

A simple population model for *Vibrio Vulnificus*
in a well mixed circulatory system

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We will be modelling *Vibrio Vulnificus*, a rather nasty marine pathogen which may be acquired either by eating uncooked shellfish, such as oysters, or through the infection of an open wound.

We shall consider the case of infection by ingestion. In particular, we will model the situation when a fixed amount of *v. vulnificus* has survived its trip through the stomach and has managed to enter the blood stream. The presence of an available iron source, such as haem, is necessary for the organism to reproduce. However, if such a source is present, extra-cellular proteins will be produced which will attack and cleave other, less readily metabolizable sources of iron, such as hemoglobin. We shall only consider the case where an initial concentration of free haem is present in the blood.

Our goal is to build the simplest population model for *v. vulnificus* in the blood stream we could which would capture the following properties.

- If *v. vulnificus* population is initially low, the infection dies out.
- If the free haem concentration is too low, the infection dies out.
- Increasing initial free haem concentration reduces the needed initial *v. vulnificus* concentration required for infection survival.

We made several simplifying assumptions:

- We assume there is an unlimited amount of hemoglobin and other iron carrying molecules for the *v. vulnificus* to break down.
- The blood is well mixed.
- We lump all usable iron-carrying molecules such as haem into a single concentration.
- We lump all extra-membrane proteins such as hemolysin and transporters such as HupA into a single concentration.
- We treat haem and haem-like molecules as a metabolizable nutrients.
- The model is deterministic. Thus values such as LD_{50} do not make sense.

Consider the following cartoon:

Our time dependent concentrations are denoted as follows:

variable	concentration of:
v	vibrio vulnificus
x	extra-membrane protein
h	haem
b	bound iron

As noted above, we will treat the haem as a nutrient. If α_1 is the optimal growth rate of the *v. vulnificus* we will write the growth rate of the *v. vulnificus* as a function of h as

$$\frac{\alpha_1 h}{\beta_1 + h}$$

Assuming a natural death rate of δ_1 we have the following equation

$$\frac{dv}{dt} = \frac{\alpha_1 h}{\beta_1 + h} v - \delta_1 v$$

We assume that when the number of iron atoms bound to a cell reaches a value f the cell will stop producing extra-cellular proteins. Otherwise the production will continue at a rate α_2 . The extra-cellular proteins are reacting at a rate γ_1 giving us the rate equation

$$\frac{dx}{dt} = \alpha_2(fv - b)^+ - \gamma_1 x$$

Letting r represent the amount of haem needed to produce one new v . vulnificus and δ_2 the rate at which bound iron is metabolized, we obtain

$$\frac{dh}{dt} = \gamma_2 x - r \frac{\alpha_1 h}{\beta_1 + h} v$$

and

$$\frac{db}{dt} = r \frac{\alpha_1 h}{\beta_1 + h} v - \delta_2 b$$

The complete model is

$$\frac{dv}{dt} = \frac{\alpha_1 h}{\beta_1 + h} v - \delta_1 v$$

$$\frac{dx}{dt} = \alpha_2 (fv - b)^+ - \gamma_1 x$$

$$\frac{dh}{dt} = \gamma_2 x - r \frac{\alpha_1 h}{\beta_1 + h} v$$

$$\frac{db}{dt} = r \frac{\alpha_1 h}{\beta_1 + h} v - \delta_2 b$$

The behavior of the system will certainly be affected by parameter values. However, our limited exploration indicates that many ranges of parameters cause the model to mimic reported results.

Some analytic results can be found quickly such as needing parameter values

$$\alpha_1 > \delta_1$$

as a necessary condition for *v. vulnificus* survival and requiring

$$h > \frac{\beta_1 \delta_1}{\alpha_1 - \delta_1}$$

for the *v. vulnificus* population to grow. However most of our work was numerical.

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