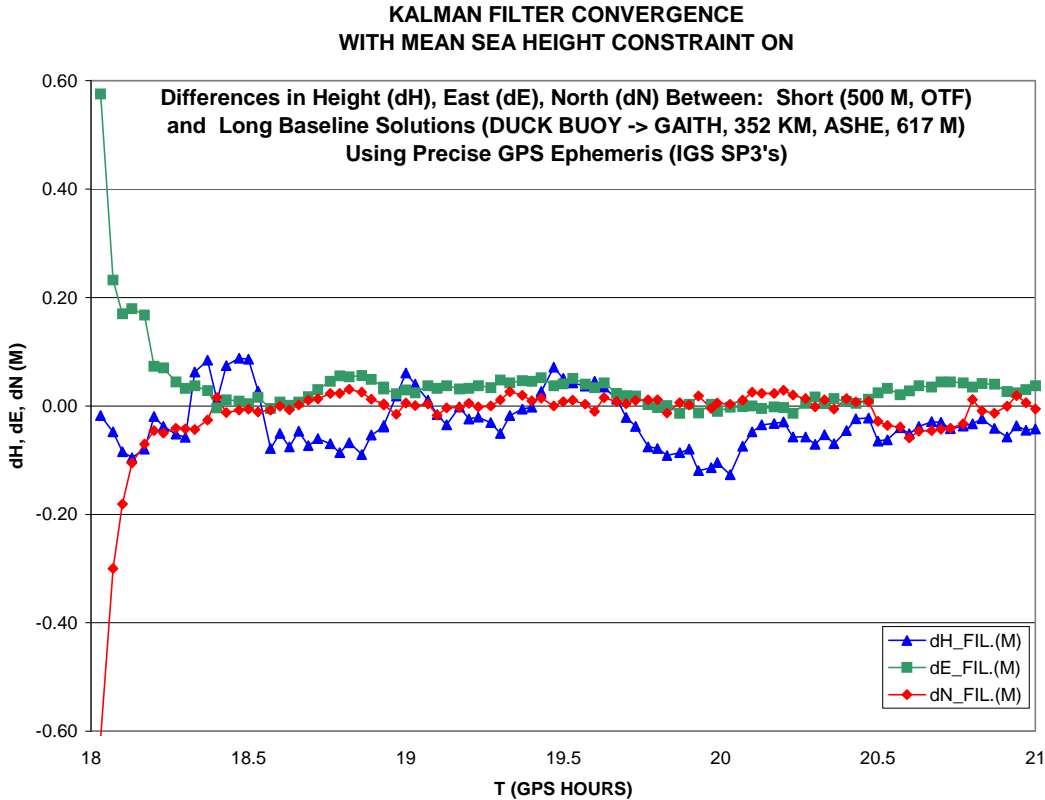
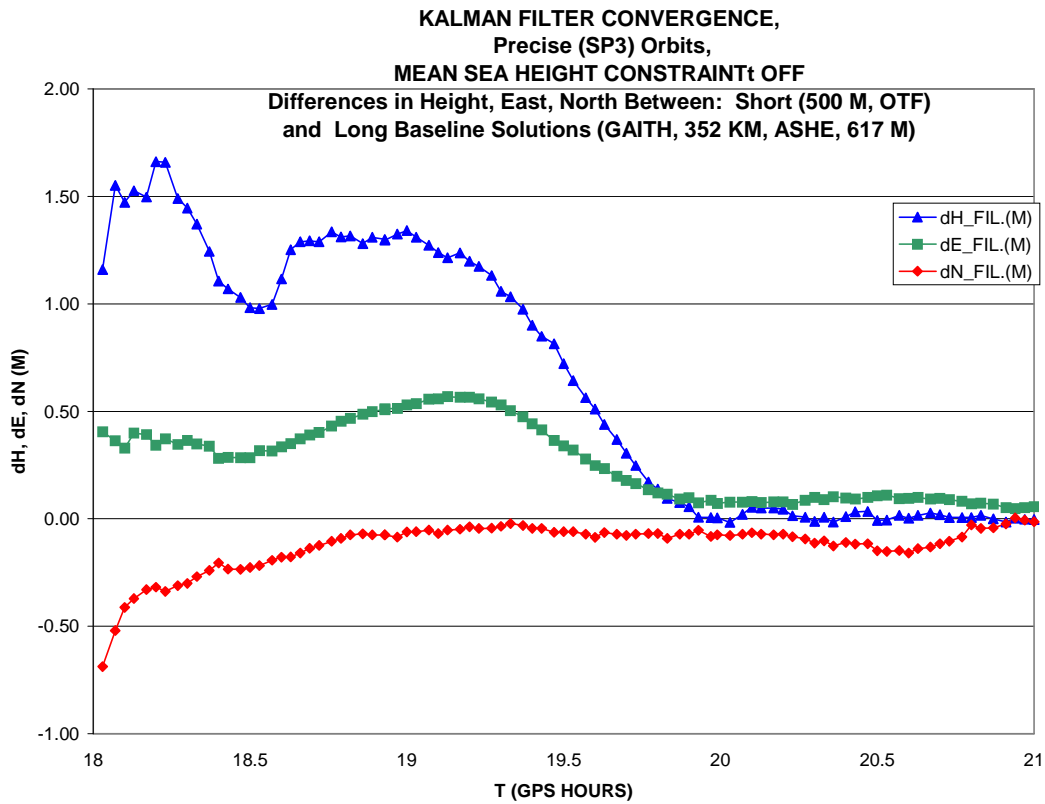
**Ocean Tide Observed at the Duck Pier Tide Gauge
(meters to the NGVD Datum)**



**Change in Mean Height of BUOY, Long & Short-Range Solutions,
(Vertical Datum: Height (Short Range) at 18:00 Hs)**



These results have been obtained by post-processing: filtering and smoothing. The following plots illustrate the convergence of the Kalman filter (as in a real time solution), and the improvement brought about by the use of the mean sea height H variability constraint described earlier on.



KALMAN FILTER CONVERGENCE
****** Adjusted Broadcast Orbits******
MEAN SEAH HEIGHT CONSTRAINT ON

