Expansion of holomorphic functions in Lamé polynomials

The following result is well known:
Let $f : D \to \mathbf{C}$ be holomorphic in the interior domain of an ellipse with foci at ± 1:

$$D = \{z \in \mathbf{C} : |z - 1| + |z + 1| < 2a\}.$$

The orthogonal expansion of f in Legendre polynomials

$$f(z) = \sum_{n=0}^{\infty} c_n P_n(z)$$

converges locally uniformly on D to f.

Is there a corresponding result for the expansion of holomorphic functions in Lamé polynomials?

The algebraic form of Lamé's differential is

$$E'' + \frac{1}{2}\left(\frac{1}{z-a} + \frac{1}{z-b} + \frac{1}{z-c}\right)E'$$
$$-\frac{\lambda + n(n+1)z}{4(z-a)(z-b)(z-c)}E = 0,$$

where $a < b < c$. For every even nonnegative integer n and every $m = 0, \ldots, \frac{n}{2}$ there exists a unique eigenvalue $\lambda = \lambda_n^m$ such that Lamé's equation admits a polynomial solution $E_n^m(z)$ which has m zeros in (a, b), $\frac{n}{2} - m$ zeros in (b, c) and no other zeros. Set

$$f_n^m(s,t) = E_n^m(s)E_n^m(t).$$

Then the system of polynomials f_n^m is orthogonal and complete over the rectangle $R = (a, b) \times (b, c)$ with respect to the inner product

$$\int_a^b \int_b^c \frac{t-s}{w(s)w(t)} f(s,t)\overline{g(s,t)}\,dt\,ds,$$

$$w(t) = (|t-a||t-b||t-c|)^{1/2}.$$

Consider the domain $G_\gamma \supset R$ defined by

$$G_\gamma = \{(s,t) \in \mathbf{C}^2 : h(s,t) < \cosh 2\gamma\},$$

where

$$h(s,t) = \frac{|s-a||t-a|}{|b-a||c-a|} + \frac{|s-b||t-b|}{|a-b||c-b|} + \frac{|s-c||t-c|}{|a-c||b-c|}.$$

Let $g : G_\gamma \to \mathbf{C}$ be a holomorphic and symmetric function. Then the orthogonal expansion of g into the products $E_n^m(s)E_n^m(t)$ converges locally uniformly to g on G_γ.

This result can be generalized to Heine-Stieltjes polynomials. There is a relationship to Dunkl's work.