Black Hole Horizons and Excision

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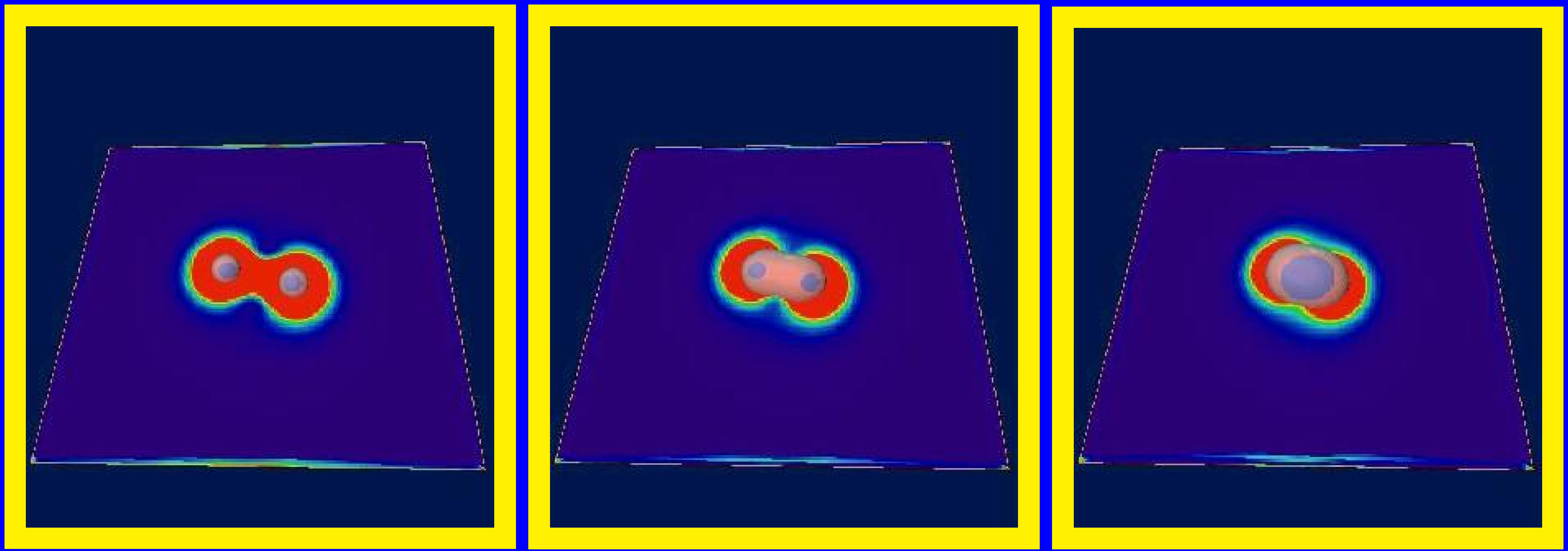
27 June 2002



The Nitty Gritty ...

- ⋆ Handling black-hole singularities: excision
- ⋆ Localizing the black-hole singularities: apparent horizons
- ⋆ Finite differencing at excision boundary
- ⋆ Open Issues

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Question: What do we do at the boundary of the excision region?

Answer: Nothing if all the field variables have outgoing (into the singularity) characteristics. That is, the excision boundary becomes the analogue of a supersonic outflow boundary in CFD.

Note: Hyperbolic formulations provide full knowledge of the characteristic properties of each field variable.

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- Find the black hole horizon
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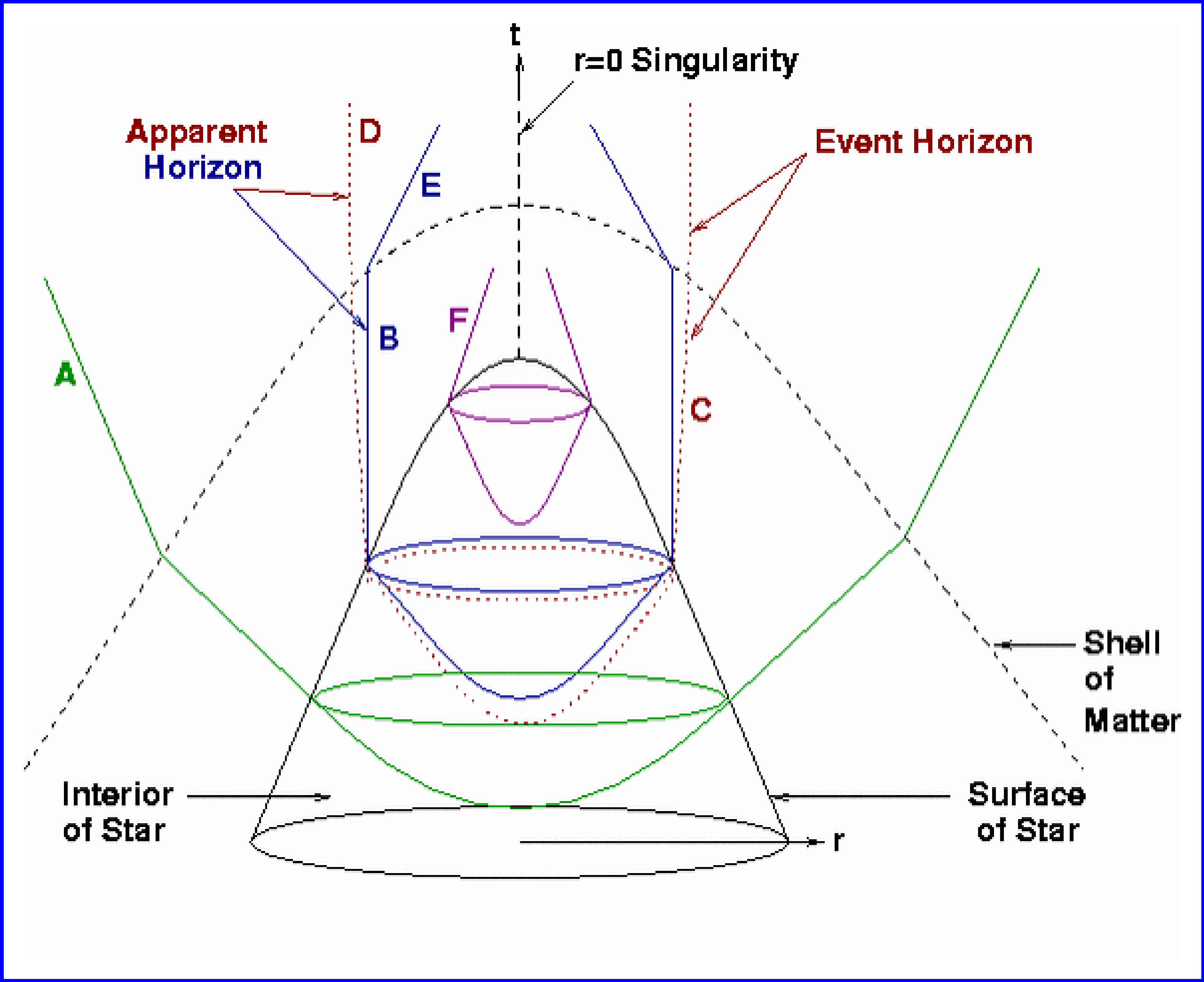
But

Event Horizon: boundary of the causal past of the collection of observers at all “distant” spacetime points

Problem: To find the event horizon, we need the complete future development of the initial data!

Any alternative?

Revisited...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Apparent
Horizon
D
E
Event Horizon
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Shell
of
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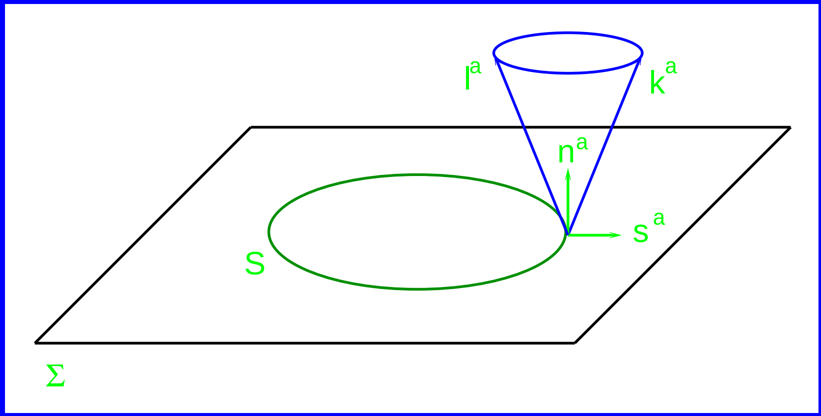
Apparent Horizons



n^a = future directed timelike normal to Σ

s^a = spacelike normal to S

$k^a = (s^a + n^a)/\sqrt{2}$ (outgoing null vector)

$l^a = (s^a - n^a)/\sqrt{2}$ (ingoing null vector)

Expansion of the outgoing null geodesics: $\Theta \equiv \nabla_a k^a$

Trapped Surface: $\Theta < 0$

Marginally Trapped Surface: $\Theta = 0$

Apparent Horizon: outermost marginally trapped surface

Theorem: If cosmic censorship holds and an apparent horizon is found, an event horizon exists outside of it. (Wald, General Relativity)

Apparent Horizon Equation

- In terms of quantities intrinsic to Σ:

$$\Theta = D_i s^i - K + s^i s^j K_{ij} = 0$$

where K_{ij} is the extrinsic curvature of Σ, K its trace and $D_i h^{ij} = 0$.

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- Methods for solving Θ equation take advantage of the horizon's S^2 topology

$$\psi = r - \rho(\theta, \phi)$$

in which $\psi = 0$ corresponds to the apparent horizon and s_i can be written:

$$s_i = \frac{D_i \psi}{||D\psi||}$$

- Note: Θ has become a 2-d problem in spherical coordinates $(\rho(\theta, \phi), \theta, \phi)$ in θ and ϕ for a 2-sphere embedded in Σ

Direct Techniques

Minimization: Nakamura, Brill and Lindquist, Eppley, NCSA/Wash U, Baumgarte and Cook ...

- Construct ψ such that we can expand $\rho(\theta, \phi)$ in terms of spherical harmonic basis functions:

$$\rho(\theta, \phi) = \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi)$$

- Solve for the coefficients for which $\Theta(a_{lm})$ reaches a minimum

Direct Discretization: Thornburg, Huq

- Solve for a ρ that yields $\Theta(\rho) = 0$ using a Newton's method

Flow Methods

Flow (Tod, Gundlach)

- Recast the Θ equation as a parabolic equation

$$\frac{\partial x^i}{\partial \lambda} = -\Theta s^i$$

- Given an initial guess, $\psi(\lambda = 0)$, with coordinates x^i and parameterized by λ
- Solve for the surface, ψ in which $\Theta = 0$.

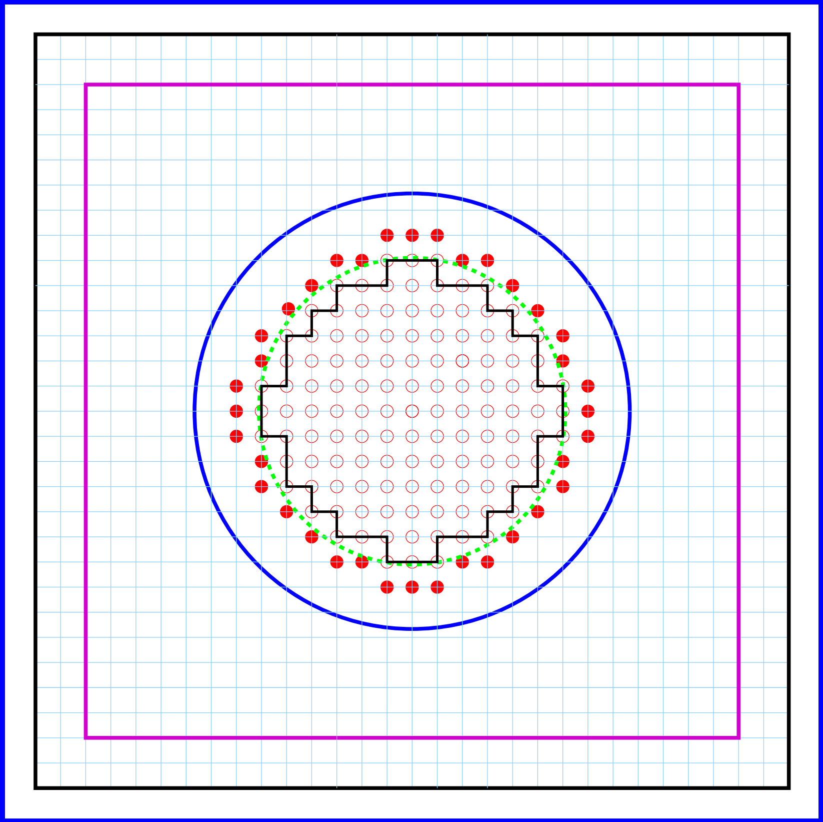
Level Flow (Pasch, DS, Diener)

- recast the equation of motion

$$\begin{aligned} \frac{\partial \psi}{\partial \lambda} &= \frac{\partial x^i}{\partial \lambda} \frac{\partial \psi}{\partial x^i} \\ \frac{\partial \psi}{\partial \lambda} &= -\Theta \|D\psi\| \end{aligned}$$

- Flow the surface ψ by iterating in λ using CN until $\psi = \epsilon$ when $\Theta = \epsilon$

Numerics of Excision

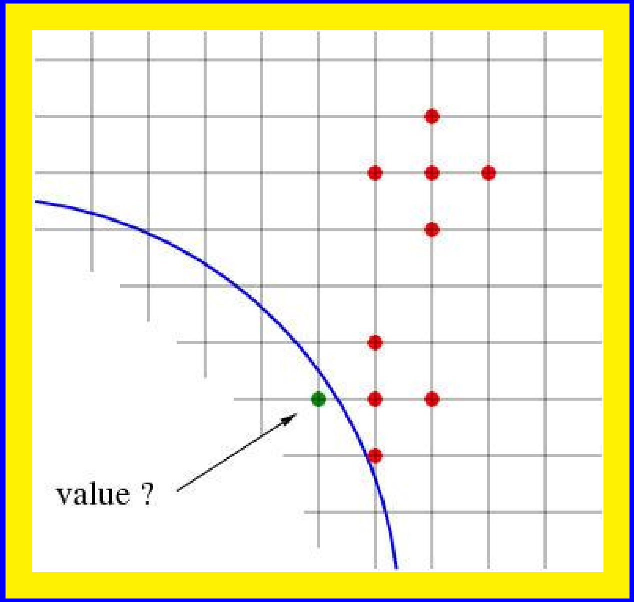


Let us view the 3+1 evolution equations in a generic form,

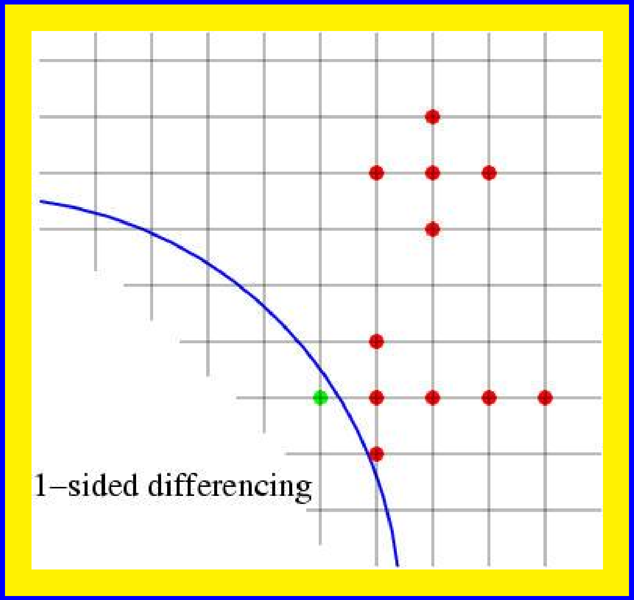
$$\partial_t \mathbf{u} = \rho(\mathbf{u}, \partial\mathbf{u}, \partial_2\mathbf{u}, \ldots)$$

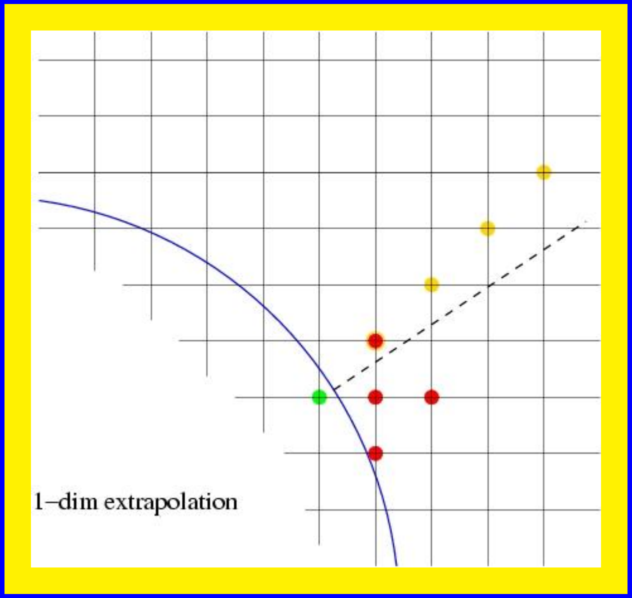
Question: Given that center finite differencing is used in the interior of the computational domain, how do we discretize the right hand side at the boundary of the excision region?

Derivatives in ρ are calculated primarily using centered finite differencing molecules.



At the excision boundary, this is not possible.

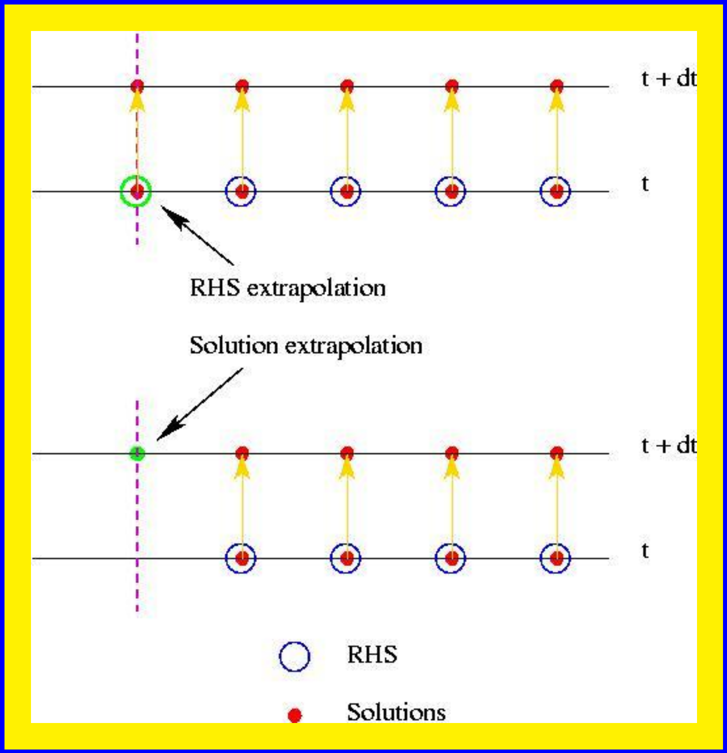




Simple Excision

Alcubierre and Bruegmann

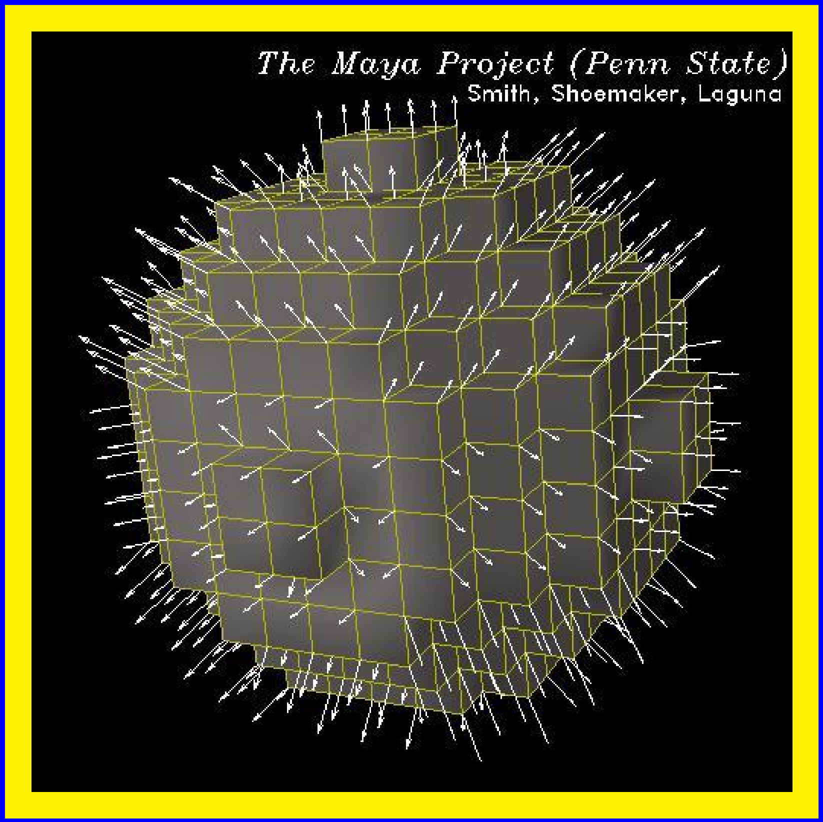
- cubical excision region
- extrapolate the r.h.s by copying ρ, 0^{th} order extrapolation, in a direction normal to the cube's faces
- imposing a boundary condition $\partial_t \partial_x \rho = 0$



Less Simple Excision

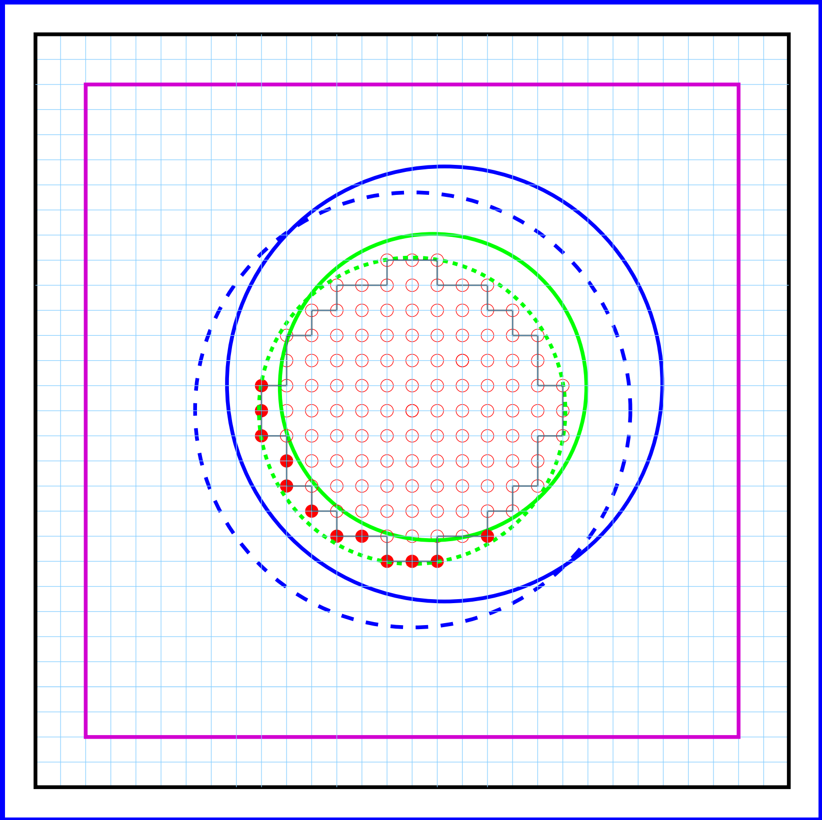
DS, Smith and Laguna, (Yo, Baumgarte, Shapiro)

- lego-sphere excision region
- extrapolate values of u to O(h^3) error in truncation or better
- carry out standard O(h^2), center differencing, where h is the discretization scale.
- extrapolation is unidirectional with a direction chosen approximately normal to the excision region



Excising Moving Black Holes

Question: What do we do with points that emerge into the computational domain as a consequence of the motion of the excision boundary?



Answer: Extrapolate + Center Differencing

Example of a Moving Black Hole

Recent toy model designed to test the excision algorithm on a trivially time-dependent problem by introducing a time-dependent coordinate change to the Schwarzschild solution in ingoing Eddington-Finkelstein coordinates.

Open Issues

- Apparent Horizons: improve computational efficiency
  - direct-solve methods are faster but require a good guess (potentially miss the outermost MTS)
  - flow methods do not require a good guess but are slow
- Excision: at this level, not a significant source of problems in 3D finite differencing codes
  - currently unidirectional extrapolation, improve to 3-d extrapolation
  - mesh conforming techniques
- Conclusion: we have a reasonable handle on excision at the current level of 3D numerics; however, as we consider more complex dynamical situations, excision will remain a fundamental issue.
- Alternatives: stuffing black holes, ...?