

**Using Linear Complementarity Techniques
to Model and Simulate Multi-Rigid-Body Dynamics
with Contact and Friction**

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Friction: an essential component of MBD

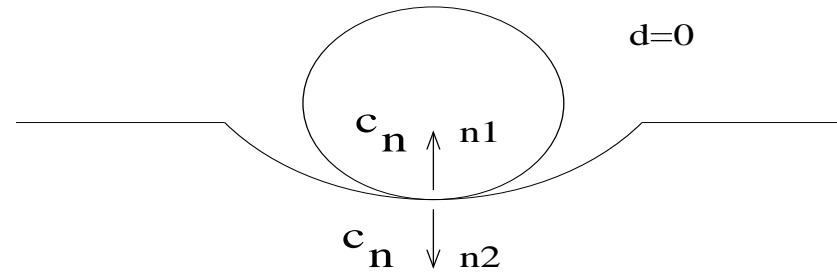
- **Robotics:** Prehensile manipulation is not possible without friction. In some devices friction is used as an active element (for example cheap, nonprehensile manipulators).
- **Virtual Reality:** The lack of it would substantially reduce the believability of a scene.
- The **Coulomb Friction Model** is the widely used model for static and dynamic friction.
- Unfortunately, Friction creates major difficulties in setting up a consistent model.

Model Requirements

- Generalized positions q and velocities v .
- No interpenetration $\Phi^{(j)}(q) \geq 0, 1 \leq j \leq n_{total}$.
- Compressive contact forces at a contact.
- Joint constraints $\Theta^{(i)}(q) = 0, 1 \leq i \leq m$.
- Coulomb friction, for friction coefficients $\mu^{(j)}$.
- Satisfaction of **acceleration based** Newton laws.
- Dynamic parameters: mass $M(q)$, external force $k(t, q, v)$.
- Impact resolution.

Normal velocity: v_n

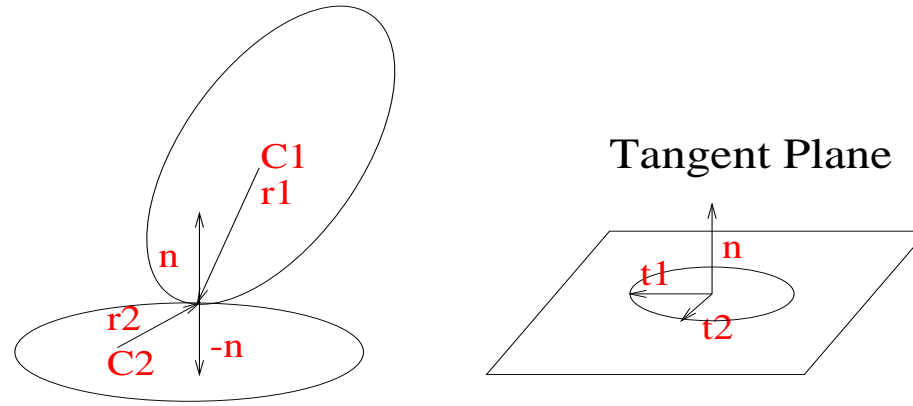
Normal impulse: c_n



Contact Model

- Contact configuration described by the (generalized) distance function $d = \Phi(q)$, which allows for some interpenetration. Feasible set: $\Phi(q) \geq 0$.
- Contact forces are compressive, $c_n \geq 0$.
- Contact forces act only when the contact constraint is exactly satisfied, or

$\Phi(q)$ is complementary to c_n or $\Phi(q)c_n = 0$, or $\Phi(q) \perp c_n$.



c_n is the normal impulse and $\beta = (\beta_1, \beta_2)^T$ is the tangential impulse;
 In generalized coordinates, q (Newton-Euler world coordinates):

$$n(q) = \begin{pmatrix} n \\ r_1 \times n \\ -n \\ r_2 \times (-n) \end{pmatrix} \quad d_i(q) = \begin{pmatrix} t_i \\ r_1 \times t_i \\ -t_i \\ r_2 \times (-t_i) \end{pmatrix}, \quad i = 1, 2.$$

Here F_c is the total contact force, $F_c = c_n n(q) + \hat{D}(q)\beta$.
 $\hat{D}(q)$ are the tangential directions, $\hat{D}(q) = [d_1(q), d_2(q)]$.

Coulomb Friction Model

- The contact impulse lies in a (circular) cone in 3D, or $\|\beta\| \leq \mu c_n$, where μ is the friction coefficient.
- When sliding exists at a contact, the tangential force is opposed to the sliding velocity, or

$$\beta = \operatorname{argmin}_{\hat{\beta}} v^T \hat{D}(q) \hat{\beta} \quad \text{subject to} \quad \|\hat{\beta}\| \leq \mu c_n.$$

- We have that the tangential velocity at the contact is v_T such that

$$|v_T| = \lambda = -v^T \hat{D}(q) \frac{\beta}{\|\beta\|}$$

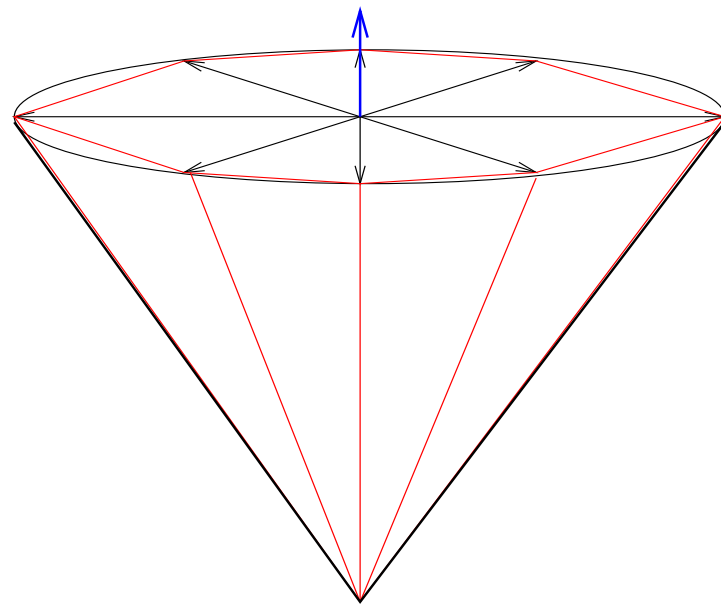
For given c_n and v , the frictional impulse has to maximize dissipation over all frictional contact impulses.

Discretized Friction Model

- d_i (GC) is the column corresponding to $t(\alpha_i)$, $\alpha_i \in [0, \pi]$, $i = 1, 2, \dots, l$, $D(q) = [d_1, d_2, \dots, d_l]$.
- To each tangential direction we attach a force $\beta_i \geq 0$, $i = 1, 2, \dots, l$. We denote by $\beta = (\beta_1, \beta_2, \dots, \beta_l)$.
- The frictional constraints become

$$\beta = \operatorname{argmin}_{\hat{\beta} \geq 0} v^T D(q) \hat{\beta} \quad \text{subject to} \quad \left\| \hat{\beta} \right\|_1 \leq \mu c_n.$$

Polygonal cone approximation to the Coulomb cone (3D).



Complementarity Formulation of Frictional Constraints

Continuous Cone: $\beta = \operatorname{argmin}_{\hat{\beta}} v^T \hat{D}(q) \hat{\beta}$ subject to $\|\hat{\beta}\| \leq \mu c_n$.

Discretized Cone: $\beta = \operatorname{argmin}_{\hat{\beta} \geq 0} v^T D(q) \hat{\beta}$ subject to $\|\hat{\beta}\|_1 \leq \mu c_n$.

Optimality Conditions: There exists a Lagrange multiplier $\lambda \geq 0$ such that

$$\lambda e + D^T v \geq 0 \quad \text{complementary to} \quad \beta \geq 0$$

$$\mu c_n - e^T \beta \geq 0 \quad \text{complementary to} \quad \lambda \geq 0$$

Here $e = [1, 1, \dots, 1]^T$. The Lagrange multiplier $\lambda \approx |v_T|$, the approximations approaches equality as the polygone approaches the circular cone.

Acceleration Formulation

$$M(q) \frac{d^2 q}{dt^2} - \sum_{i=1}^m \nu^{(i)} c_\nu^{(i)} - \sum_{j=1}^p \left(n^{(j)}(q) c_n^{(j)} + D^{(j)}(q) \beta^{(j)} \right) = k(t, q, \frac{dq}{dt})$$

$$\Theta^{(i)}(q) = 0,$$

$$\Phi^{(j)}(q) \geq 0,$$

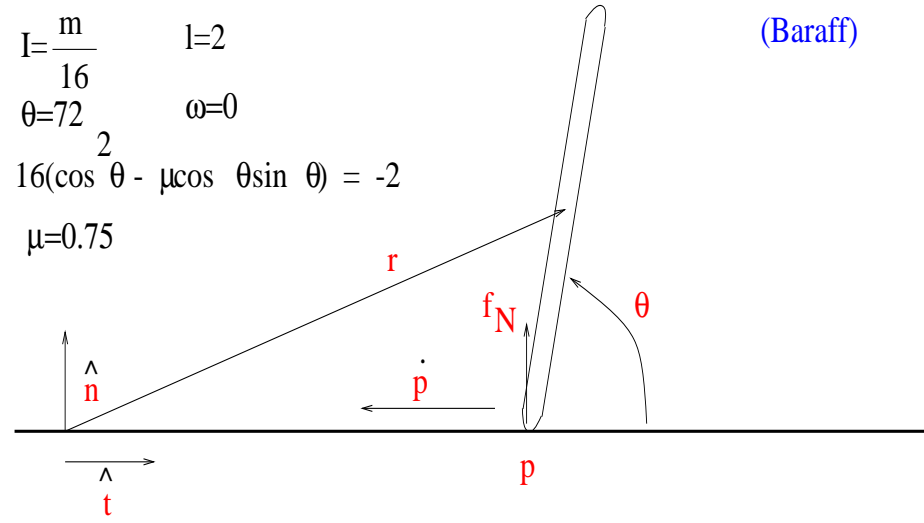
$$i = 1 \dots m$$

compl. to $c_n^{(j)} \geq 0, \quad j = 1 \dots p$

$$\beta = \operatorname{argmin}_{\hat{\beta}^{(j)}} v^T D(q)^{(j)} \hat{\beta}^{(j)}$$

subject to $\left\| \hat{\beta}^{(j)} \right\| \leq \mu^{(j)} c_n^{(j)}, \quad j = 1 \dots p$

We use the Coulomb Friction model, nondiscretized. In 2 dimensions the polygonal model and the Coulomb Friction model are equivalent.



$$p = r - \frac{l}{2} \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$$

Constraint: $\hat{n}p \geq 0$ (defined everywhere).

$$\hat{n}\ddot{p} = -g + f_N \left(\frac{1}{m} + \frac{l}{2I} (\cos^2(\theta) - \mu \sin(\theta) \cos(\theta)) \right)$$

$$\hat{n}\ddot{p}_a = -g - \frac{f_N}{m}$$

Painleve Paradox: No classical solutions!

Approaching Frictional Inconsistency

Assume that the system has a classical solution. Formulate the Euler method, half-explicit in velocities, with polyhedral approximation to the friction cone. Linearize the geometrical constraints.

$$M(v^{l+1} - v^{(l)}) - \sum_{i=1}^m \nu^{(i)} c_\nu^{(i)} - \sum_{j \in \mathcal{A}} (n^{(j)} c_n^{(j)} + D^{(j)} \beta^{(j)}) = hk$$

$$\nu^{(i)T} v^{l+1} = 0, \quad i = 1..m$$

$$\rho^{(j)} = n^{(j)T} v^{l+1} \geq 0, \quad \text{compl. to } c_n^{(j)} \geq 0, \quad j \in \mathcal{A}$$

$$\sigma^{(j)} = \lambda^{(j)} e^{(j)} + D^{(j)T} v^{l+1} \geq 0, \quad \text{compl. to } \beta^{(j)} \geq 0, \quad j \in \mathcal{A}$$

$$\zeta^{(j)} = \mu^{(j)} c_n^{(j)} - e^{(j)T} \beta^{(j)} \geq 0, \quad \text{compl. to } \lambda^{(j)} \geq 0, \quad j \in \mathcal{A}.$$

Here $\nu^{(i)} = \nabla \Theta^{(i)}$, $n^{(j)} = \nabla \Phi^{(j)}$. h is the time step. The set \mathcal{A} consists of the active constraints. Forces are replaced by impulses!

Matrix Form of the Integration Step

$$\begin{bmatrix} M & -\tilde{\nu} & -\tilde{n} & -\tilde{D} & 0 \\ \tilde{\nu}^T & 0 & 0 & 0 & 0 \\ \tilde{n}^T & 0 & 0 & 0 & 0 \\ \tilde{D}^T & 0 & 0 & 0 & \tilde{E} \\ 0 & 0 & \tilde{\mu} & -\tilde{E}^T & 0 \end{bmatrix} \begin{bmatrix} v^{(l+1)} \\ \tilde{c}_\nu \\ \tilde{c}_n \\ \tilde{\beta} \\ \tilde{\lambda} \end{bmatrix} + \begin{bmatrix} -Mv^{(l)} - hk \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tilde{\rho} \\ \tilde{\sigma} \\ \tilde{\zeta} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{c}_n \\ \tilde{\beta} \\ \tilde{\lambda} \end{bmatrix}^T \begin{bmatrix} \tilde{\rho} \\ \tilde{\sigma} \\ \tilde{\zeta} \end{bmatrix} = 0, \quad \begin{bmatrix} \tilde{c}_n \\ \tilde{\beta} \\ \tilde{\lambda} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \tilde{\rho} \\ \tilde{\sigma} \\ \tilde{\zeta} \end{bmatrix} \geq 0.$$

Linear Complementarity Problems (LCP)

$$s = \mathcal{M}x + q, s \geq 0, x \geq 0, s^T x = 0.$$

- Examples: Linear and Quadratic Programming.
- Important classes of matrices: **PSD** ($x^T \mathcal{M}x \geq 0, \forall x$) and **copositive** ($x^T \mathcal{M}x \geq 0, \forall x \geq 0$).
- LCP's involving copositive matrices do not have a solution in general.
- Let \mathcal{M} be copositive. If, $x \geq 0$ and $x^T \mathcal{M}x = 0$ implies $q^T x \geq 0$, then the **LCP** has a solution that can be found by Lemke's algorithm.

Theorem

Consider a (mixed) **LCP** of the form

$$\begin{pmatrix} 0 \\ 0 \\ s \end{pmatrix} = \begin{pmatrix} M & -F & -H \\ F^T & 0 & 0 \\ H^T & 0 & N \end{pmatrix} \begin{pmatrix} x \\ y \\ \lambda \end{pmatrix} + \begin{pmatrix} -k \\ 0 \\ 0 \end{pmatrix}$$
$$s \geq 0; \quad \lambda \geq 0; \quad \lambda^T s = 0.$$

If M is a positive definite matrix, N a copositive matrix ($x \geq 0 \Rightarrow x^T N x \geq 0$) then the above **LCP** has a solution. Lemke's algorithm will always find a solution λ of the **LCP** obtained by eliminating x and y . A solution (x, y, λ) of the original **LCP** can be recovered by solving for x and y in the first two rows of the mixed **LCP**.

The time-stepping method is guaranteed to have a solution!

Accommodating Stiffness

- The scheme is based on an explicit Euler scheme and as such cannot accommodate stiffness well (such as systems with very large damping or elastic forces).
- A stiff method should also accommodate the case where there are no contacts and joints. So it should also apply to

$$\begin{aligned}\frac{dq}{dt} &= v; \\ M(q)\frac{dv}{dt} &= k(q, v).\end{aligned}$$

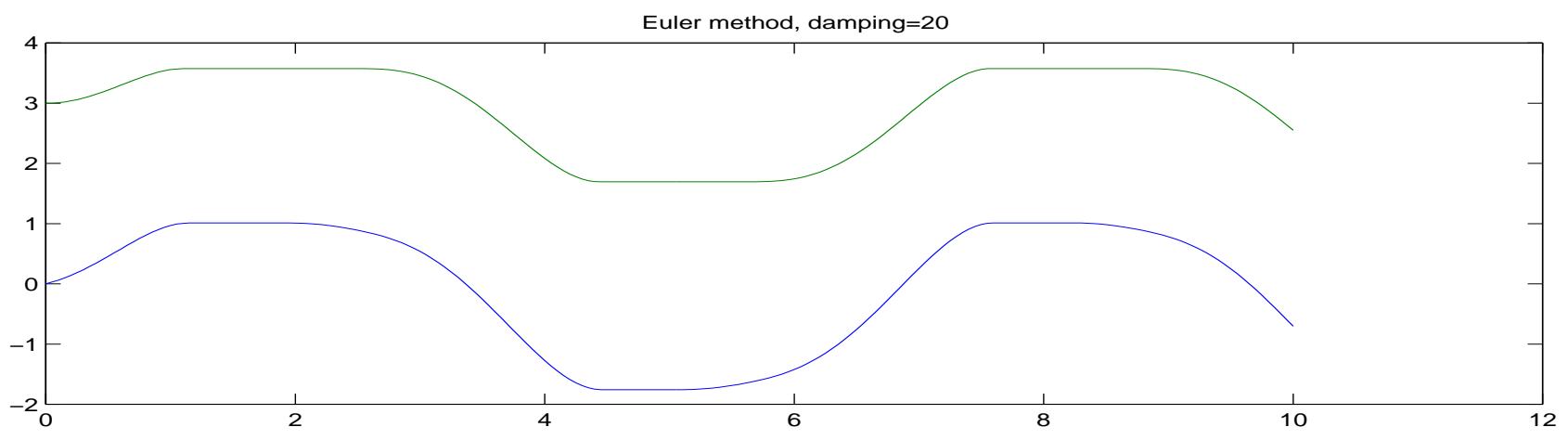
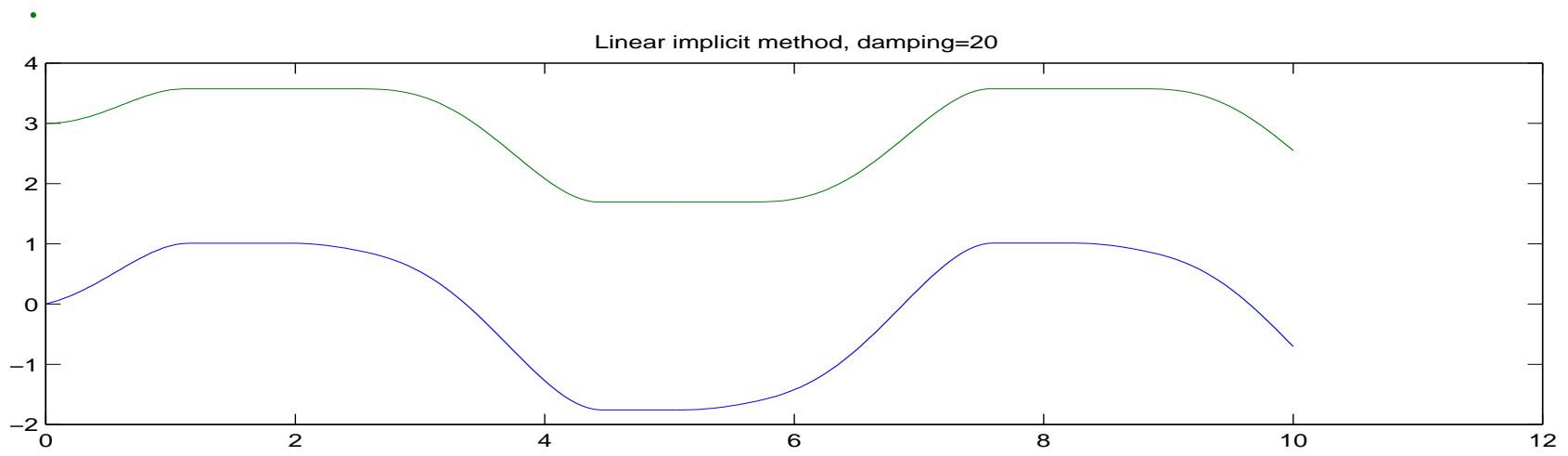
- However, we are still interested in an explicit scheme since otherwise the scheme for the case including contacts would translate into a **nonlinear complementarity problem**.



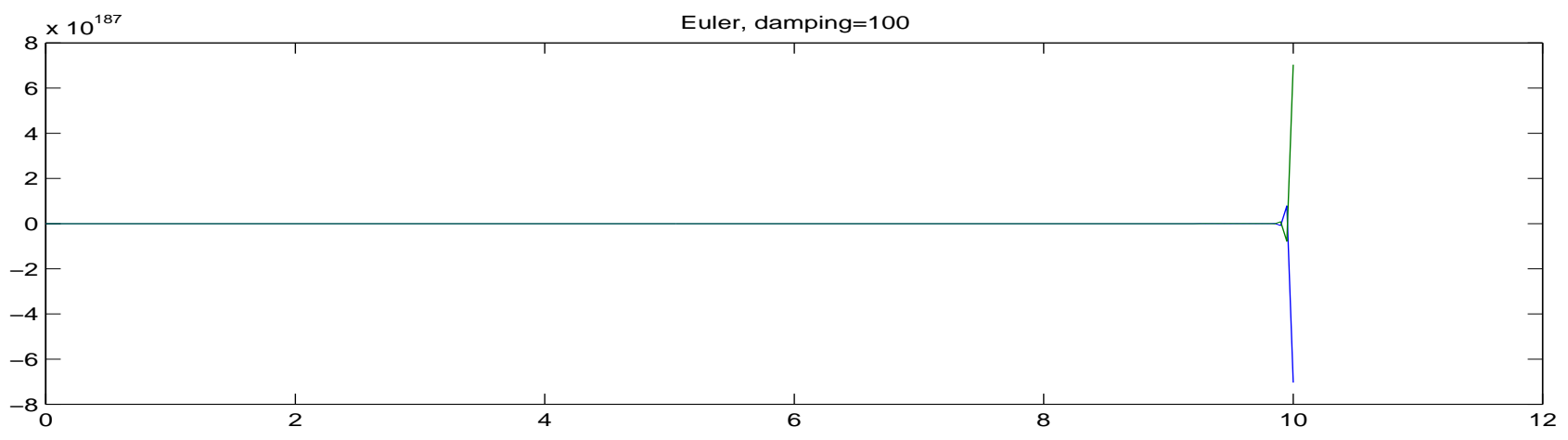
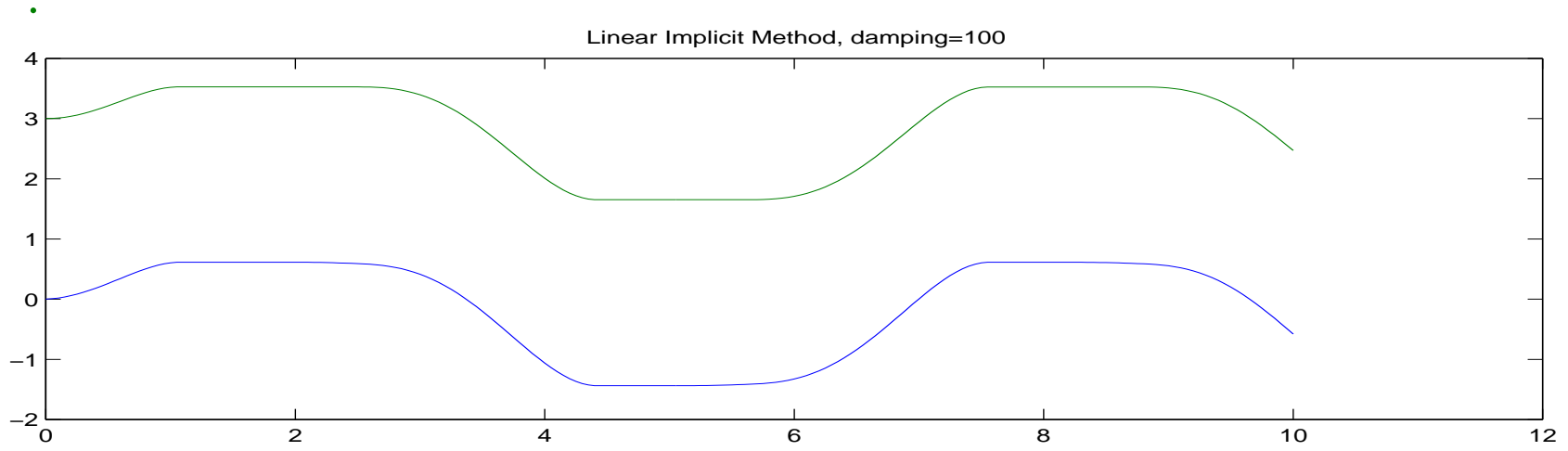
Example of a run on a stiff problem

- Identical objects, of mass 1 and with $\mu = 0.4$.
- Initial distance between objects is 3.
- An External force $F = 20\cos(t)$ acts on the object in the left.
- Time step 0.05, integration interval 10.
- The damper exerts a force $F_D = \delta (-\dot{x}_1 + \dot{x}_2)$ on the first object and $-F_D$ on the second object.

Results for $\delta = 20$



Results for $\delta = 100$, note instability



Linearly Implicit Schemes

$$\begin{aligned}
 q^{(n+1)} &= q^{(n)} + h v^{(n+1)}, \\
 M(q^{(n)}) \frac{v^{(n+1)} - v^{(n)}}{h} &= k(q^{(n)}, v^{(n)}) + h \nabla_q k(q^{(n)}, v^{(n)}) v^{(n+1)} \\
 &\quad + \nabla_v k(q^{(n)}, v^{(n)}) (v^{(n+1)} - v^{(n)}),
 \end{aligned}$$

or, after solving for $v^{(n+1)}$,

$$q^{(n+1)} = q^{(n)} + h v^{(n+1)},$$

$$\begin{aligned}
 v^{(n+1)} &= \left[M(q^{(n)}) - h^2 \nabla_q k(q^{(n)}, v^{(n)}) - h \nabla_v k(q^{(n)}, v^{(n)}) \right]^{-1} \times \\
 &\quad \left[M(q^{(n)}) v^{(n)} + h k(q^{(n)}, v^{(n)}) - h \nabla_v k(q^{(n)}, v^{(n)}) v^{(n)} \right]
 \end{aligned}$$

Well-posedness of the method

- To apply the theorem, it will be essential to have

$$\widehat{M} = \left[M \left(q^{(n)} \right) - h^2 \nabla_q k \left(q^{(n)}, v^{(n)} \right) - h \nabla_v k \left(q^{(n)}, v^{(n)} \right) \right] \succ 0,$$

and not only invertible.

- If $k(q, v) = -\nabla U(q) - \Gamma(v)$, where $\Gamma(v)$ is a damping-type force, then near an equilibrium point one could expect $\nabla_{qq}U(q) \succeq 0$ and $\nabla_v \Gamma(v) \succeq 0$.
- However, positive definiteness of \widehat{M} cannot generally be ensured for moderate values of h when the linear system has eigenvalues with a large negative real part.

Damping and elastic forces

- Most stiff forces in rigid multibody dynamics originate in springs and dampers attached between two points of the multibody system.
- For that case, we have $k(t, q, v) = k_s(t, q, v) + k_1(t, q, v)$, where

$$k_s(t, q, v) = - \sum_{i=1}^{n_\gamma} \gamma_i \phi^{(i)}(q) \nabla_q \phi^{(i)}(q) - \sum_{j=1}^{n_\delta} \delta_j \nabla_q \psi^{(j)}(q) \left(\nabla_q \psi^{(j)T}(q) v \right)$$

Here $\gamma_i, i = 1, \dots, n_\gamma$ are spring constants and $\delta_j, j = 1, \dots, n_\delta$ are the damper constants. $\phi^{(i)}(q)$ and $\psi^{(j)}(q)$ describe distances between points in the system. $k_1(t, q, v)$ are the nonstiff forces.

- We can then approximate, for the purpose of the linearly implicit method

$$\begin{aligned} \nabla_q k(t, q, v) &\approx - \sum_{i=1}^{n_\gamma} \gamma_i \nabla_q \phi^{(i)}(q) \nabla_q \phi^{(i)T}(q), \\ \nabla_v k(t, q, v) &\approx - \sum_{j=1}^{n_\delta} \delta_j \nabla_q \psi^{(j)}(q) \nabla_q \psi^{(j)T}(q). \end{aligned}$$

Linearly Implicit LCP

$$\begin{bmatrix} \widehat{M} & -\tilde{\nu} & -\tilde{n} & -\tilde{D} & 0 \\ \tilde{\nu}^T & 0 & 0 & 0 & 0 \\ \tilde{n}^T & 0 & 0 & 0 & 0 \\ \tilde{D}^T & 0 & 0 & 0 & \tilde{E} \\ 0 & 0 & \tilde{\mu} & -\tilde{E}^T & 0 \end{bmatrix} \begin{bmatrix} v^{(l+1)} \\ \tilde{c}_\nu \\ \tilde{c}_n \\ \tilde{\beta} \\ \tilde{\lambda} \end{bmatrix} + \begin{bmatrix} -Mv^{(l)} - h\widehat{k} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tilde{\rho} \\ \tilde{\sigma} \\ \tilde{\zeta} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{c}_n \\ \tilde{\beta} \\ \tilde{\lambda} \end{bmatrix}^T \begin{bmatrix} \tilde{\rho} \\ \tilde{\sigma} \\ \tilde{\zeta} \end{bmatrix} = 0, \quad \begin{bmatrix} \tilde{c}_n \\ \tilde{\beta} \\ \tilde{\lambda} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \tilde{\rho} \\ \tilde{\sigma} \\ \tilde{\zeta} \end{bmatrix} \geq 0.$$

$$\widehat{M} = M + h^2 \sum_{i=1}^{n_\gamma} \gamma_i \nabla_q \phi(q) \nabla_q \phi^T(q) + h \sum_{i=1}^{n_\delta} \delta_i \nabla_q \psi(q) \nabla_q \psi^T(q) \succ 0.$$

Properties of the linearly implicit scheme

- The scheme continues to be well defined for any values of h : the LCP is solvable.
- As $\delta \rightarrow \infty$, and $\gamma \rightarrow \infty$ the solution to the linearly implicit LCP approaches the solution of the nonstiff LCP that has the additional equality constraints $\nabla_q \phi^{(i)}(q^{(l)})^T v^{(l+1)} = -h \phi^{(i)}(q^{(l)})$ and $\nabla_q \psi^{(j)}(q^{(l)}) v^{(l+1)}(q) = 0$, whenever the limit system has a pointed friction cone. Stiff links behave like joints, for large stiffness parameters!
- Denoting $\hat{w} = \left(v^{(l)} + khM(q^{(l)})^{-1}k_1(t^{(l)}, q^{(l)}, v^{(l)}) \right)^T$, we have

$$v^{(l+1)T} M(q^{(l)}) v^{(l+1)} + \sum_{i=1}^{n_\gamma} \gamma_i \left(\phi^{(i)T}(q^{(l)}) + h \nabla_q \phi^{(i)}(q^{(l)})^T v^{(l+1)} \right)^2 \leq$$

$$+ \hat{w}^T M(q^{(l)}) \hat{w} + \sum_{i=1}^{n_\gamma} \gamma_i \left(\phi^{(i)}(q^{(l)}) \right)^2.$$

This ensures the stability of the linear model, as in the unconstrained case.

Collision Assumptions

- The collision within a system of bodies consists of
 - * **Compression Phase:** interpenetration is prevented by compression impulses from each constraint involved in the collision (even joints).
 - * **Decompression Phase:** A proportion of e_i (elasticity coefficient) from the normal compression impulse is restituted to the system by each contact constraint Φ_i (**Poisson hypothesis**).
Interpenetration is prevented by decompression impulses.
- The compression/decompression phases following an imminent interpenetration detection are simultaneous for all the bodies involved.

Impact Model: Compression Phase

Collision are instantaneous. Since we have a force-velocity approach, compression can be interpreted as a regular time-step with $h = 0$. Same solvability results apply.

$$M(v^c - v^-) - \sum_{i=1}^m \nu^{(i)} c_{\nu}^{c(i)} - \sum_{j=1}^p (n^{(j)} c_n^{c(j)} + D^{(j)} \beta^{c(j)}) = 0$$

$$\nu^{(i)T} v^c = 0, \quad i = 1..m$$

$$n^{(j)T} v^c \geq 0, \quad \text{compl to} \quad c_n^{c(j)} \geq 0, \quad j = 1..p$$

$$\lambda^{c(j)} e^{(j)} + D^{(j)T} v^c \geq 0, \quad \text{compl. to} \quad \beta^{c(j)} \geq 0, \quad j = 1..p$$

$$\mu^{(j)} c_n^{c(j)} - e^{(j)T} \beta^{c(j)} \geq 0, \quad \text{compl. to} \quad \lambda^{c(j)} \geq 0, \quad j = 1..p$$

Impact Model: Decompression Phase

Poisson Hypothesis $F^r = \sum_{j=1}^p e_j n^{(j)} c_n^{c(j)}$.

$$M(v^+ - v^c) - \sum_{i=1}^m \nu^{(i)} c_\nu^{x(i)} - \sum_{j=1}^p (n^{(j)} c_n^{x(j)} + D^{(j)} \beta^{x(j)}) = F^r$$

$$\nu^{(i)T} v^+ = 0,$$

$$i = 1..m$$

$$n^{(j)T} v^+ \geq 0,$$

compl to

$$c_n^{x(j)} \geq 0, \quad j = 1..p$$

$$\lambda^{x(j)} e^{(j)} + D^{(j)T} v^+ \geq 0,$$

compl. to

$$\beta^{x(j)} \geq 0, \quad j = 1..p$$

$$\mu^{(j)} c_n^{x(j)} - e^{(j)T} \beta^{x(j)} \geq 0,$$

compl. to

$$\lambda^{x(j)} \geq 0, \quad j = 1..p$$

Decompression Solution for a Particular Case

Assumptions

- **(a)** The contacts are frictionless.
- **(b)** All new contacts generated by collision have the same elasticity coefficient ϵ .
- **(c)** The elasticity coefficients characterizing the other contacts are less than ϵ , $e_j \leq \epsilon$, $1 \leq j \leq p$.
- **(d)** The pre-collision velocities satisfy the contact constraints exactly, $(n^{(j)}(q^-))^T v^- = 0$.

Solution and Properties

- **(a)** Just the compression phase is solved by **LCP**.
- **(b)** $v^+ = (1 + \epsilon)v^c - v^-$, $c_n^{x(i)} = (\epsilon - e_i)c_n^{c(i)}$.
- **(c)** $v^{+T} M v^+ \leq v^- M v^-$, the kinetic energy does not increase after the collision (desirable, but not guaranteed for other cases).
- **(d)** This decompression resolution can be used as a general strategy where computational efficiency is required.

Algorithm

$v = v^0, q = q^0; time = 0;$

while ($time < T$)

$q_{new} = q + hv;$

Find $(v_{new}, \tilde{c}_\nu, \tilde{c}_n, \tilde{\beta}, \tilde{\lambda}) \in \mathcal{L}(v, hk)$ (k at (q, v) , the rest at q_{new});

if (no collision detected between $time$ and $time + h$)

$time = time + h, q = q_{new}, v = v_{new};$

else

Estimate the collision data $time_{new}, q_{new}$ and v^- ;

Find $(v^c, \tilde{c}_\nu^c, \tilde{c}_n^c, \tilde{\beta}^c, \tilde{\lambda}^c) \in \mathcal{L}(v^-, 0);$

Find $(v^+, \tilde{c}_\nu^x, \tilde{c}_n^x, \tilde{\beta}^x, \tilde{\lambda}^x) \in \mathcal{L}(v^c, F^r)$ (or $v^+ = (1 + e)v^c - v^-$);

$time = time_{new}, v = v^+, q = q_{new};$

end if

end while

LCP contact list

- The initial point is assumed to be feasible for all constraints.
- At each regular (non-collision) step the contact list consists of the union of the set of contacts that the LCP has decided to maintain ($v_n = 0$) at the previous step with the set of contacts that exhibit interpenetration ($\Phi^i(q) < 0$).
- When a collision is detected ($\Phi^i(q)$ changes sign from + to -), the contacts for which impact is imminent are added to the contact list.
- The decompression phase uses the same contact list as the compression phase.

Conclusions

- We present a complementarity-based model for multi-rigid body with contact and friction that is guaranteed to be solvable for the most common types of stiff forces.
- The model is based on a discretization of the friction cone and can be as close to the Coulomb model as desired.
- Stiffness is accommodated by means of a linearly implicit scheme for the case of damping forces. In the limit, stiff links behave like joints.
- If the mass matrix $M(q^{(l)})$ is constant and the elastic forces are linear, then the velocity stays bounded at all times. This recovers the analogue of the stability result for differential equations.
- These conclusions were validated with several simulations, where **PATH** was used to solve the LCP.

Future work

- Higher order schemes between collisions and discontinuities. Extrapolation is attractive since it comes with a minor loss of stability and it adapts very well to this context.
- Alternative friction models, that solve convex subproblems, while maintaining most physical properties of this model. We are currently working on a mixed penalty complementarity framework.
- Interface this approach with enhanced geometrical approaches that compute signed distance functions and feasible configuration fast.
- If a projection is used, how can energy balance be maintained?
- Can a fixed timestep scheme be used, which solves only one LCP per iteration?

Simulations for the cannonball arrangement, $h=0.05$

Problem	Bodies	Contacts	μ	CPU time (s)
1	10	21	0.2	0.04
2	10	21	0.8	0.03
3	21	52	0.2	0.28
4	21	52	0.8	0.20
5	36	93	0.2	0.81
6	36	93	0.8	0.82
7	55	146	0.2	2.10
8	55	146	0.8	2.07
9	210	574	0.0	0.80
10	210	574	0.2	174.29
11	210	574	0.8	FAIL