

# Characterization of GPS Carrier Phase Multipath and Mitigation through the use of Multiple Antennas

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# WHY IS CARRIER PHASE MULTIPATH IMPORTANT?

- It is not reduced through differential processing (highly localized)
- It can reach magnitudes of 0.25 of a wavelength
- It generally causes a systematic error in the measurements
- It can affect the accuracy of positioning and attitude determination
- It can significantly affect the ability to resolve integer ambiguities

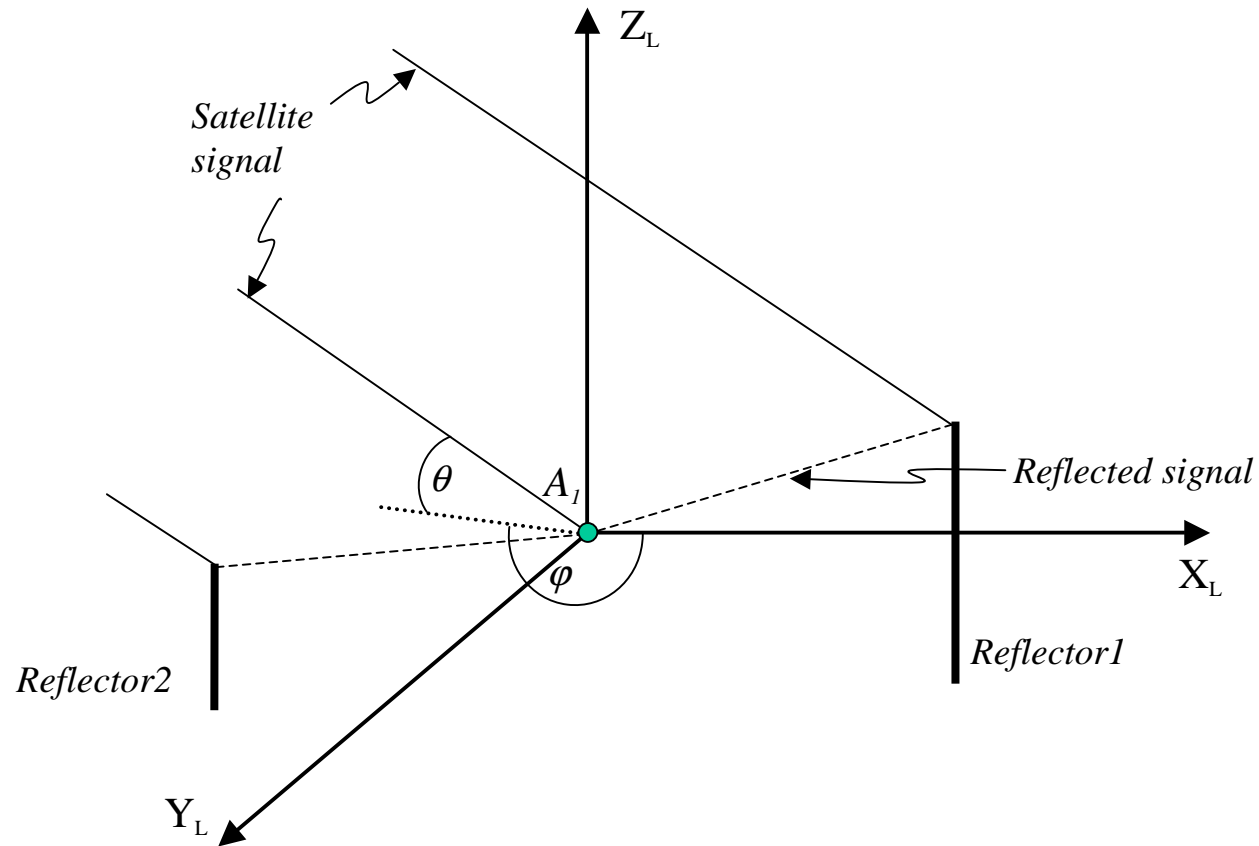
# OUTLINE

- Objective
- Multipath parameters
- Multipath from antenna-reflector geometry
- Simulation description
- Simulation results
- Multipath mitigation through multi-antennas

# OBJECTIVES OF CARRIER PHASE MULTIPATH CHARACTERIZATION

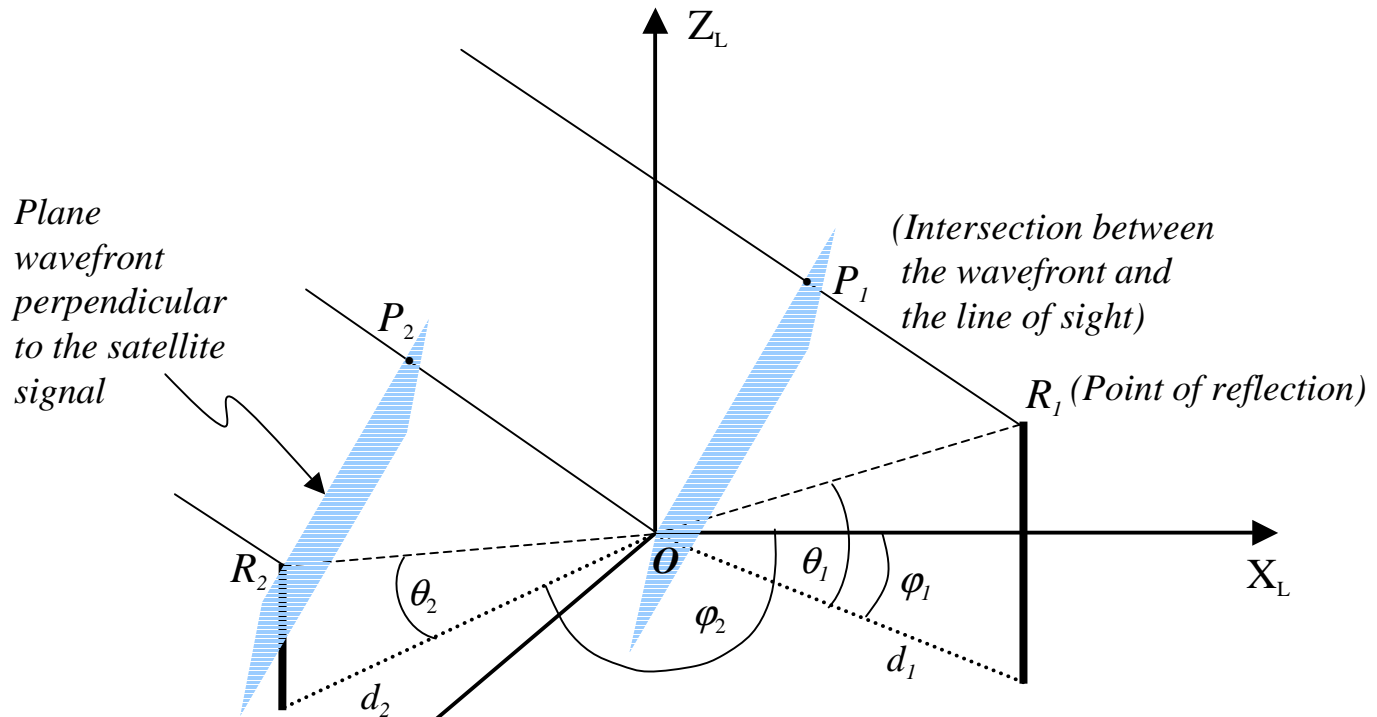
- Relate carrier phase multipath
  - amplitude, phase, phase rate
- With
  - antenna-reflector geometry
  - satellite dynamics
  - signal frequency
  - SNR
- In a
  - multi-antenna system
  - multi-reflector environment
- Using
  - mathematical models
  - simulation models
- Mitigate
  - Using multiple antennas

# MULTIPATH FROM GEOMETRY (1/3)



- $\theta$  : elevation of direct signal
- $\phi$  : azimuth of direct signal

# MULTIPATH FROM GEOMETRY (2/3)



- $\theta_k$  : elevation of reflected signal
- $\phi_k$  : azimuth of reflected signal
- $d_k$  : distance between antenna and reflector in the horizontal plane
- path delay :  $P_1R_1+R_1O$  (case 1) and  $R_2O-P_2O$  (case 2)

# MULTIPATH FROM GEOMETRY (3/3)

- Differential path delay

$$a_k = c * \tau_k = d_k \left( \frac{1}{\cos \theta_k} - \tan \theta_k \sin \theta - \cos \theta \cos(\varphi - \varphi_k) \right)$$

- $c$  is the velocity of light

- Reflected signal relative phase

$$\gamma_k = \frac{2\pi a_k}{\lambda_L}$$

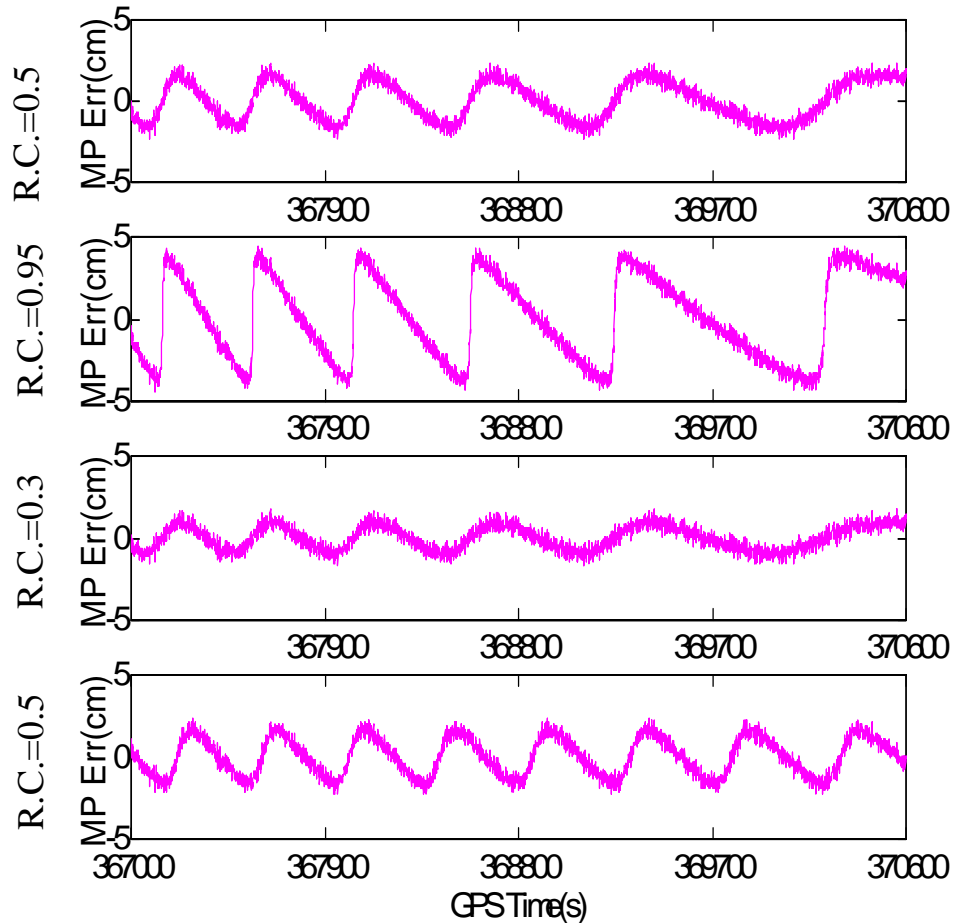
- Path delay, phase and phase rate depend upon
  - antenna-reflector distance
  - antenna-reflector geometry
  - satellite line-of-sight
  - signal wavelength

# MULTIPATH SIMULATION SOFTWARE

- Carrier phase multipath simulation and mitigation
- Inputs
  - reflector parameters
  - antenna parameters
- Outputs
  - true carrier phase
  - measured carrier phase
  - estimated carrier phase
- Multi-antenna configuration
- Multi-reflector environment
- Satellite trajectory from stored ephemeris

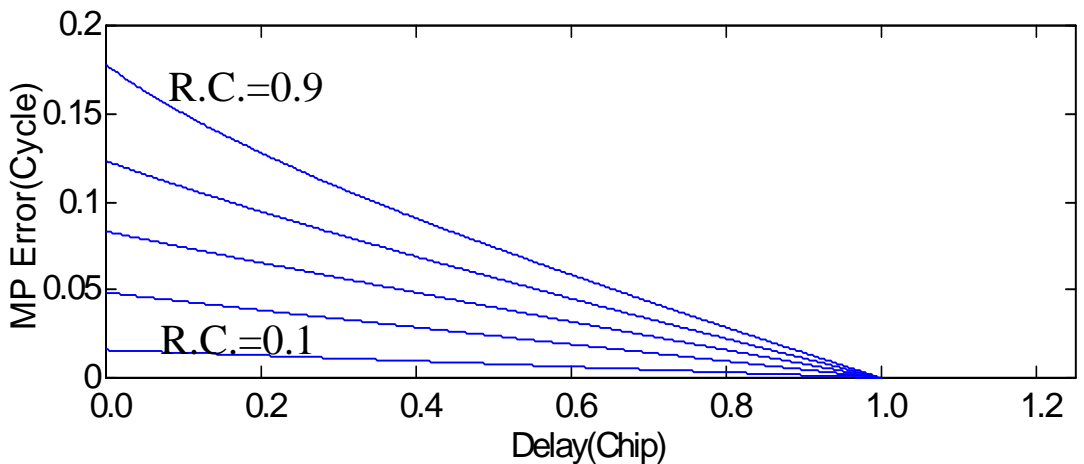
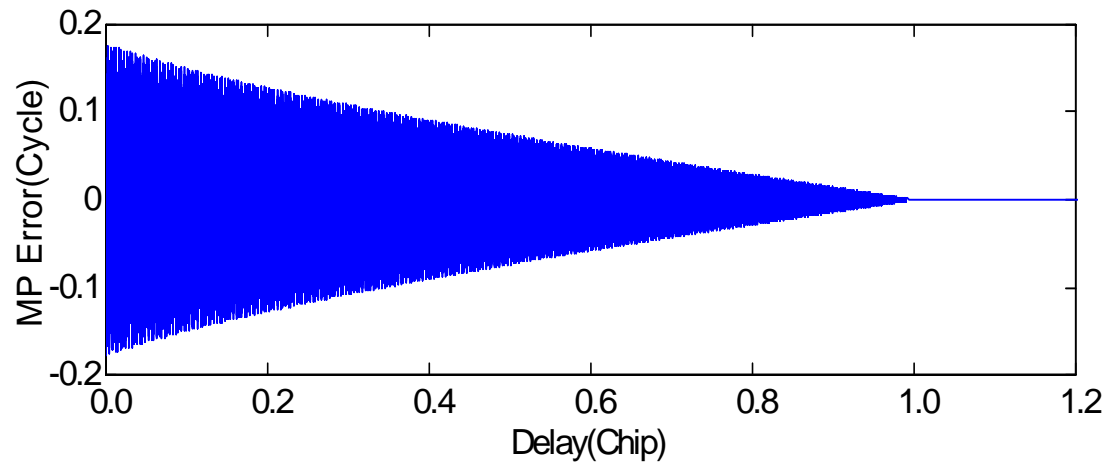
# MULTIPATH VS. REFLECTION COEFFICIENT

- Reflection Coefficient (R.C.): 0.5, 0.95, 0.3 and 0.5
- Antenna-reflector distance = 5m
- In the last case reflector location altered
- Error sinusoidal for low R.C.
- Error inverted sawtooth for high R.C.
- Highly dependent on antenna-reflector geometry



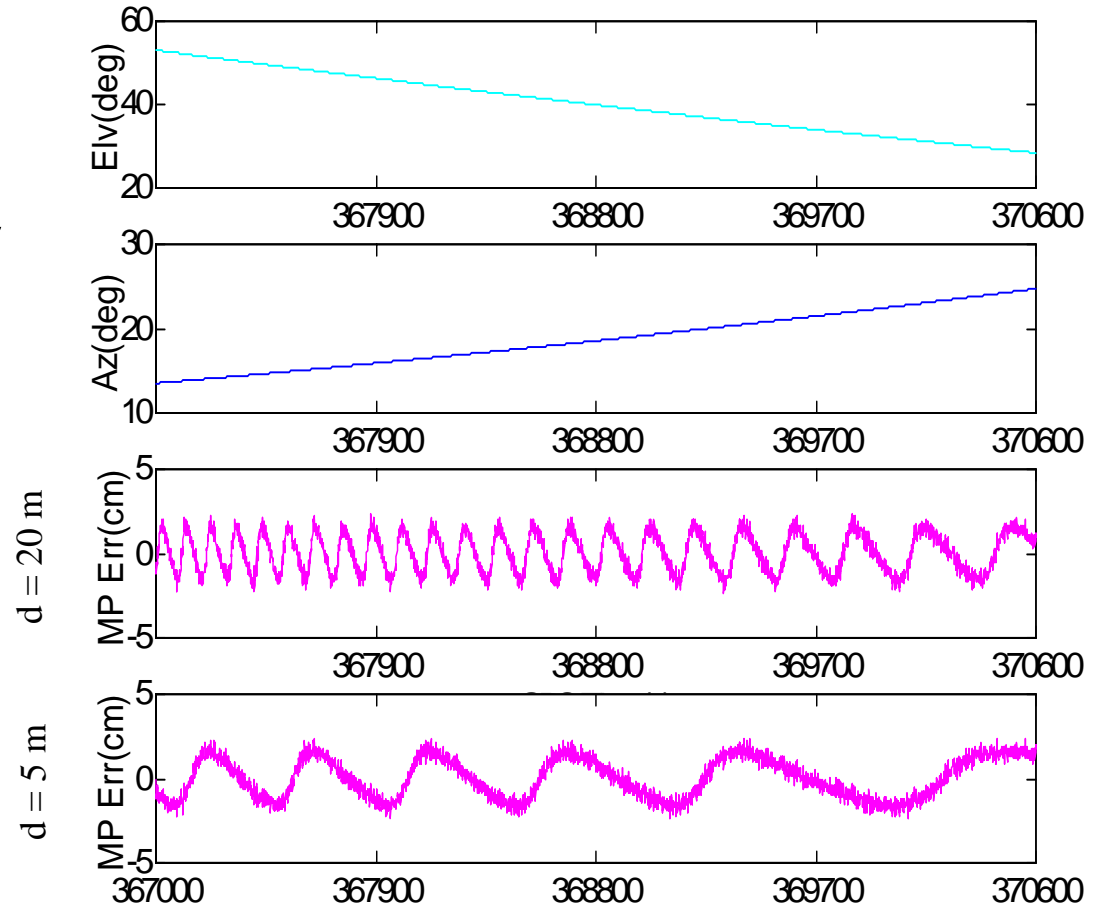
# MULTIPATH VS. DISTANCE

- Amplitude inversely proportional to distance
- Zero mean
- Envelope size proportional to R.C.
- Affected by path delay up to one chip



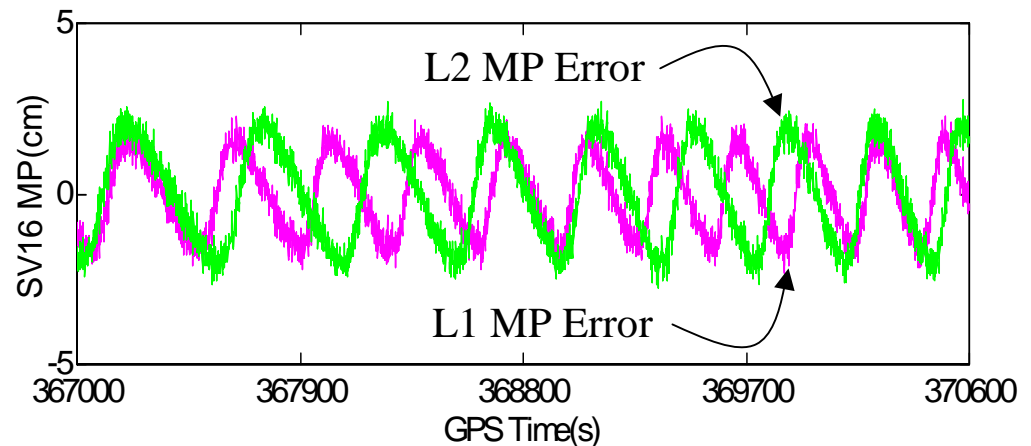
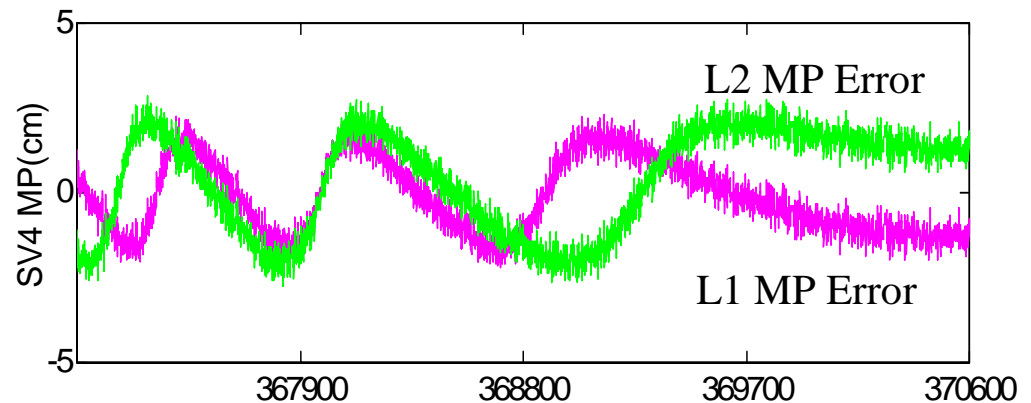
# MULTIPATH VS. SATELLITE DYNAMICS

- Reflector at 20 m and 5 m from the antenna
- Change in multipath due to satellite dynamics only
- Typical for stationary user



# MULTIPATH VS. SIGNAL FREQUENCY

- Errors in L1 and L2 carrier due to multiple reflectors for satellites 4 and 16
- L1 multipath has higher frequency
- L1 and L2 have the same phase error amplitude
- Highly dependent on antenna-reflector and line-of-sight vectors



# MULTIPATH SPATIAL CORRELATION (1/2)

- Multipath phase at an antenna is related to the phase in a close-by antenna by
  - their relative geometry
  - direction of the multipath signal

- For example, phase at Antenna 1

$$\gamma_1 = \gamma_0 + \frac{2\pi a_{01} \cos(\varphi_1 - \phi_{01})}{\lambda \cos \theta_1}$$

where

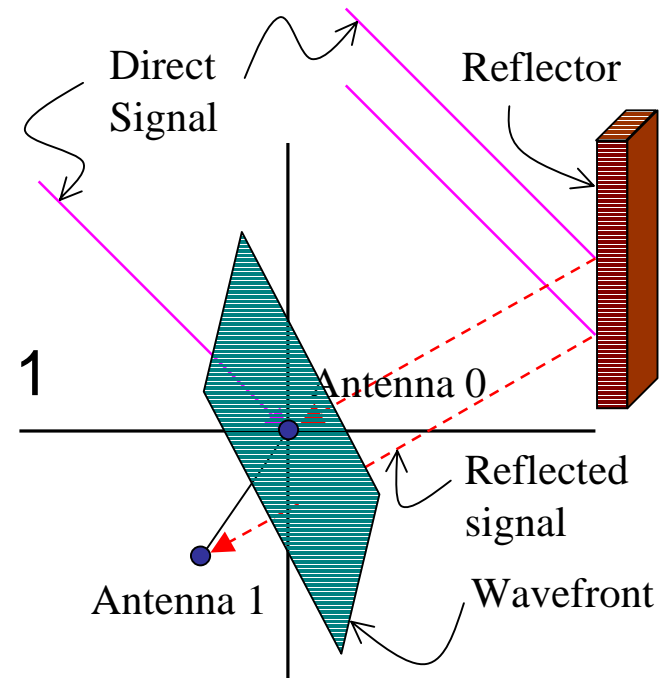
$\gamma_0$  is multipath phase at Antenna 0

$a_{01}$  is distance between 0 and 1

$\varphi_1$  is azimuth of multipath signal

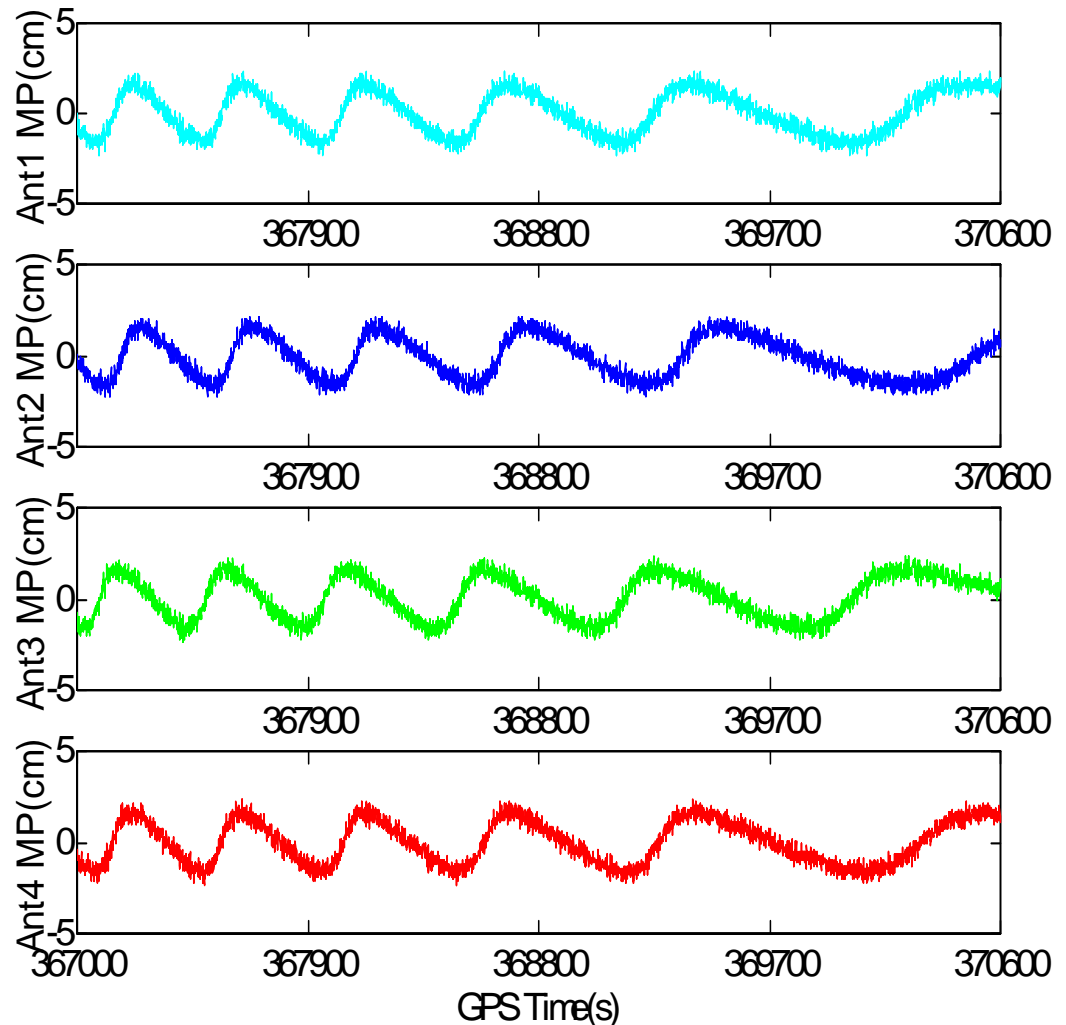
$\phi_{01}$  is azimuth of vector 0-1 and

$\theta_1$  is elevation of multipath signal



# MULTIPATH SPATIAL CORRELATION (2/2)

- Antennas 5 to 10 cm away from each other
- Reflector distance approximately 5 m
- Spatially correlated
- Spatial correlation may be used for mitigation
- Larger size reflector causes higher correlation



# MULTIPATH PHASE RATE (1/2)

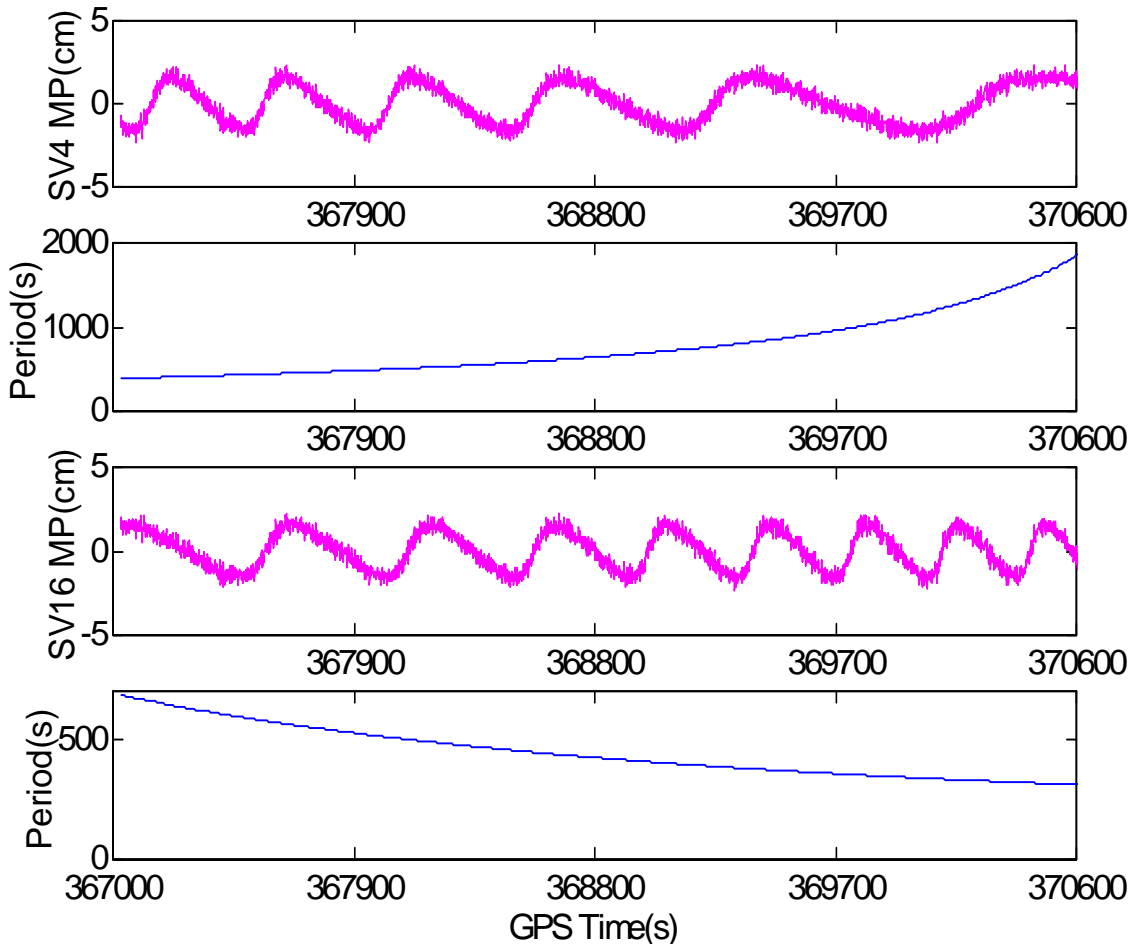
- Multipath phase rate:

$$\frac{\delta\gamma}{\delta t} = \frac{2\pi d_1}{\lambda_L} \left( \begin{aligned} & \left\{ \sin(\theta) \cos(\varphi - \varphi_1) - \cos(\theta) \tan(\theta_1) \right\} \frac{\delta\theta}{\delta t} \\ & + \left\{ \cos(\theta) \sin(\varphi - \varphi_1) \right\} \frac{\delta\varphi}{\delta t} \end{aligned} \right)$$

- directly proportional to the antenna-reflector distance
- inversely proportional to the carrier wavelength
- directly proportional to the satellite elevation rate
- directly proportional to the satellite azimuth rate
- dependent upon antenna-reflector vector
- dependent upon line-of-sight vector

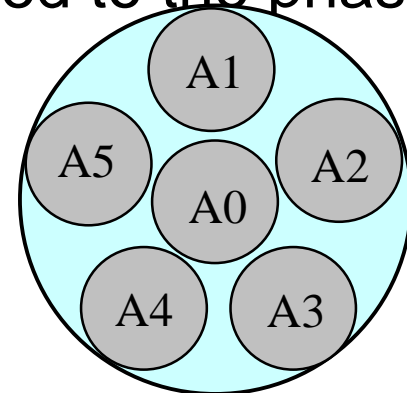
# MULTIPATH PHASE RATE (2/2)

- Multipath error for satellites 4 and 16 and their estimated periods
- Phase rate estimated from antenna-reflector geometry



# MULTIPATH MITIGATION TECHNIQUE (1/4)

- A virtual reflector with time varying parameters is equivalent to all the reflectors in the system
- If normalized reflection coefficient and multipath phase of the virtual reflector are estimated, total multipath error can be computed
- Normalized reflection coefficient is almost the same for all antennas in a closely-spaced antenna assembly
- Multipath phase at an antenna is related to the phase at another antenna by
  - their relative geometry
  - direction of the multipath signal



# MULTIPATH MITIGATION TECHNIQUE (2/4)

- For example, phase at Antenna 1

$$\gamma_1 = \gamma_0 + \frac{2\pi a_{01} \cos(\phi_0 - \phi_{01})}{\lambda \cos \theta_0}$$

where

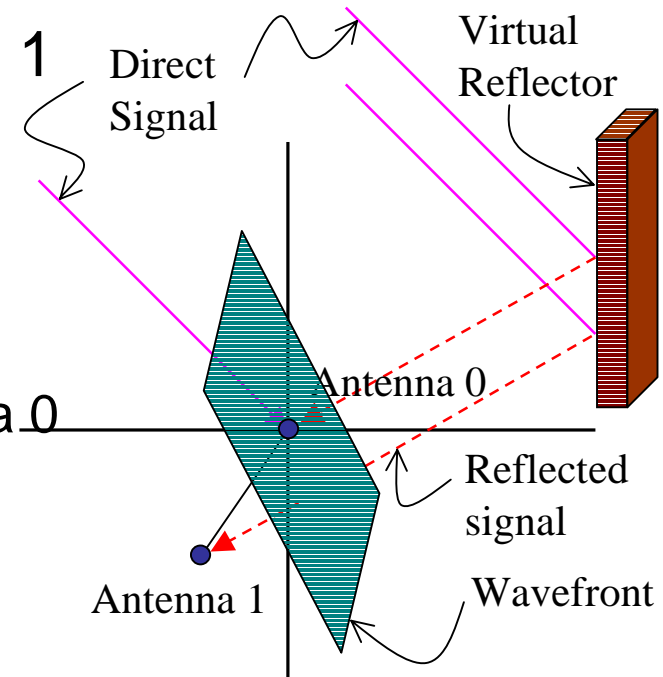
$\gamma_0$  is multipath phase at Antenna 0

$a_{01}$  is distance between 0 and 1

$\phi_0$  is azimuth of multipath signal

$\phi_{01}$  is azimuth of vector 0-1 and

$\theta_0$  is elevation of multipath signal



# MULTIPATH MITIGATION TECHNIQUE (3/4)

- A Kalman filter can be developed to estimate the multipath signal parameters at the reference antenna (generally the center antenna in the assembly)
- State vector:

$$\begin{bmatrix} \alpha \\ \gamma_0 \\ \theta_0 \\ \varphi_0 \end{bmatrix} = \begin{bmatrix} \text{Reflection coefficient} \\ \text{Multipath signal phase} \\ \text{Multipath signal elevation} \\ \text{Multipath signal azimuth} \end{bmatrix}$$

- Though the parameters are for a single multipath signal, the filter estimates the composite multipath due to all reflectors in the environment

# MULTIPATH MITIGATION TECHNIQUE (4/4)

- Single difference carrier phase between closely-spaced antennas (6-7cm apart):

$$\Delta\Psi_{0,i} = \Delta\rho_{0,i} + \Delta N_{0,i}\lambda + c\Delta t_{0,i} + \varepsilon_{\phi 0,i} + \varepsilon_{MP0,i}$$

- In this,
  - range difference due to spatial separation ( $\Delta\rho$ ) can be removed
  - phase due to carrier cycle ambiguity ( $\Delta N\lambda$ ) can be removed
  - clock bias error ( $c\Delta t$ ) negligible if driven by a common clock
- The resultant residual contains difference of multipath ( $\varepsilon_{MP}$ ) and carrier phase noise ( $\varepsilon_{\phi}$ )
- Measurement vector consists of single difference carrier phase residual  $[\Delta\Psi_{0,1}, \dots, \Delta\Psi_{0,m-1}]^T$

# FIELD TEST DESCRIPTION

- A special antenna array was assembled with six antennas mounted close together (aprox. 6 cm apart) on an aluminum plate
- Three NovAtel BeeLine™ receivers were used for data collection. The BeeLine™ is an 8+8 channel (L1-L1) receiver
- All receivers were driven by a common clock
- Data was collected for several sessions on the roof of the Engineering building at the University of Calgary

# ANTENNA ASSEMBLY



# TEST SET-UP: PART 1



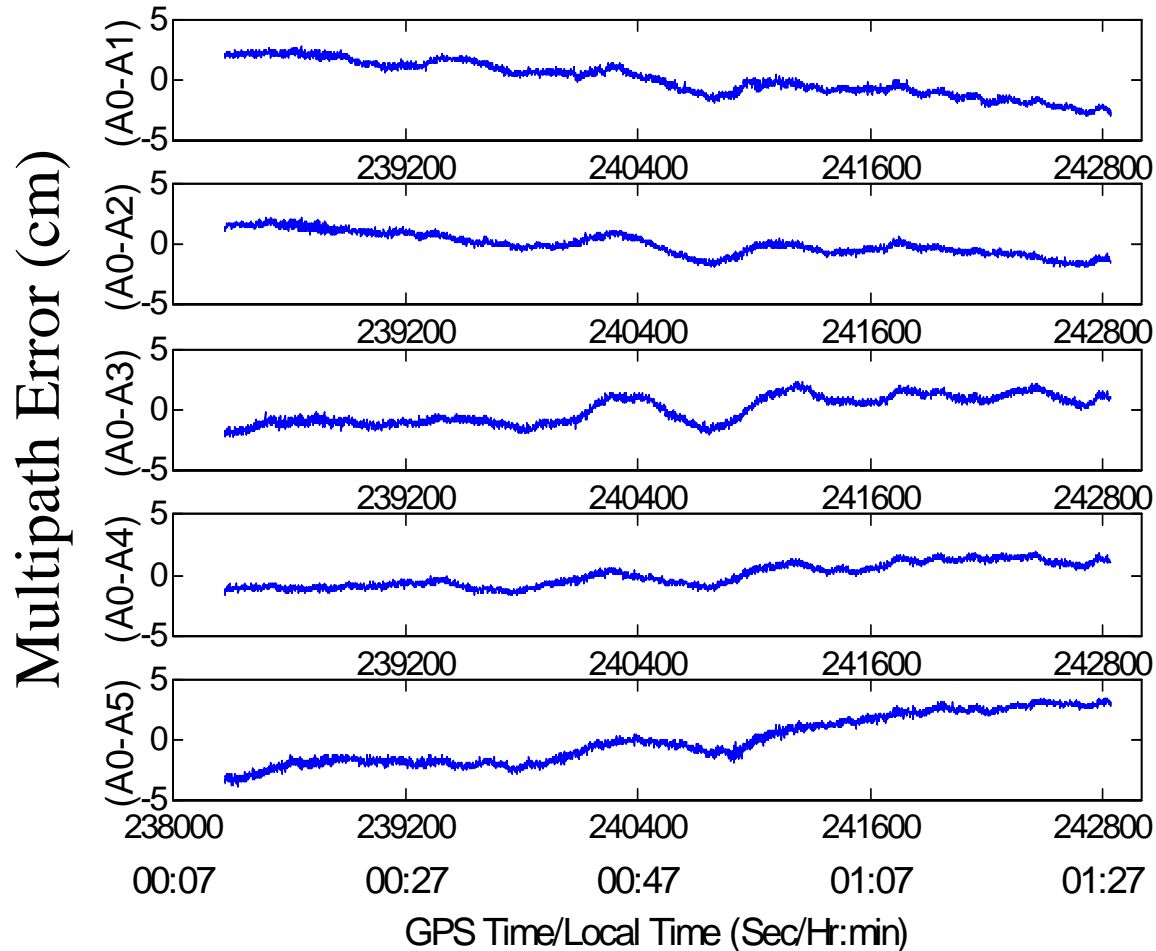
# TEST PROCEDURE

- Establish the relative geometry of the antennas in the array
- Form single difference carrier phase residuals and input to the filter
- Estimate parameters of the virtual reflector by the filter
- Compute multipath error at each antenna due to virtual reflector and remove it from measurement
- Form single difference carrier phase residuals with corrected measurements
- Compute RMS values of single difference carrier phase residuals before and after multipath correction

# SINGLE DIFFERENCE FOR SV 21

## No Multipath Correction

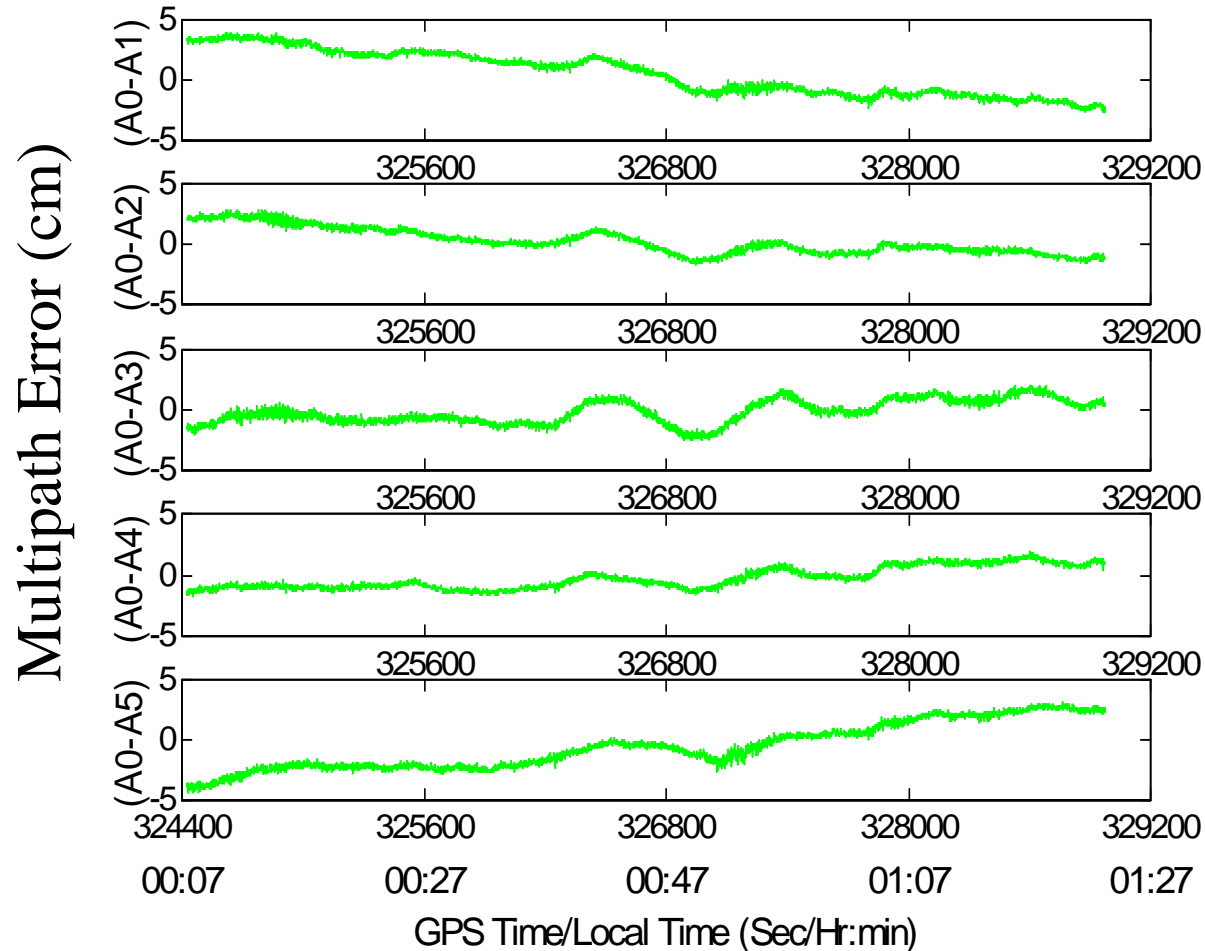
- Data from August 25, 1998 (RMS = 1.3 cm)



# SINGLE DIFFERENCE FOR SV 21

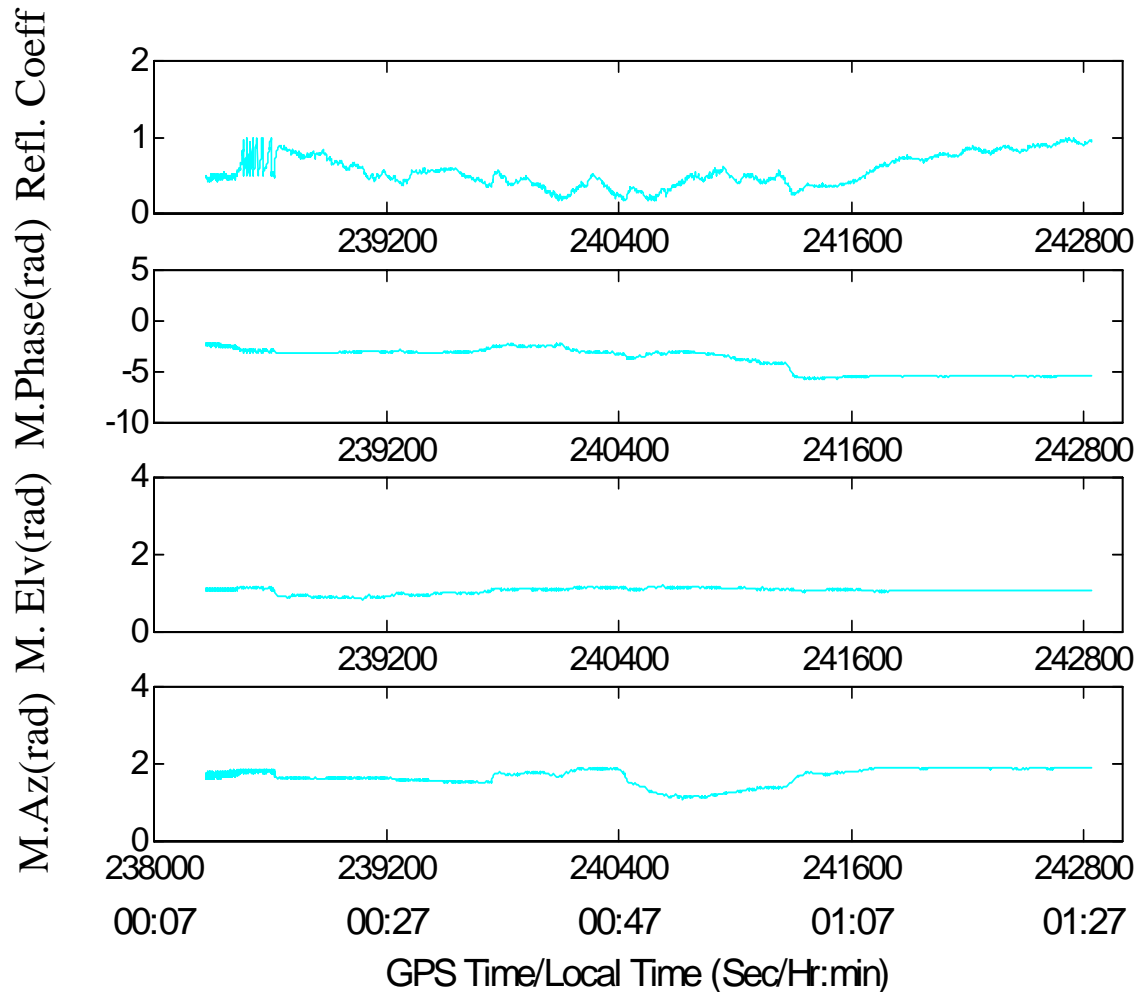
## No Multipath Correction

- Data from August 26 (RMS = 1.3 cm, correlation = 84%)



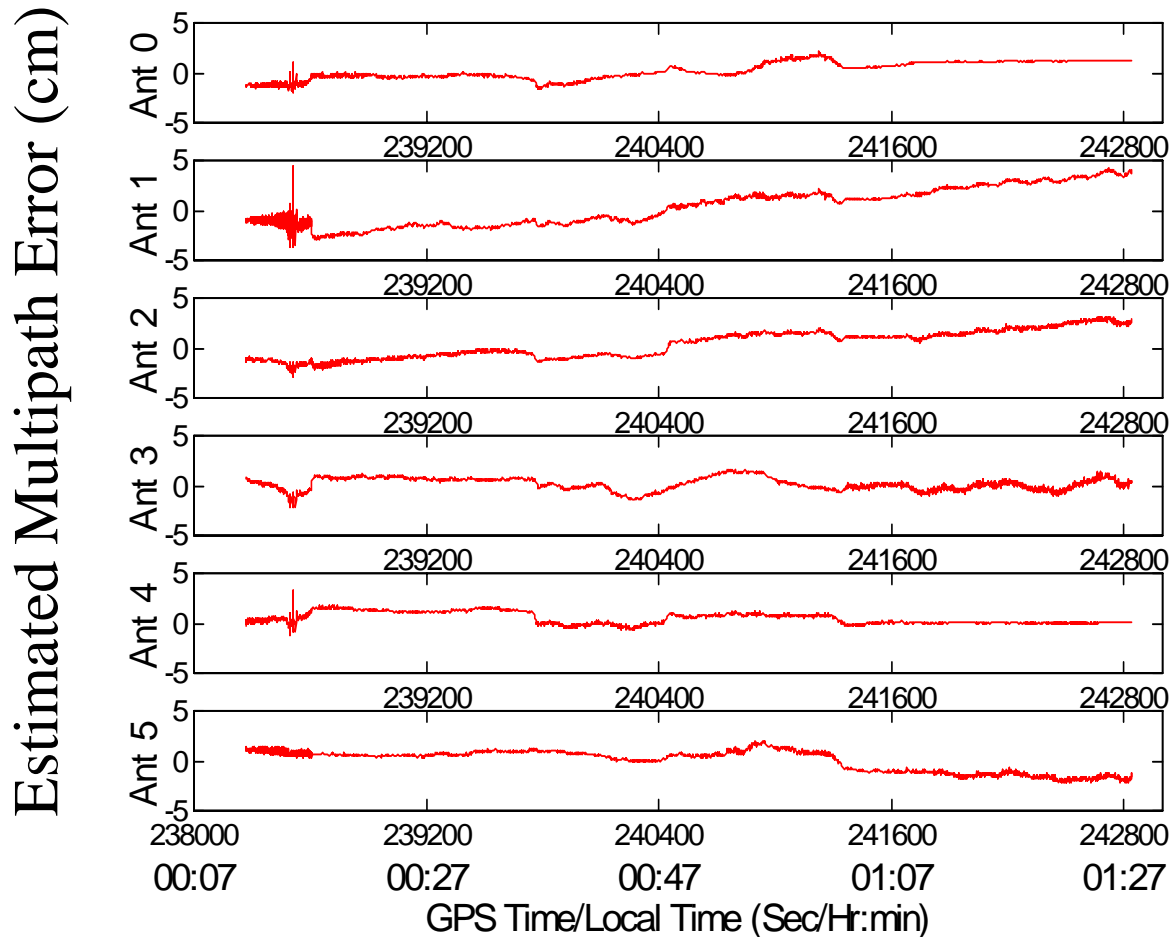
# ESTIMATED PARAMETERS OF THE VIRTUAL REFLECTOR FOR SV 21

- Data from August 25



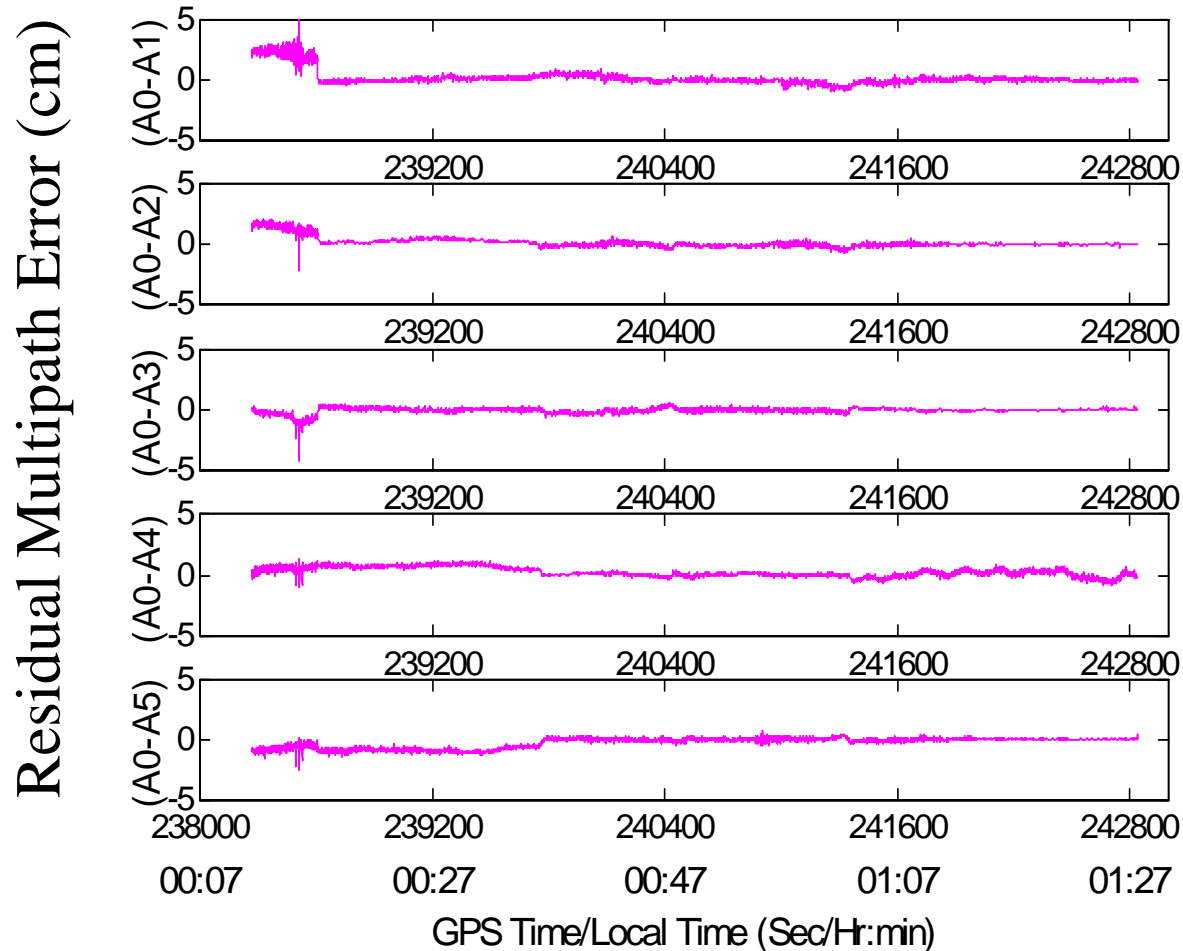
# ESTIMATED MULTIPATH ERROR FOR SV 21

- Confirms that this technique estimates composite multipath



# SINGLE DIFFERENCE FOR SV 21 After Multipath Correction

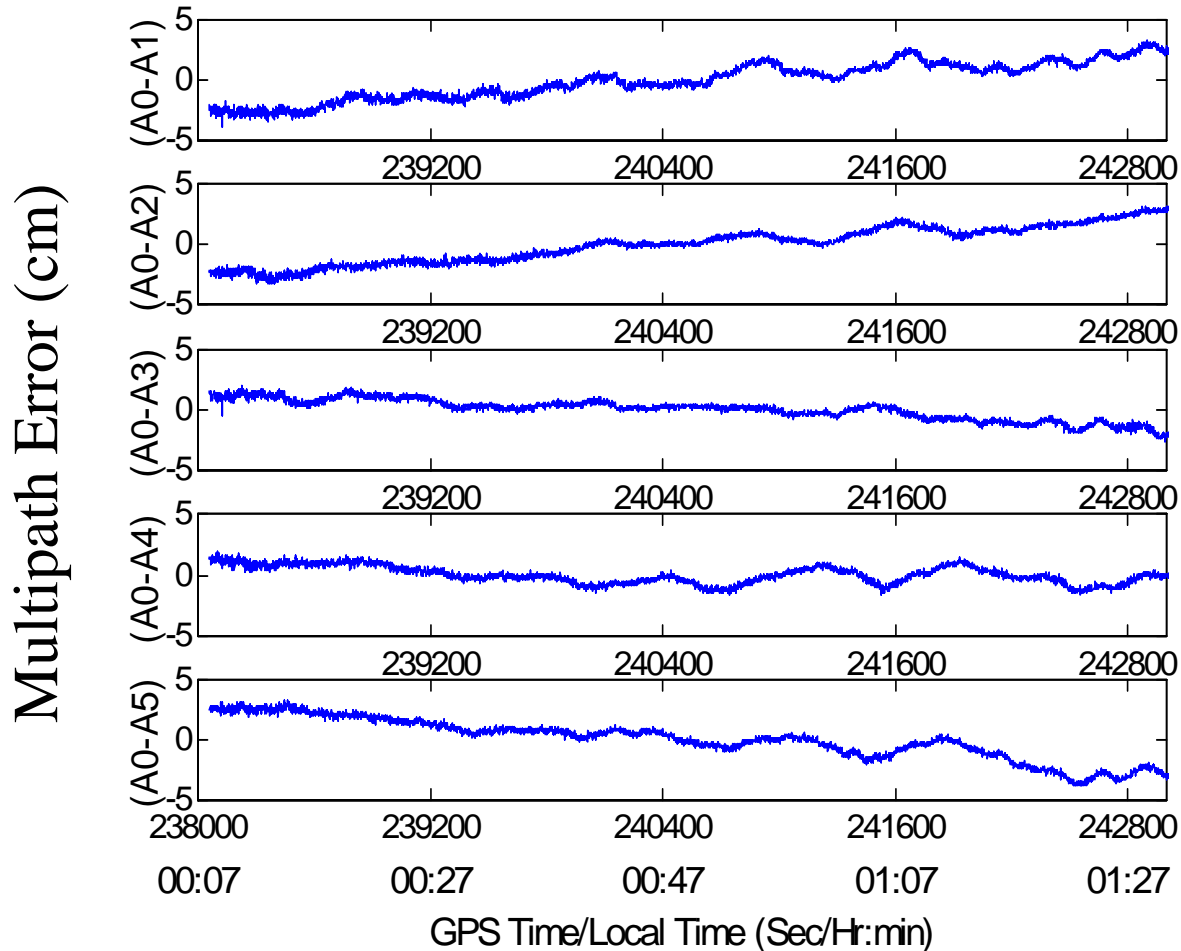
- RMS = 0.3 cm and improvement = 76.2%



# SINGLE DIFFERENCE FOR SV 31

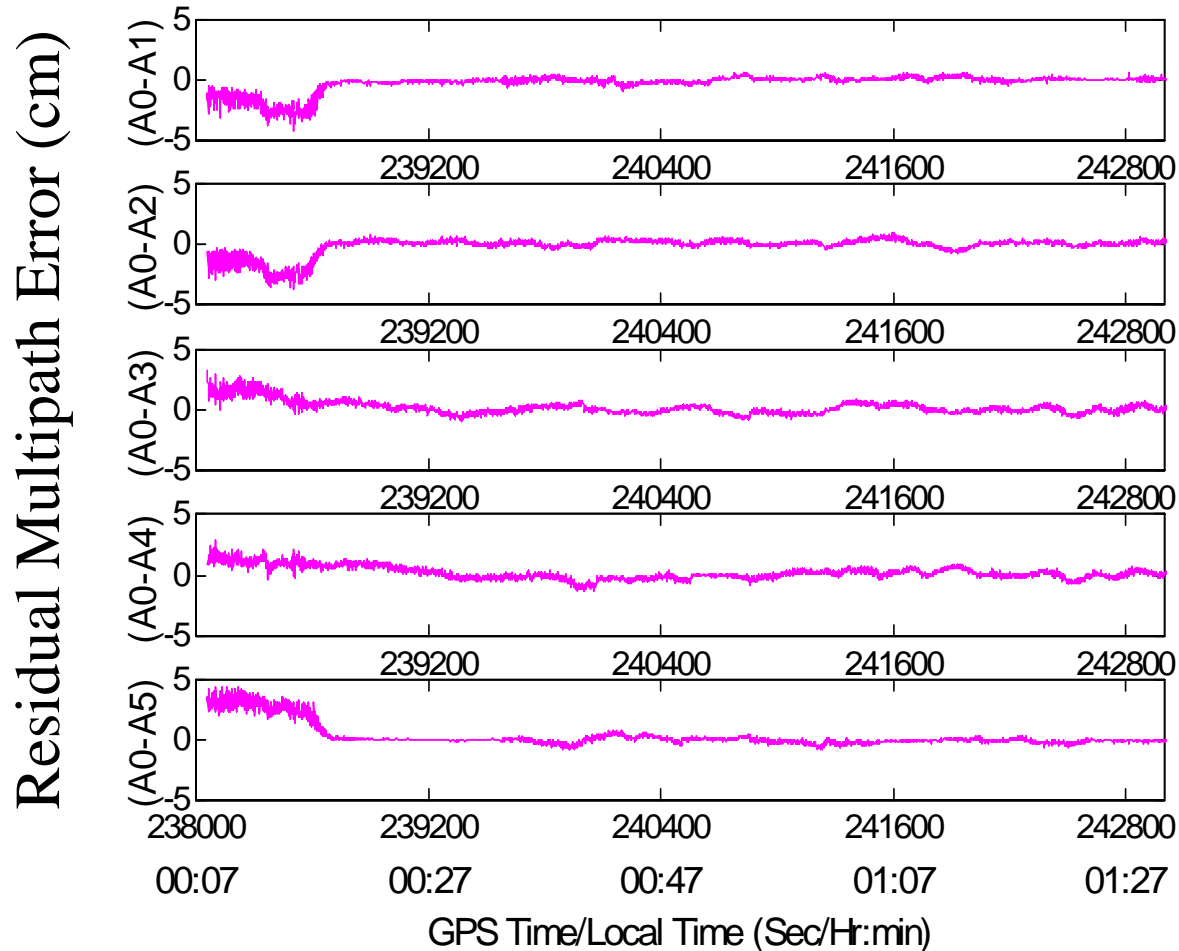
## No Multipath Correction

- Data from August 25, 1998 (RMS = 1.3 cm)



# SINGLE DIFFERENCE FOR SV 31 After Multipath Correction

- RMS = 0.3 cm and improvement = 78.5%

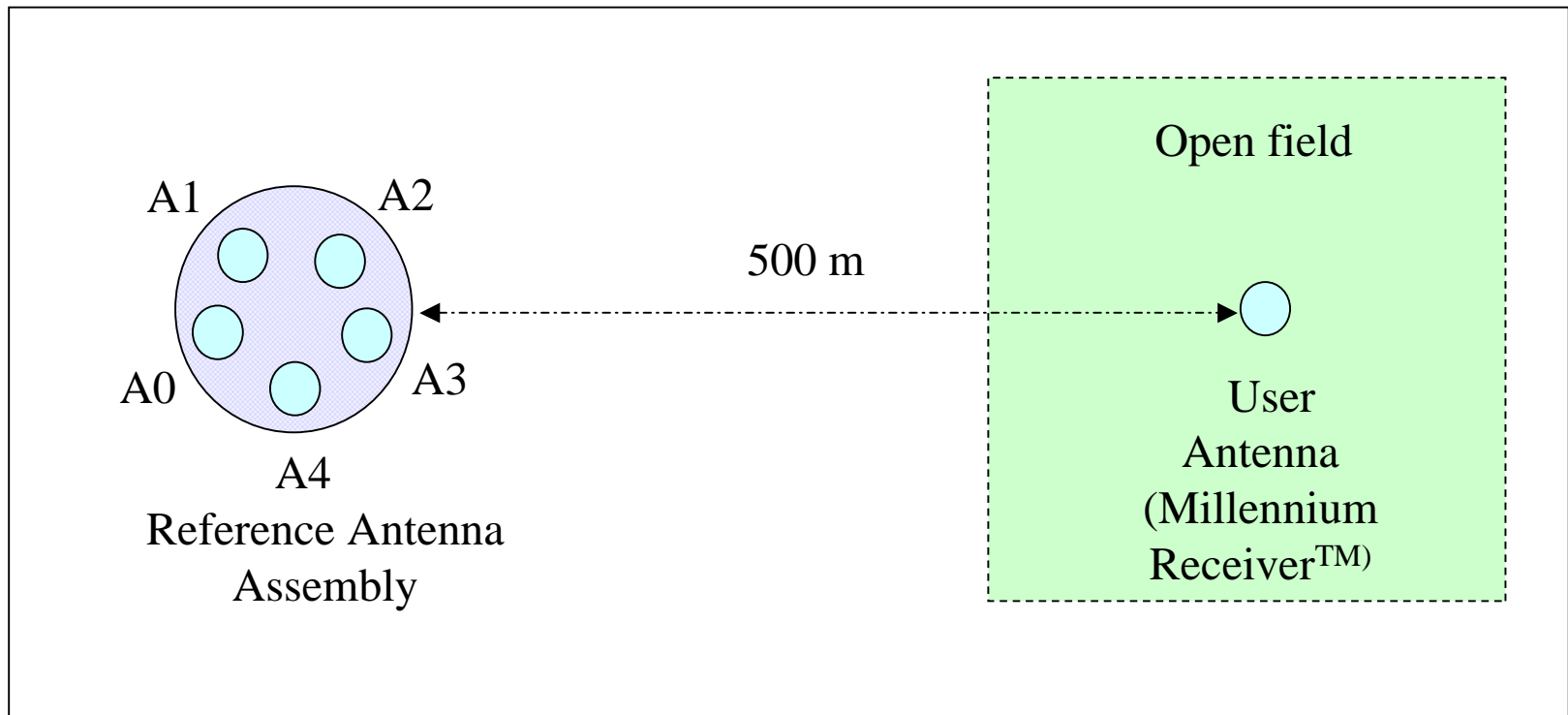


# MEASUREMENT DOMAIN TEST RESULTS

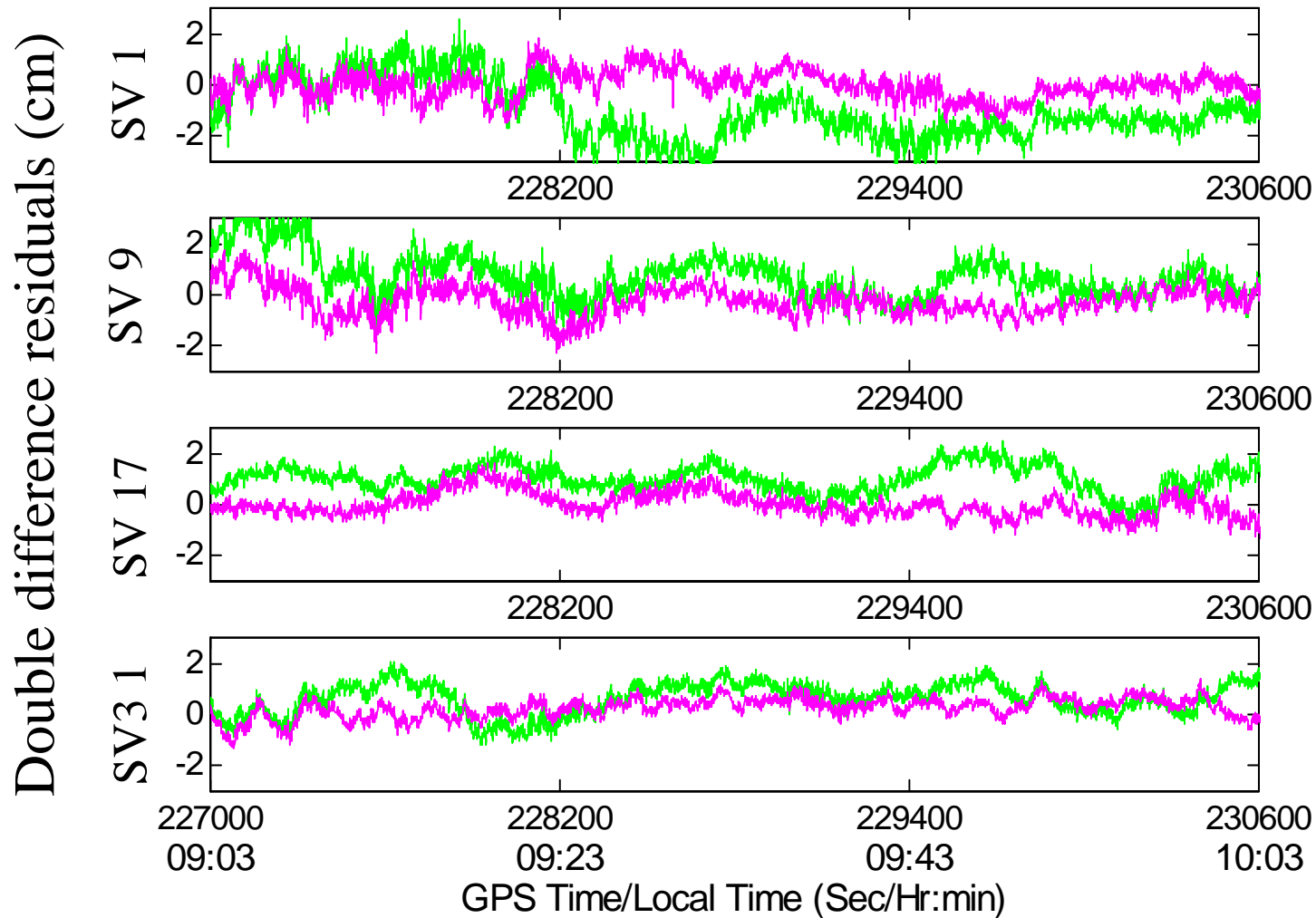
- RMS value of single difference carrier phase residual before and after multipath correction
- Average improvement = 73%

SV ID	August 25					August 26				
	Before correction		After Correction		Improve-ment	Before correction		After correction		Improve-ment
	Mean (cm)	RMS (cm)	Mean (cm)	RMS (cm)	%	Mean (cm)	RMS (cm)	Mean (cm)	RMS (cm)	%
17	0.1	1.8	0.0	0.4	77.4	0.1	1.7	0.0	0.4	78.0
21	0.0	1.3	0.0	0.3	76.2	-0.1	1.3	-0.0	0.2	82.0
23	-0.2	1.7	0.3	0.6	65.3	-0.2	1.7	0.3	0.8	60.0
31	0.0	1.3	0.0	0.3	78.5	0.0	1.3	0.0	0.5	63.5

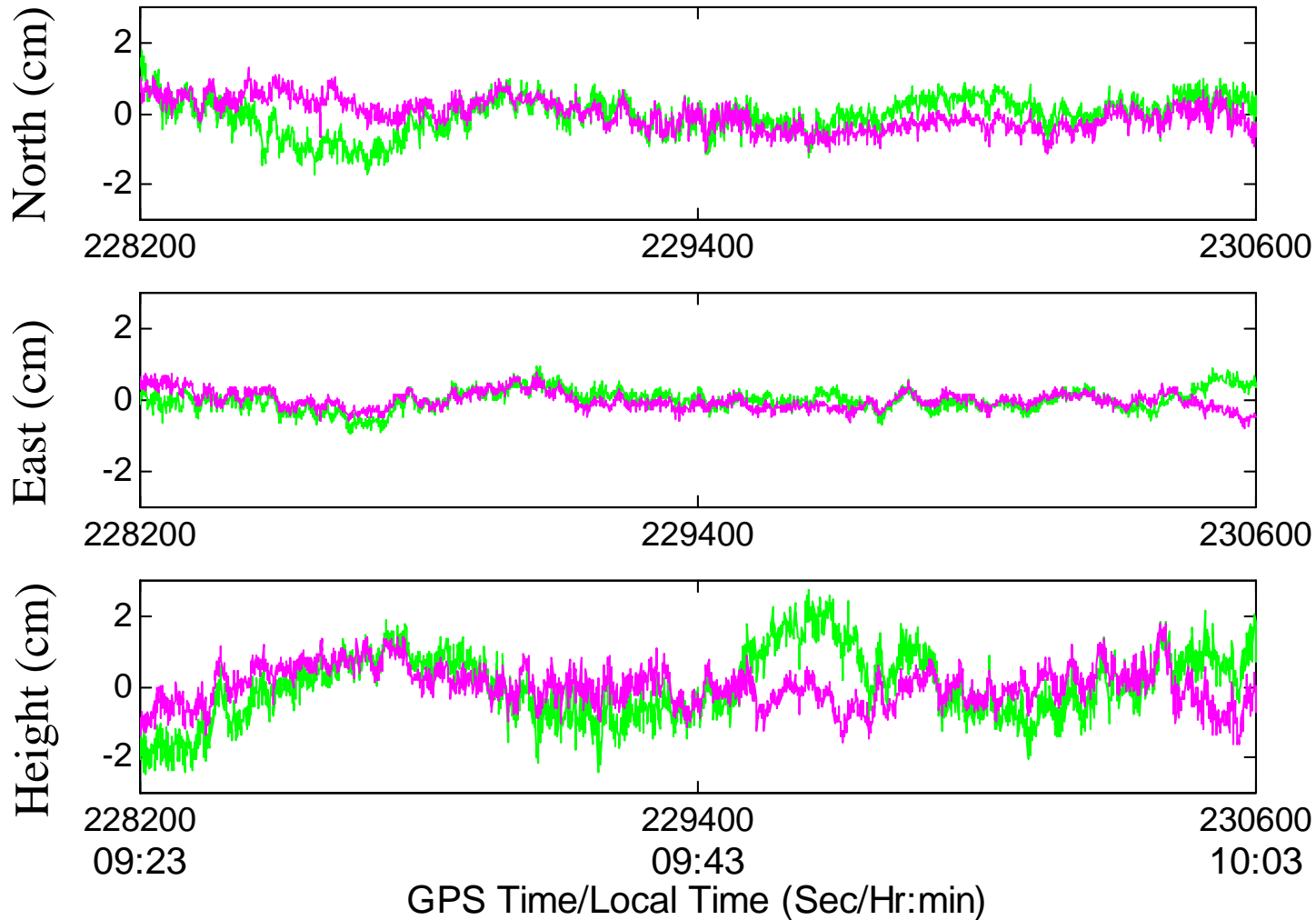
# DIFFERENTIAL TEST SET-UP: PART 2



# DD CARRIER PHASE RESIDUALS BETWEEN ANTENNA 1 AND THE USER



# USER POSITION ERROR



# TEST STATISTICS

## DD MEASUREMENT DOMAIN

SV No.	Elevation Angle	Antenna No.	Multipath $\sigma$ <i>before</i> <i>correction</i> (cm) [3]	Multipath $\sigma$ <i>after</i> <i>correction</i> (cm) [4]	Multipath Reduction ([3]-[4])/[3] (%)
9	13° - 27°	A0	0.80	0.49	38.91
		A1	0.88	0.64	27.40
		A2	0.45	0.48	-5.12
		A3	0.53	0.57	-7.16
		A4	0.75	0.45	40.11
31	23° - 35°	A0	0.55	0.26	52.54
		A1	0.58	0.57	1.20
		A2	0.84	0.55	34.75
		A3	0.84	0.32	61.90
		A4	0.58	0.48	17.02

# TEST STATISTICS

## POSITION DOMAIN

Antenna No.	3D position error due to multipath + noise $\sigma$ (cm) [4]		3D position error due to multipath only $\sigma$ (cm) [5]		Improvement in 3D position accuracy $(5[a]-5[b])/5[a]$ (%)
	<i>Before</i> [a]	<i>After</i> [b]	<i>Before</i> [a]	<i>After</i> [b]	
A0	1.20	0.77	0.97	0.30	69.34
A1	1.29	0.84	1.08	0.46	57.79
A2	1.01	1.08	0.72	0.87	-6.93
A3	1.19	1.04	0.96	0.76	20.47
A4	1.40	0.88	1.21	0.52	57.22

Nominal position error due to phase noise = PDOP\*standard deviation of phase noise

# SUMMARY

- Carrier phase multipath was characterized in terms of reflector, distance, frequency
- A method to mitigate the total effect of carrier phase multipath due to all sources was developed using multiple, closely-spaced antennas
- The technique was shown to be highly effective reducing the multipath error residuals up to 60% and the position error up to 70%
- This has applications to reference stations and for attitude determination (being investigated)

# REFERENCES AND WEB PAGE

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- Ray, J.K., M.E. Cannon and P. Fenton (2000), Code and Carrier Multipath Mitigation Using a Multi-Antenna System, IEEE Transactions on Aerospace and Electronic Systems, (in press).
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- Ray, J.K., and M.E. Cannon (1999), Characterization of Carrier Phase Multipath, Proceedings of the ION National Technical Meeting, San Diego, January 25-27, pp. 343-352.
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<http://www.geomatics.ucalgary.ca/GPSRes/GPSResIndex.html>