# Friction rendering and perception

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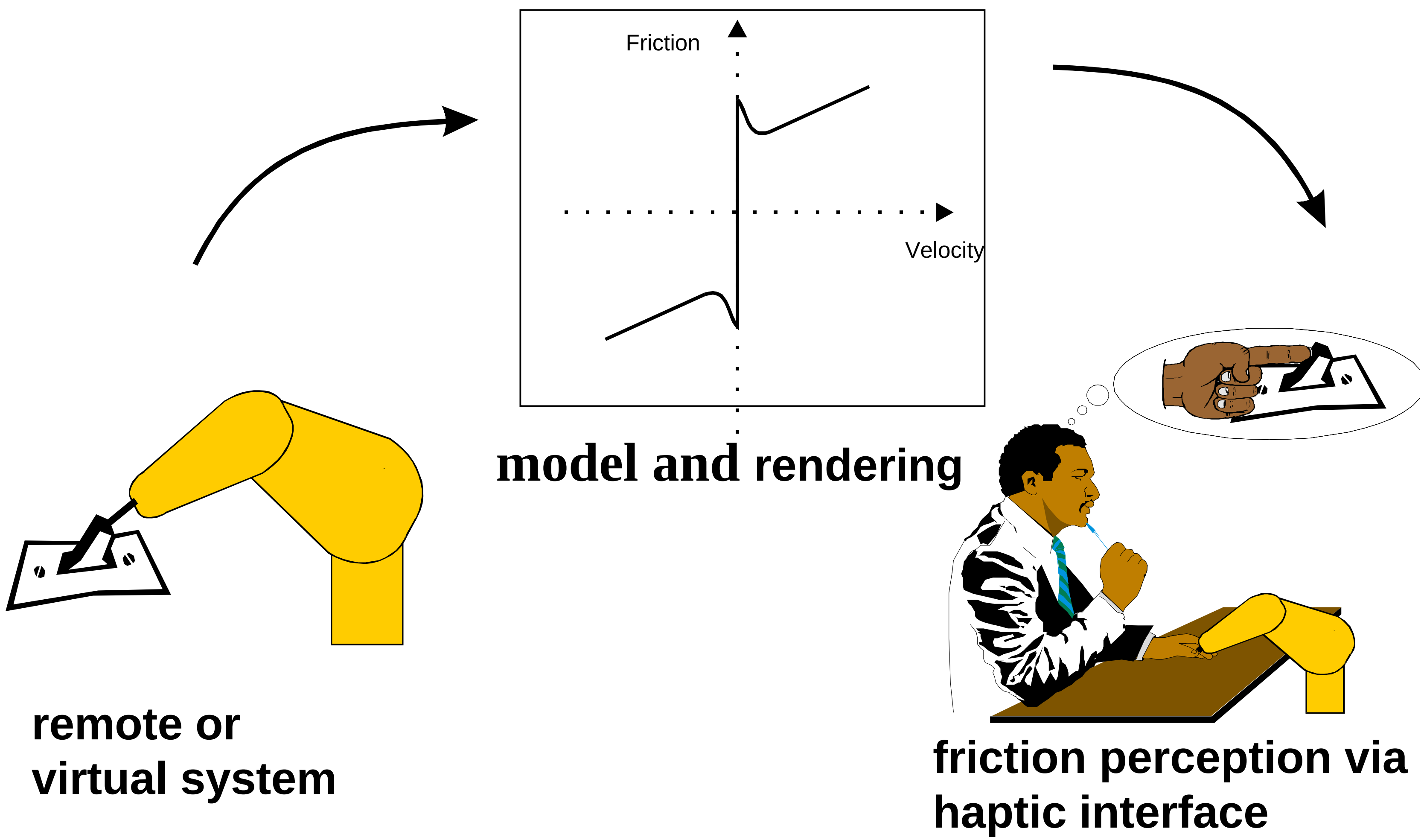
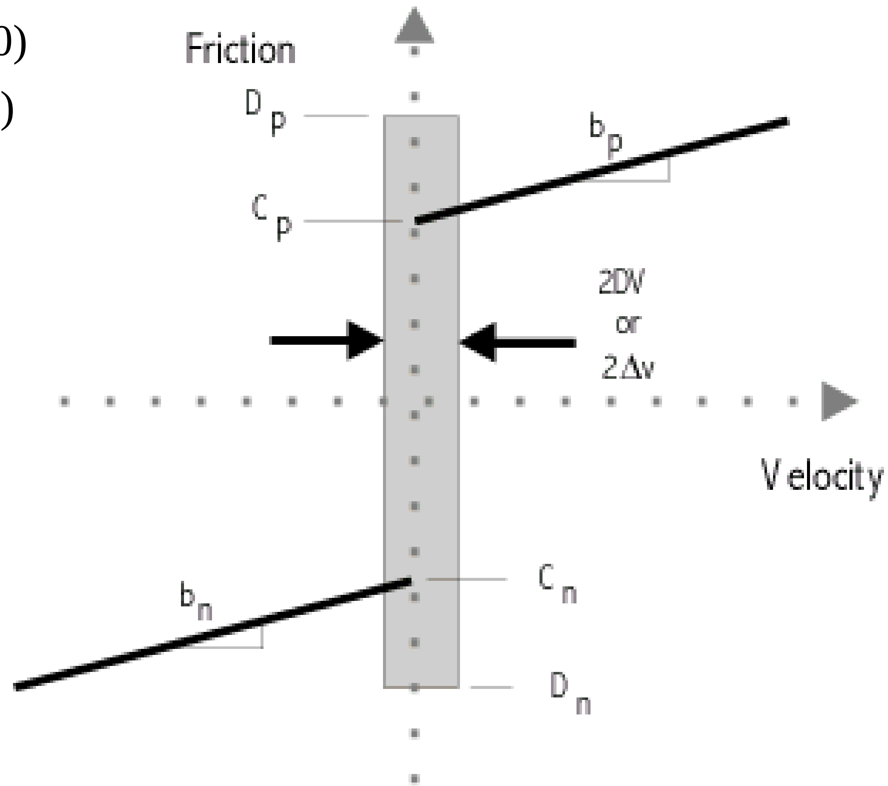


Friction models

# The Karnopp Friction Model

$$F_{friction}(\dot{x}, F_a) = \begin{cases} C_n \operatorname{sgn}(\dot{x}) + b_n \dot{x} & (\dot{x} < -\Delta v) \\ \max(D_n, F_a) & (-\Delta v < \dot{x} < 0) \\ \min(D_p, F_a) & (0 < \dot{x} < \Delta v) \\ C_p \operatorname{sgn}(\dot{x}) + b_p \dot{x} & (\dot{x} > \Delta v) \end{cases}$$

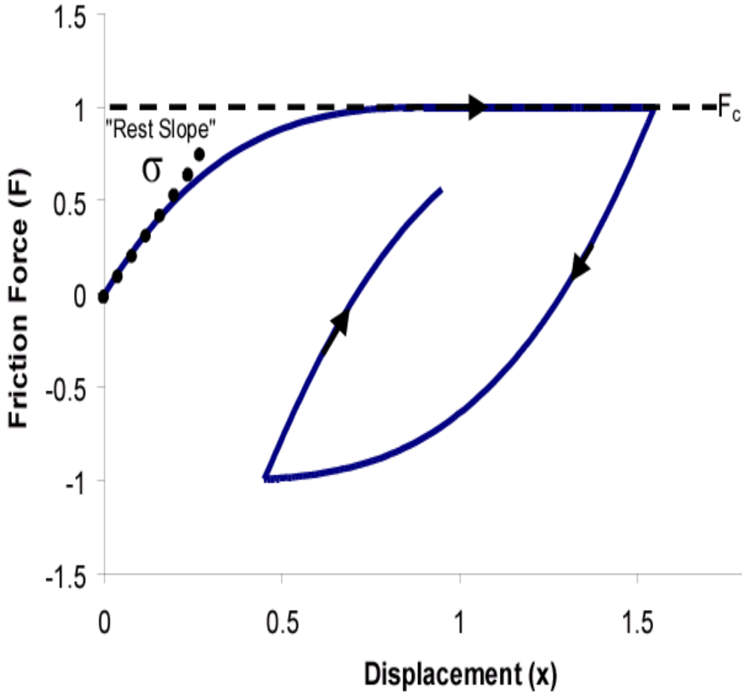
- Small threshold around zero velocity
- Designed to allow stiction
- Asymmetric to match experimental data



# The Dahl Friction Model

$$\frac{dF(x)}{dx} = \sigma\left|1 - \frac{F}{F_C}\operatorname{sgn}\dot{x}\right|^i \operatorname{sgn}\left(1 - \frac{F}{F_C}\operatorname{sgn}\dot{x}\right)$$

$$\frac{dF(x)}{dt} = \frac{dF(x)}{dx}\frac{dx}{dt}$$

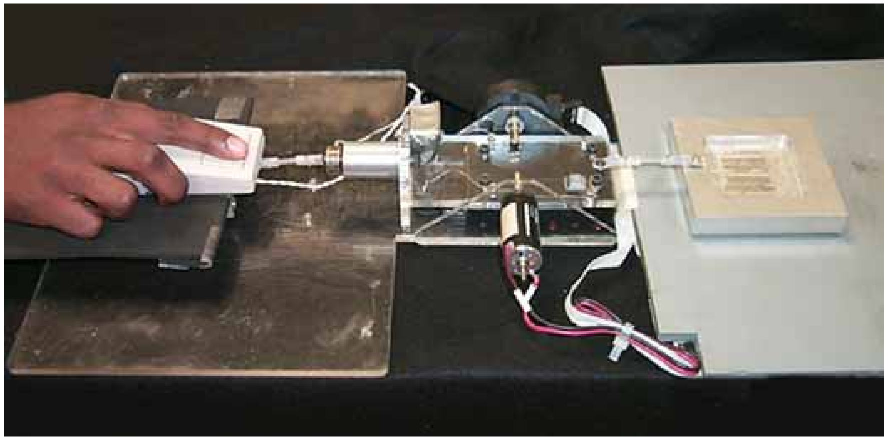


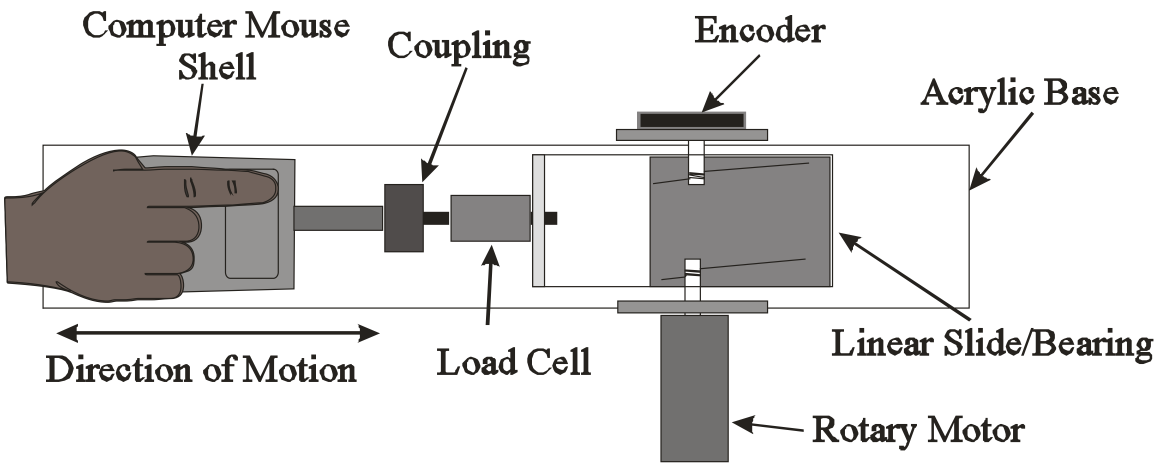

- Based on position
- Designed to allow pre-sliding displacement
- Made asymmetric by replacing $F_c$ with $D_p$ (positive velocity.) and $D_n$ (negative velocity)

Friction models

# Experimental Apparatus

- Force Sensor located upstream of the actuator
- Special fixed distance velocity sensing
- Accelerometer for inertia estimation
- Can add optional real friction as desired





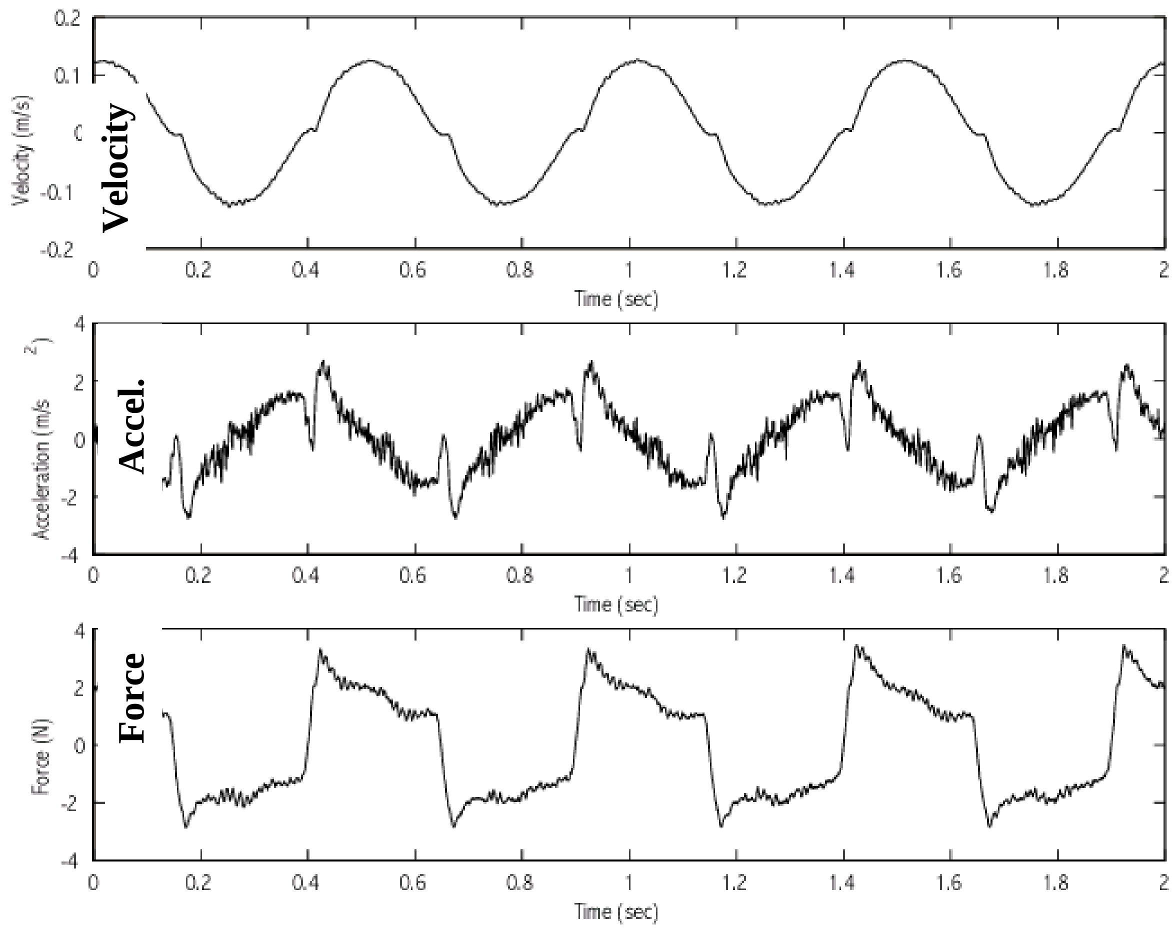
# Identification Procedure

- Model the force/motion interaction
- Move system over a range of velocities of interest (0.5 to 3 Hz)
- Record force/motion variables included in the model
- Solve for unknown parameters of the system model.

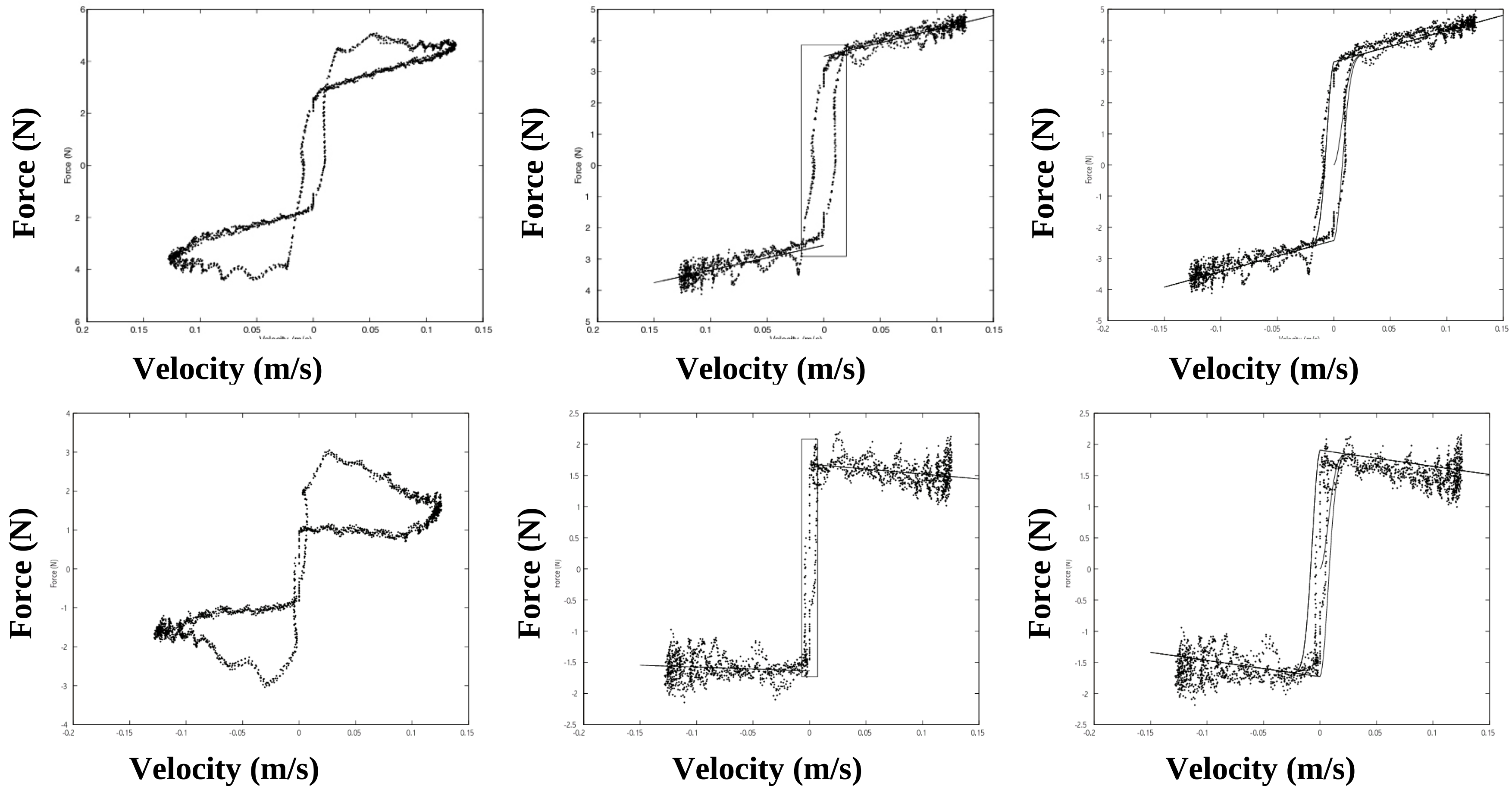
## Friction models

# Record force/motion variables included in the model

Evidence of sticking can be seen during velocity reversals.



# Model fitting results




Unadjusted data

Data adjusted for mass
Karnopp fit

Data adjusted for mass
Dahl fit

# Model Comparison

- Karnopp
  - Simple linear expression leads to few numerical difficulties
  - Can accurately capture stiction, but not pre-sliding displacement
- Dahl
  - Expressed with a differential equation making optimization more difficult. ( Convergence and initial values)
  - Captures pre-sliding displacement for more elastic materials, but does not represent stiction

Rendering

# Algorithm for Implementing Karnopp Friction

```
if state = STUCK
   friction = -Kp*(x – x_stuck);
   if abs(friction) > F_static
      state = SLIDING;
      friction = -sign(x - x_stuck)*F_dynamic -B*v;
   endif
else
   if abs(v)< DV
      state = STUCK;
      x_stuck = x + sign(ff_old)*(F_static/Kp);
      ff = -Kp*(x – x_stuck);
   else
      ff = -sign(v)*F_dynamic – B*v;
   endif
endif
ff_old = friction;
```

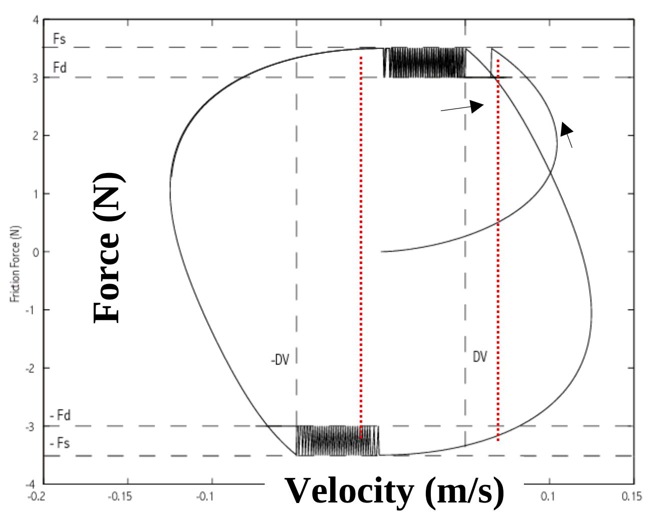
# Three regimes of interest

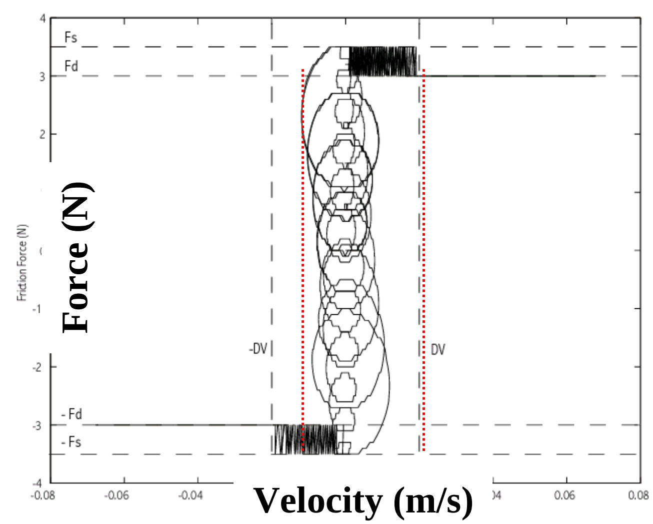
- Pre-sliding Displacement
  - How does the system behave while in the stuck state?
- Stick-slip and Sliding
  - How does the system transition between the stuck state and the sliding state?
- Free Motion
  - How does the system behave in the absence applied force?

# Effects of $K_p$ on Rendering

(sinusoidal input force)

***UNSTABLE***



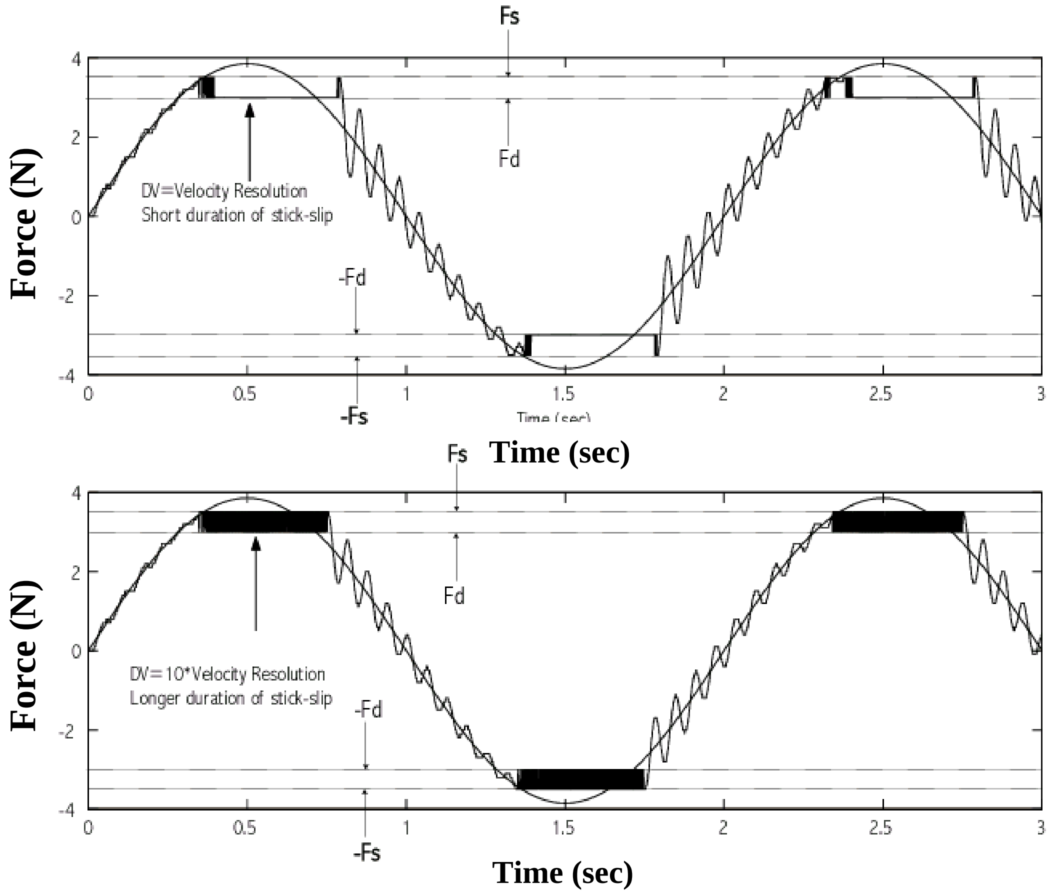

*$K_p$ too large*

*$K_p$ too low*

*$K_p$ properly tuned*

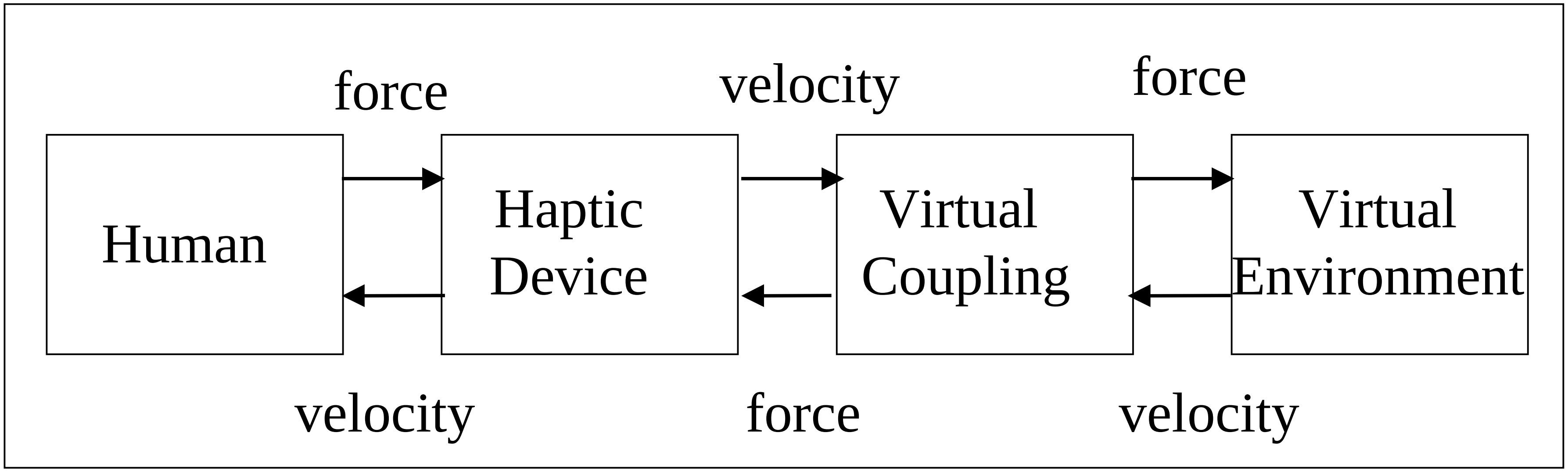
# Effect of *DV* on the duration of the stick-slip regime

**A larger velocity threshold, *DV*, results in more stick-slip oscillations**



Rendering

# The virtual coupling



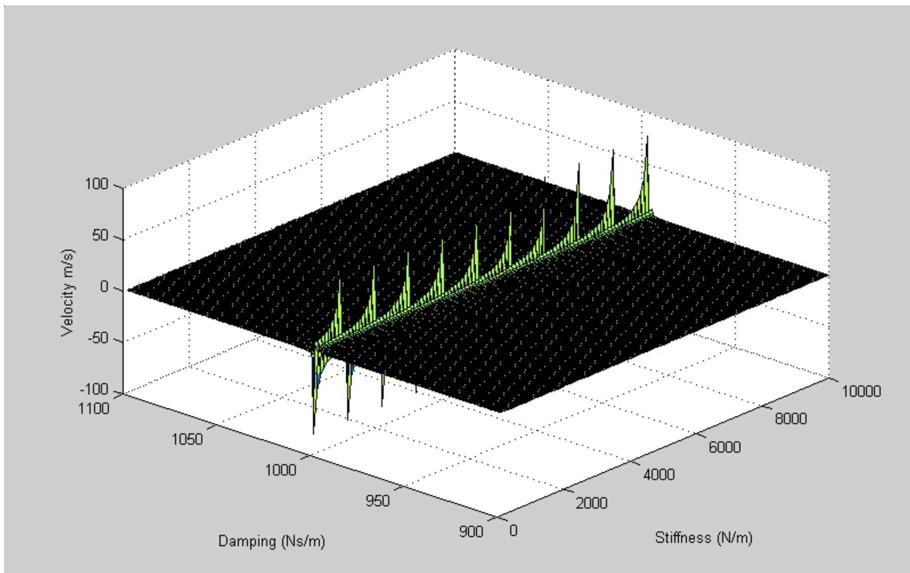

Can simulate both the friction and inertia of the block without estimating either the user's input acceleration or the user's input force.

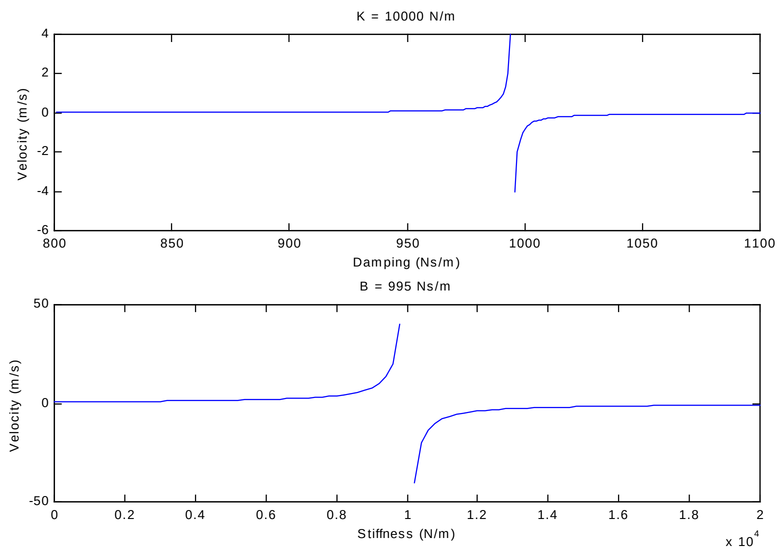
Rendering

# Choosing parameters for the virtual coupling

Limit cycle velocity amplitude

$$v = \frac{-2F_s T}{KT^2 + 2BT - 4M}$$



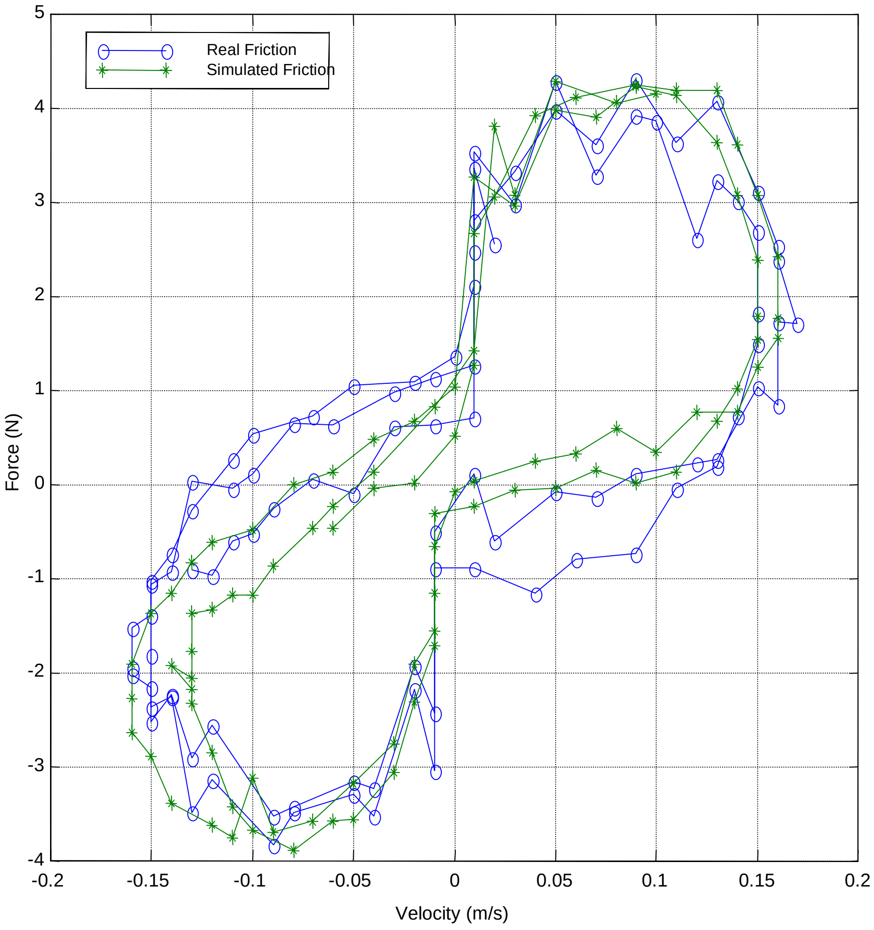




M = 0 .5 kg, F= 4.0 N, T = 0 .001s

Rendering

# Real vs. Simulated Friction

- **Aluminum on Teflon**
- **Friction Force 2N**
- **Inertia 1 kg**
- **User supplied input motion**



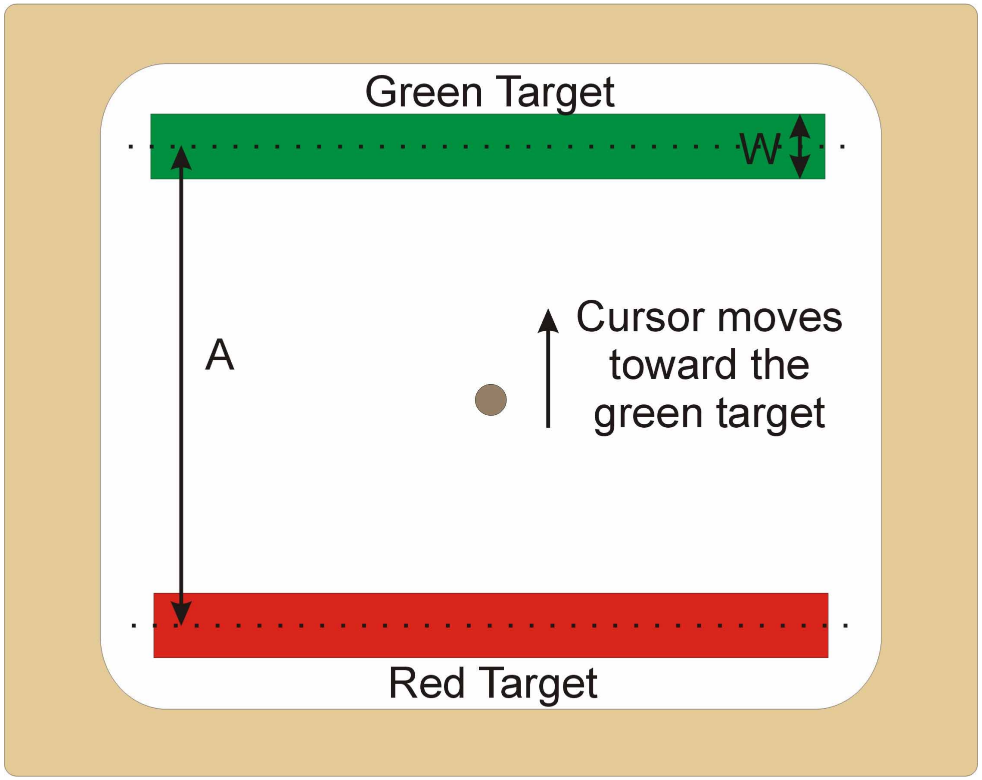
# Rendering Summary

- Varying the proportional gain affects both the stability and realism of the haptic friction rendering.
- Karnopp model for rendering can produced stick-slip oscillations.
- The free motion limit cycle is avoided with proper parameter selection.
- The virtual coupling allows for rendering of more than just friction.

**Perception**

# Real and simulated friction: effect on human performance in Fitts task

- Subjects were instructed to enter the green target and click the mouse button.
- The red and green target alternate location after each valid click.
- Difficulty varied by changing widths and distances.
- Level and type of friction also varied: baseline, real, simulated, viscous, high stiction.

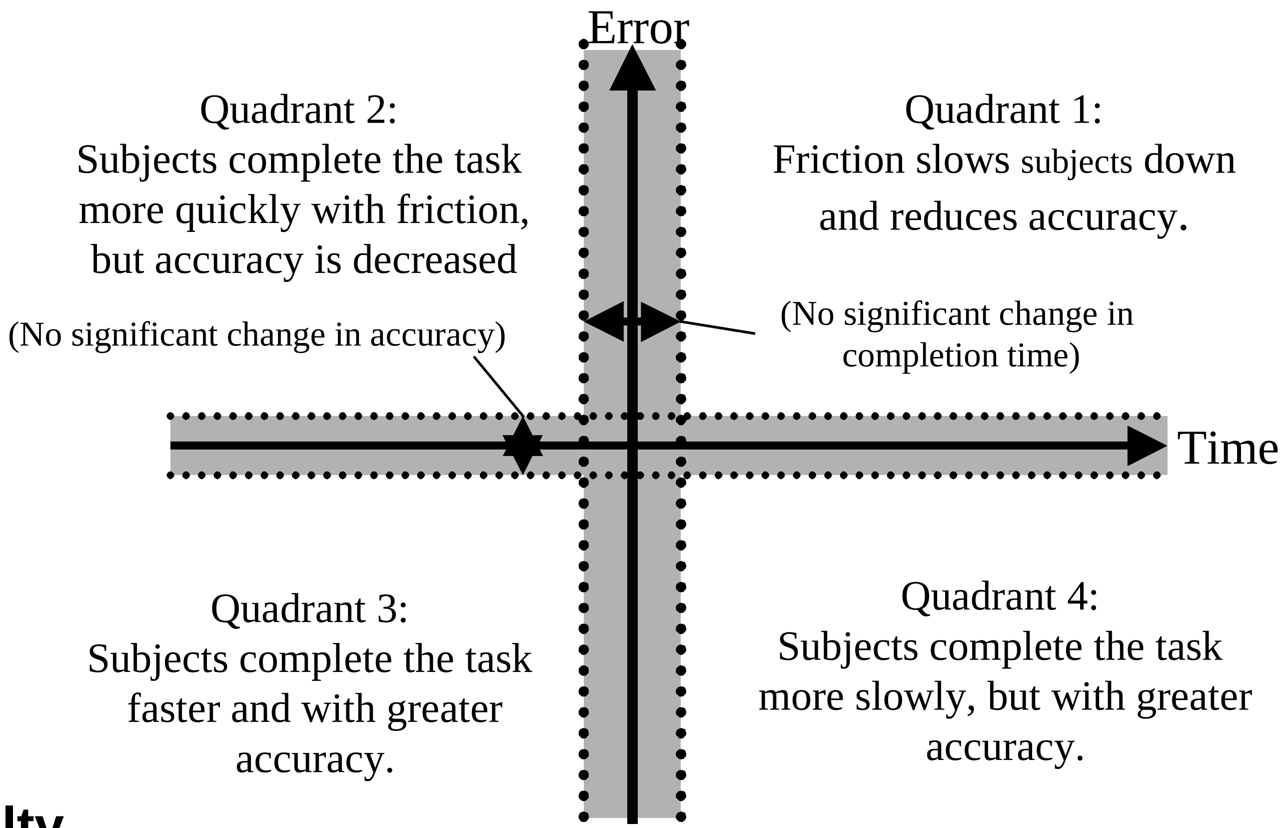


**Perception**

# Fitts' Test Data

Comparing baseline performance with:

- •real friction
- •simulated friction,
- •viscous damping,
- •high stiction




**20 Subjects**
**9 Indices of difficulty**

# Fitts' Test Results

Real and Simulated Friction as well as Viscous damping improve subject time performance in the most difficult tasks. High stiction degrades performance

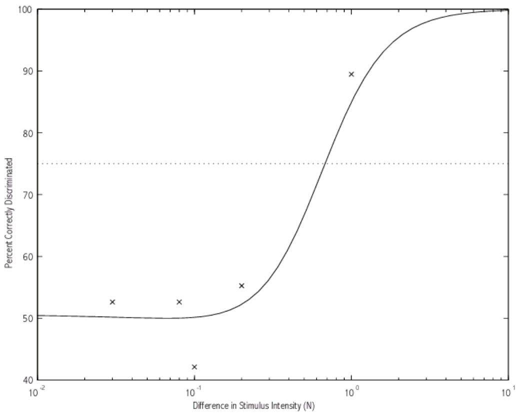
| Time | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *Index* | *Real* | | *Simulated* | | *High Stiction* | | *Viscous* | |
| | *Better* | *Worse* | *Better* | *Worse* | *Better* | *Worse* | *Better* | *Worse* |
| 1 | 10 | 4 | 14 | 3 | 9 | 11 | 15 | 3 |
| 2 | 6 | 10 | 5 | 9 | 3 | 9 | 8 | 7 |
| 3 | 8 | 4 | 5 | 5 | 5 | 9 | 7 | 4 |
| 4 | 10 | 5 | 10 | 7 | 1 | 19 | 9 | 8 |
| 5 | 3 | 8 | 6 | 4 | 3 | 9 | 6 | 5 |
| 6 | 5 | 6 | 3 | 4 | 3 | 7 | 2 | 5 |
| 7 | 15 | 3 | 13 | 3 | 8 | 10 | 12 | 5 |
| 8 | 8 | 6 | 12 | 3 | 8 | 9 | 11 | 3 |
| 9 | 9 | 4 | 7 | 5 | 8 | 5 | 10 | 4 |
| **Totals** | 74 | 50 | 75 | 43 | 48 | 88 | 80 | 44 |

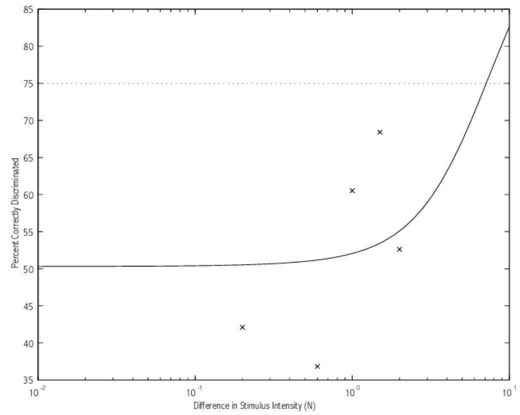
# Perception Experiments

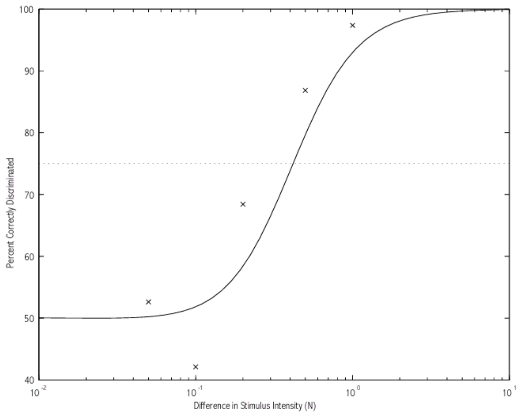
- Subjects were asked to discern between two differing levels of friction.
  - They were forced to select which case was different than the base case
- Case A: The variant had both higher static and dynamic friction than the base case
- Case B: The variant only had higher static friction than the base case
- Case C: The variant had both lower static and dynamic friction than the base case

**Perception**

# Perception Experiment Results




**Case A**
**JND ≅ 0.78N**

**Case B**
**JND ≅ 3.14N**

**Case C**
**JND ≅ 0.36N**

# Conclusions

- Rendering
  - The Karnopp model is well suited for haptic friction rendering
    - Simple to implement
    - Can capture pre-sliding displacement
  - The addition of a virtual coupling allows rendering of mass without causality issues
  - Proper selection of coupling parameters prevents the onset of limit cycles
- Perception
  - The addition of a moderate amount of friction helps subjects in difficult targeting tasks
  - Very sticky friction degrades subject performance
  - Subjects more readily distinguish small changes in dynamic friction than changes in static friction

# Future Work

- Better velocity sensing
- Extensions to two dimensions
- Capture other frictional effects
  - rising static friction
  - varied frequency stick-slip