

Central Limit Theorem:

$$\frac{d\mathbf{x}}{dt} = \varepsilon \mathbf{F}(\mathbf{x}, t) + \varepsilon^2 \mathbf{G}(\mathbf{x}, t)$$

Introduce scaled time $s = \varepsilon^2 t$, so

$$\frac{d\mathbf{x}}{ds} = \frac{1}{\varepsilon} \mathbf{F}(\mathbf{x}, s/\varepsilon^2) + \mathbf{G}(\mathbf{x}, s/\varepsilon^2).$$

Special case:

$$\text{let } F_i = \sum_k F_i^k(\mathbf{x}, s) \eta_k(s/\varepsilon^2).$$

$$\text{Define } C_{km} = \int_{-\infty}^{\infty} \langle \eta_k(t) \eta_m(t+t') \rangle dt .$$

Then, in the limit of long times ($t \rightarrow \infty$) and small ε ($\varepsilon \rightarrow 0$; we'll get to what ε is later), the conditional pdf $p(\mathbf{x}, s | \mathbf{x}_o, s_o)$ satisfies a Fokker-Planck equation *in the scaled coordinates*, with

$$\mathbf{S}(\mathbf{x}, s) \mathbf{S}^T(\mathbf{x}, s) = \mathbf{F}(\mathbf{x}, s/\varepsilon^2) \mathbf{C} \mathbf{F}^T(\mathbf{x}, s/\varepsilon^2)$$