

Computing Initial Data I: Formalisms and History

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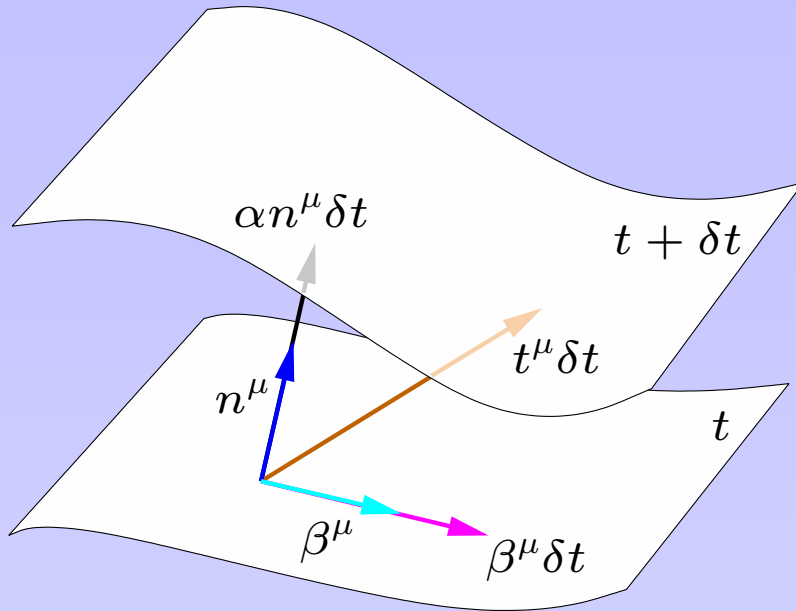
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Abstract

In the first of two talks on the computation of initial data, we will look at some of the formalisms used for posing the constraint equations of general relativity as a boundary-value problem. This process requires making well-motivated choices for which of the initial-data quantities are constrained and which can be freely specified. After looking at the general formalisms used to construct initial data, we will review the approaches that have been used to date in constructing black-hole and neutron-star initial data. Finally we will look at some of the current issues, from a physicist perspective, that are at the forefront of initial-data research.

A related [review article](#) is online at [Living Review in Relativity](#)[23]

The 3 + 1 Decomposition



Lapse : α

Spatial metric : γ_{ij}

Shift vector : β^i

Extrinsic Curvature : K_{ij}

Time vector : $t^\mu = \alpha n^\mu + \beta^\mu$

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \quad K_{\mu\nu} = -\frac{1}{2}\gamma_\mu^\alpha \gamma_\nu^\beta \mathcal{L}_n g_{\alpha\beta}$$

Constraint equations

$$\bar{R} + K^2 - K_{ij}K^{ij} = 16\pi\rho$$

$$\bar{\nabla}_j (K^{ij} - \gamma^{ij}K) = 8\pi j^i$$

$$S_{\mu\nu} \equiv \gamma_\mu^\alpha \gamma_\nu^\beta T_{\alpha\beta}$$

$$j_\mu \equiv -\gamma_\mu^\nu n^\alpha T_{\nu\alpha}$$

$$\rho \equiv n^\mu n^\nu T_{\mu\nu}$$

$$T_{\mu\nu} = S_{\mu\nu} + 2n_{(\mu}j_{\nu)} + n_\mu n_\nu \rho$$

Evolution equations

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \bar{\nabla}_i \beta_j + \bar{\nabla}_j \beta_i$$

$$\partial_t K_{ij} = -\bar{\nabla}_i \bar{\nabla}_j \alpha + \alpha \left[\bar{R}_{ij} - 2K_{il}K_j^\ell + K K_{ij} - 8\pi S_{ij} + 4\pi \gamma_{ij}(S - \rho) \right]$$

$$+ \beta^\ell \bar{\nabla}_\ell K_{ij} + K_{il} \bar{\nabla}_j \beta^\ell + K_{jl} \bar{\nabla}_i \beta^\ell$$

Degrees of Freedom

Kinematical variables

- Lapse α : 1 degree of freedom
- Shift β^i : 3 degrees of freedom

Initial-data variables

- Metric γ_{ij} : 6 degrees of freedom
- Extrinsic curv. K_{ij} : 6 degrees of freedom

Simple counting arguments

Between γ_{ij} and K_{ij} , there are 12 degrees of freedom at each point in space.

Dynamical

Gravitational waves have two polarization states. So, there are $2 + 2 = 4$ *dynamical* degrees of freedom.

(1st order form)

Gauge

In a 4-D spacetime, there are 4 initial coordinate gauge choices. So there are $3 + 1 = 4$ *gauge* degrees of freedom.

Constrained

We have 4 constraint equations and there are
 $12 - 4 - 4 = 4$
remaining *constrained* degrees of freedom.

Conformal Transverse-Traceless Decomposition

York–Lichnerowicz conformal decomposition of the 3-metric[46, 72, 54]:

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \left\{ \begin{array}{l} \psi : 1 \text{ constrained DOF} \\ \tilde{\gamma}_{ij} : \left\{ \begin{array}{l} 3 \text{ spatial gauge DOF} \\ 2 \text{ dynamical DOF} \end{array} \right. \end{array} \right.$$

Conformal TT decomposition of extrinsic curvature[73, 74]:

$$K^{ij} = \psi^{-10} \left[(\tilde{\mathbb{L}}X)^{ij} + \tilde{Q}^{ij} \right] + \frac{1}{3} \gamma^{ij} K \left\{ \begin{array}{l} \tilde{Q}^{ij} : \left\{ \begin{array}{l} \tilde{\nabla}_j \tilde{Q}^{ij} = \tilde{Q}^i_i = 0 \\ 2 \text{ dynamical DOF} \end{array} \right. \\ X^i : \left\{ \begin{array}{l} (\tilde{\mathbb{L}}X)^{ij} \equiv \tilde{\nabla}^i X^j + \tilde{\nabla}^j X^i - \frac{2}{3} \tilde{\gamma}^{ij} \tilde{\nabla}_k X^k \\ 3 \text{ constrained DOF} \end{array} \right. \\ K : 1 \text{ temporal gauge DOF} \end{array} \right.$$

Specifying Initial Data

Freely specified degrees of freedom

$$\tilde{\gamma}_{ij} \Leftarrow \begin{cases} (2) \text{ initial dynamical ("wave")} \text{ content} \\ (3) \text{ initial spatial gauge choices} \end{cases}$$

$$\tilde{Q}^{ij} \Leftarrow (2) \text{ initial dynamical ("wave")} \text{ content}$$

$$K \Leftarrow (1) \text{ initial temporal gauge choice}$$

Freely specified *dynamical gauge freedom*

$\beta^i \Leftarrow$ how spatial coordinates evolve

$\alpha \Leftarrow$ how time coordinate evolves

Constrained degrees of freedom (Conf. TT)

$$\psi \Leftarrow \begin{cases} (1) \text{ Hamiltonian constraint} \\ \tilde{\nabla}^2 \psi - \frac{1}{8} \psi \tilde{R} - \frac{1}{12} \psi^5 K^2 + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} = -2\pi \psi^5 \rho \end{cases}$$

$$X^i \Leftarrow \begin{cases} (3) \text{ Momentum constraint} \\ \tilde{\Delta}_{\mathbb{L}} X^i = \frac{2}{3} \psi^6 \tilde{\nabla}^i K + 8\pi \psi^{10} j^i \end{cases}$$

$$\tilde{A}^{ij} \equiv (\tilde{\mathbb{L}} X)^{ij} + \tilde{Q}^{ij}$$

$$\tilde{\Delta}_{\mathbb{L}} X^i \equiv \tilde{\nabla}_j (\tilde{\mathbb{L}} X)^{ij}$$

Boundary conditions

The constraints form a set of 4 coupled nonlinear PDEs for (ψ, X^i) that require the specification of boundary conditions at spatial infinity and any interior boundaries.

General Initial-Data Decompositions

Conformal TT[75, 77]:

$$\begin{aligned} \gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \quad : \quad K^{ij} = \psi^{-10} \tilde{A}^{ij} + \frac{1}{3} \psi^{-4} \tilde{\gamma}^{ij} K \quad : \quad \tilde{A}^{ij} \equiv (\tilde{\mathbb{L}}V)^{ij} + \tilde{M}^{ij} \\ \tilde{\nabla}^2 \psi - \frac{1}{8} \psi \tilde{R} - \frac{1}{12} \psi^5 K^2 + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} = -2\pi \psi^5 \rho \\ \tilde{\Delta}_{\mathbb{L}} V^i - \frac{2}{3} \psi^6 \tilde{\nabla}^i K = -\tilde{\nabla}_j \tilde{M}^{ij} + 8\pi \psi^{10} j^i \end{aligned}$$

$$\tilde{\alpha} = \frac{1}{2}$$

Physical TT[53, 54, 55]:

$$\begin{aligned} \gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \quad : \quad K^{ij} = \psi^{-4} \left(\tilde{A}^{ij} + \frac{1}{3} \tilde{\gamma}^{ij} K \right) \quad : \quad \tilde{A}^{ij} \equiv (\tilde{\mathbb{L}}V)^{ij} + \psi^{-6} \tilde{M}^{ij} \\ \tilde{\nabla}^2 \psi - \frac{1}{8} \psi \tilde{R} - \frac{1}{12} \psi^5 K^2 + \frac{1}{8} \psi^5 \tilde{A}_{ij} \tilde{A}^{ij} = -2\pi \psi^5 \rho \\ \tilde{\Delta}_{\mathbb{L}} V^i + 6(\tilde{\mathbb{L}}V)^{ij} \tilde{\nabla}_j \ln \psi + \psi^{-6} \tilde{\nabla}_j \tilde{M}^{ij} = \frac{2}{3} \tilde{\nabla}^i K + 8\pi \psi^4 j^i \end{aligned}$$

$$\alpha = \frac{1}{2}$$

\tilde{M}^{ij} is symmetric-tracefree, but not divergenceless. The new variable V^i incorporates the solution of the constraints and the decomposition of \tilde{M}^{ij} into \tilde{Q}^{ij} .

Conformal Thin Sandwich(TS)[76]:

$$\begin{aligned} \gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \quad : \quad K^{ij} = \psi^{-10} \tilde{A}^{ij} + \frac{1}{3} \psi^{-4} \tilde{\gamma}^{ij} K \quad : \quad \tilde{A}^{ij} \equiv \frac{1}{2\tilde{\alpha}} \left((\tilde{\mathbb{L}}\beta)^{ij} + \tilde{u}^{ij} \right) \\ \tilde{\alpha} \equiv \psi^{-6} \alpha \quad : \quad \tilde{\nabla}^2 \psi - \frac{1}{8} \psi \tilde{R} - \frac{1}{12} \psi^5 K^2 + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} = -2\pi \psi^5 \rho \\ \tilde{\Delta}_{\mathbb{L}} \beta^i - (\tilde{\mathbb{L}}\beta)^{ij} \tilde{\nabla}_j \tilde{\alpha} - \frac{4}{3} \tilde{\alpha} \psi^6 \tilde{\nabla}^i K = \tilde{\alpha} \tilde{\nabla}_j \left(\frac{1}{\tilde{\alpha}} \tilde{u}^{ij} \right) + 16\pi \tilde{\alpha} \psi^{10} j^i \end{aligned}$$

α and β^i are the lapse and shift. \tilde{u}^{ij} is symmetric-tracefree.

Special Initial-Data Decompositions

- The Conformal TS, Conformal TT, & Physical TT decompositions are widely used, but are not unique. In particular, different conformal weightings of the extrinsic curvature are used.
- For stationary situation (i.e. axisymmetric, time independent) like isolated neutron stars, different decompositions are used.

$$ds^2 = -\psi^{-4}dt^2 + \psi^4[A^2(dr^2 + r^2d\theta^2) + B^2r^2 \sin^2 \theta(d\phi + \beta^\phi dt)^2]$$

- ★ The 4 constraints yield 2 independent equations that fix ψ and β^ϕ .
- ★ Stationarity ($\partial_t \gamma_{ij} = 0, \partial_t K_{ij} = 0$) yields 2 independent equations that fix A and B .

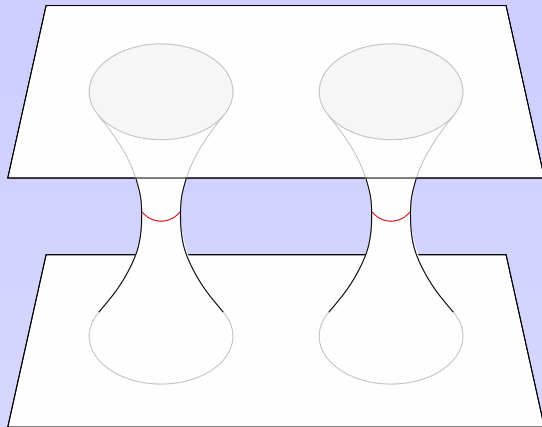
“Traditional” Black-Hole Data

Conformal TT with conformal flatness and maximal slicing

$$\left. \begin{array}{l} \tilde{\gamma}_{ij} = f_{ij} \text{ (flat)} \\ \tilde{M}^{ij} = 0 \\ K = 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \tilde{\Delta}_{\perp} X^i = 0 \\ \tilde{\nabla}^2 \psi + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} = 0 \end{array} \right. \Rightarrow \begin{array}{l} \text{Bowen-York solution [14, 45]} \\ \text{Analytic solutions for } \tilde{A}^{ij} \end{array}$$

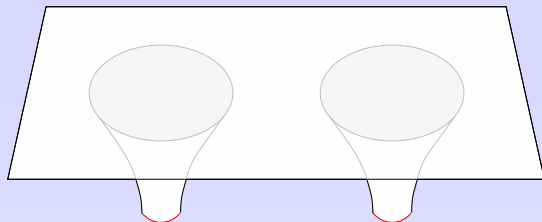
Three general solution schemes

Conformal Imaging-[52, 14, 21, 26]



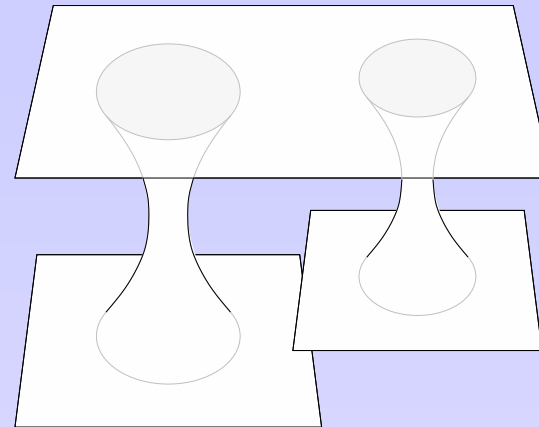
Inversion
symmetry
inner-BC

Apparent Horizon BC-[62]



Apparent
horizon
inner-BC

Puncture Method-[47, 17, 15]



No inner-BC:
singular
behavior
factored out

All methods can produce very general configurations of multiple black holes, but are fundamentally limited by choices of $\tilde{\gamma}_{ij} = f_{ij}$ and Bowen-York \tilde{A}^{ij} .

Black-Hole Data

- The “traditional” black hole initial data approach was motivated by *computational convenience*, not by any strong physical arguments. Research focused on:
 - methods for solving the Hamiltonian constraint for one or two holes. [77, 20, 21, 26, 1, 42, 57, 33]
 - understanding the physical content (and limitations) of initial data containing one or two holes. [77, 30, 21, 25]
 - finding solutions that represented two black holes in nearly circular orbits. [22, 2, 57]
- Early departures from the “traditional approach” gave up the assumption of conformal flatness so that initial data containing strong waves could be constructed. However the initial data was assumed to be time-symmetric ($K_{ij} = 0$) or \tilde{A}_{ij} was found analytically, so only the Hamiltonian constraint had to be solved. [5, 16]
- Recent work has generalized many of these assumption and requires solving the full set of 4 (or more) coupled, non-linear elliptic PDEs. [51, 6, 50, 38, 41, 56]

Neutron-Star Data[59]

- Single, stationary neutron stars have been studied extensively. The matter is assumed to be in hydrostatic equilibrium. Research has focused on:
 - various numerical methods.[13, 19, 18, 43, 27, 29, 60, 12, 37, 8, 10]
 - rigid and differential rotation.[69, 44, 27]
 - studies with various equations of state.[34, 28]
- Binary systems have also been studied. These are usually based on the assumption that the binary is nearly in equilibrium (quasi-equilibrium). Research has focused on:
 - various numerical methods.[70, 71, 9, 3, 39, 40]
 - corotating (tidally locked) binaries.[9, 3, 4, 48, 65]
 - irrotational binaries.[61, 58, 36, 11, 49, 63, 64]

Current Issues

- Current initial data schemes do not represent, with sufficient detail, the *physical configurations* we are most interested in — **astrophysical compact binary inspiral**.
 - The conformal-flatness assumption does not represent rapidly spinning objects well. It also contributes to unphysical gravity waves in orbiting binaries.[35, 57]
 - The Bowen-York analytic extrinsic curvature does not model the physics of binary inspiral well.[41, 56, 31]
- ★ Boosted Kerr — Use a superposition of analytic black hole metrics and extrinsic curvatures as trial data and solve the constraints to correct them.[51, 50]
 - This approach lacks well motivated boundary conditions and yields results that are only marginally better than “Bowen-York” methods at representing astrophysical binary inspiral.[56]
- ★ Forced-equilibrium — Enforce a “helical Killing-vector” yielding data with equal amounts of ingoing and outgoing radiation – then subtract away the unphysical ingoing radiation. ($\partial_t \gamma_{ij} = \partial_t K_{ij} = 0$)[7, 32, 68, 67, 66]
 - Questions about boundary conditions, etc. . .

Current Issues

- ★ Quasi-equilibrium — For sufficiently separated binaries, the timescale for orbital decay is much larger than the orbital period. Assume **only** $\partial_t \tilde{\gamma}_{ij} = \partial_t \text{Tr}K = 0$.
 - For black holes, use topology and quasi-equilibrium to fix boundary conditions on the constrained data.[38, 41] *Leads to overdetermined boundary conditions — only satisfied in true equilibrium.*
 - For black holes, use the principle of *quasi-equilibrium* to derive boundary conditions for the constrained data.[24] *Leads to complicated, elliptic boundary conditions. May not be well posed.*
 - How do we choose $\tilde{\gamma}_{ij}$?[65, 56]
 - * Can we use post-Newtonian metrics to improve the choice of $\tilde{\gamma}_{ij}$?
 - * Can we use a perturbative evolution to iteratively improve the choice of $\tilde{\gamma}_{ij}$?

Quasi-Equilibrium with Black Hole Excision[24]

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \quad K^{ij} = \psi^{-10} \tilde{A}^{ij} + \frac{1}{3} \gamma^{ij} K \quad \tilde{A}^{ij} = \frac{\psi^6}{2\alpha} (\tilde{\mathbb{L}}\beta)^{ij} \quad \partial_t \tilde{\gamma}_{ij} = 0$$

$$\tilde{\nabla}^2 \psi - \frac{1}{8} \psi \tilde{R} - \frac{1}{12} \psi^5 K^2 + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} = 0$$

$$\tilde{\Delta}_{\mathbb{L}} \beta^i - (\tilde{\mathbb{L}}\beta)^{ij} \tilde{\nabla}_j \ln \alpha \psi^{-6} = \frac{4}{3} \alpha \tilde{\nabla}^i K$$

$$\tilde{\nabla}^2 (\alpha \psi) - (\alpha \psi) \left[\frac{1}{8} \tilde{R} + \frac{5}{12} \psi^4 K^2 + \frac{7}{8} \psi^{-8} \tilde{A}_{ij} \tilde{A}^{ij} \right] = \psi^5 \beta^i \tilde{\nabla}_i K \quad \partial_t K = 0$$

$$\tilde{s}^k \tilde{\nabla}_k \ln \psi|_S = -\frac{1}{4} (\tilde{h}^{ij} \tilde{\nabla}_i \tilde{s}_j - \psi^2 J)|_S \quad \theta = 0$$

$$\beta^i|_S = \begin{cases} \alpha \psi^{-2} \tilde{s}^i|_S & \text{corotation} \\ \alpha \psi^{-2} \tilde{s}^i|_S + \Omega \tilde{h}^i_j \left(\frac{\partial}{\partial \phi} \right)^j \Big|_S & \text{irrotation} \end{cases} \quad \begin{matrix} \mathcal{L}_\zeta \theta = 0 \\ \sigma_{ij} = 0 \end{matrix}$$

$$J \tilde{s}^i \tilde{\nabla}_i \alpha|_S = -\psi^2 (J^2 - JK + \tilde{\mathcal{D}}) \alpha|_S \quad \mathcal{L}_\zeta \theta = 0$$

$$\begin{aligned} \psi|_{r \rightarrow \infty} &= 1 \\ \beta^i|_{r \rightarrow \infty} &= \Omega \left(\frac{\partial}{\partial \phi} \right)^i \\ \alpha|_{r \rightarrow \infty} &= 1 \end{aligned}$$

The only remaining freedom in the system is the choice of the orbital angular velocity Ω , the initial spatial gauge and the initial dynamical (“wave”) content found in $\tilde{\gamma}_{ij}$, and the initial temporal gauge in K .

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