Implementing Stochastic Volatility with Jumps: Risk Management & Hedging Strategies

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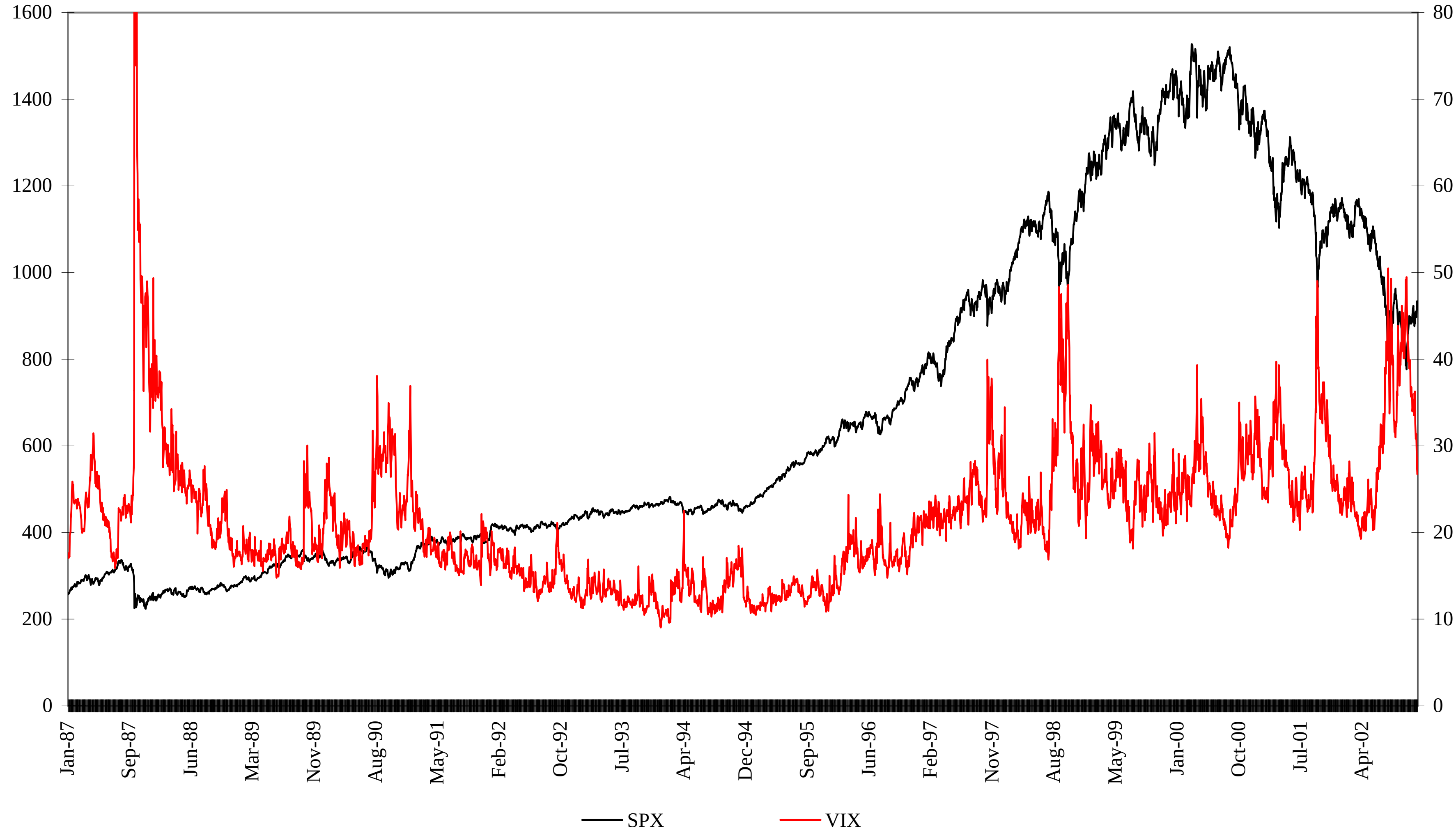
MORGAN STANLEY & CO.

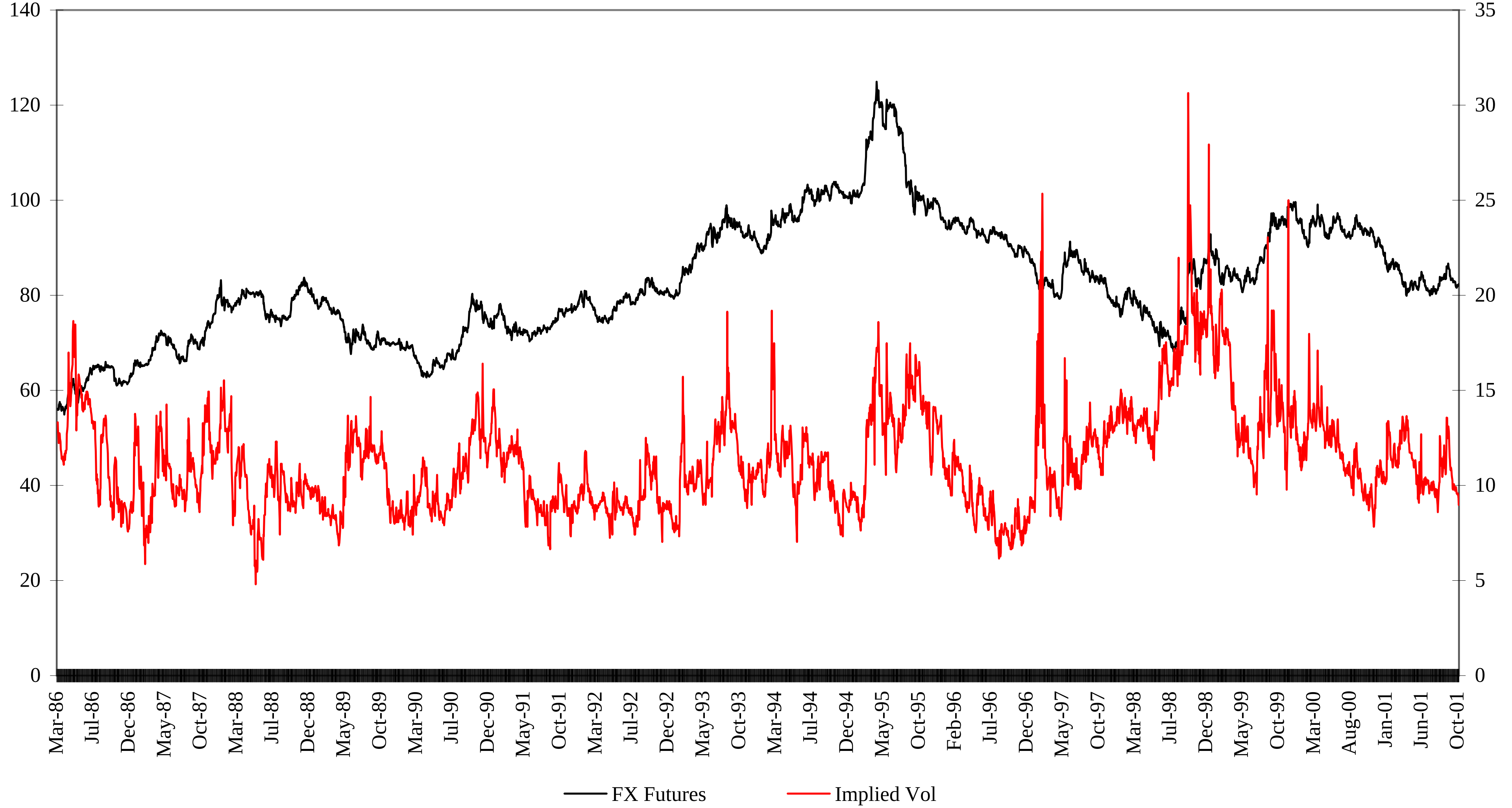
Stochastic Volatility and Jumps: Risk Management and Hedging Strategies

- Volatility Risk and Jump Risk (Gap Risk)
- Discipline and Risk Management
- The Role of Models
  - Greeks: delta, gamma, kappa/vega, theta, PV01
  - Stress Tests and Scenario Analyses: revalue portfolios for extreme, but plausible, market changes
  - Valuation and Relative Value Trading
  - Additional Tools for Understanding Market Dynamics and Risks
- New Markets: CDO’s and Basket Default Swaps
  - Elements of gap risk
  - Stress Tests: what happens to tranches when one or several names go into financial distress?
  - Model Correlation for Relative Value Trading

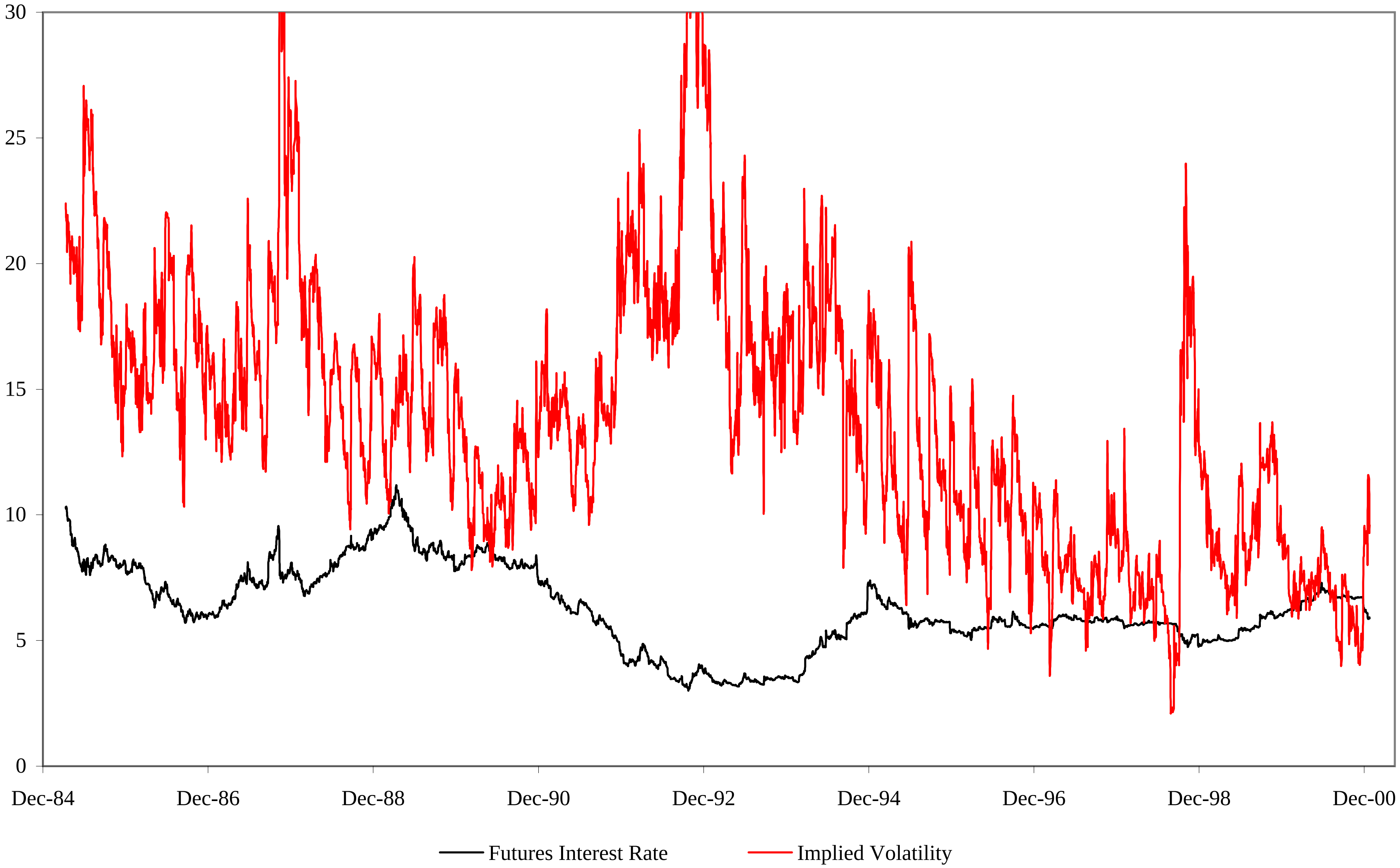
S&P 500 Stock Price Index and the VIX Volatility Index



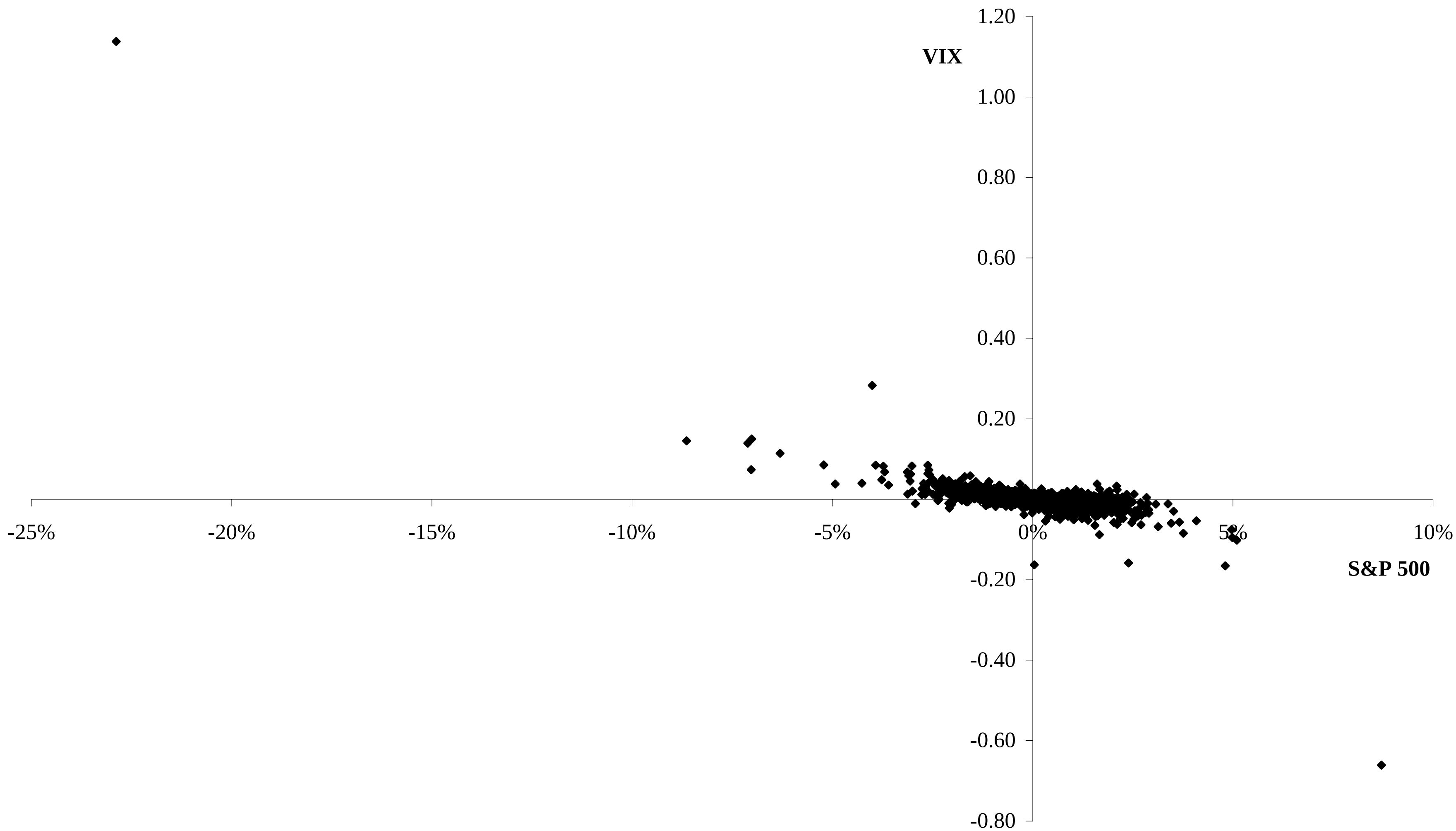
USD/JPY Futures FX Rate and Implied Volatility



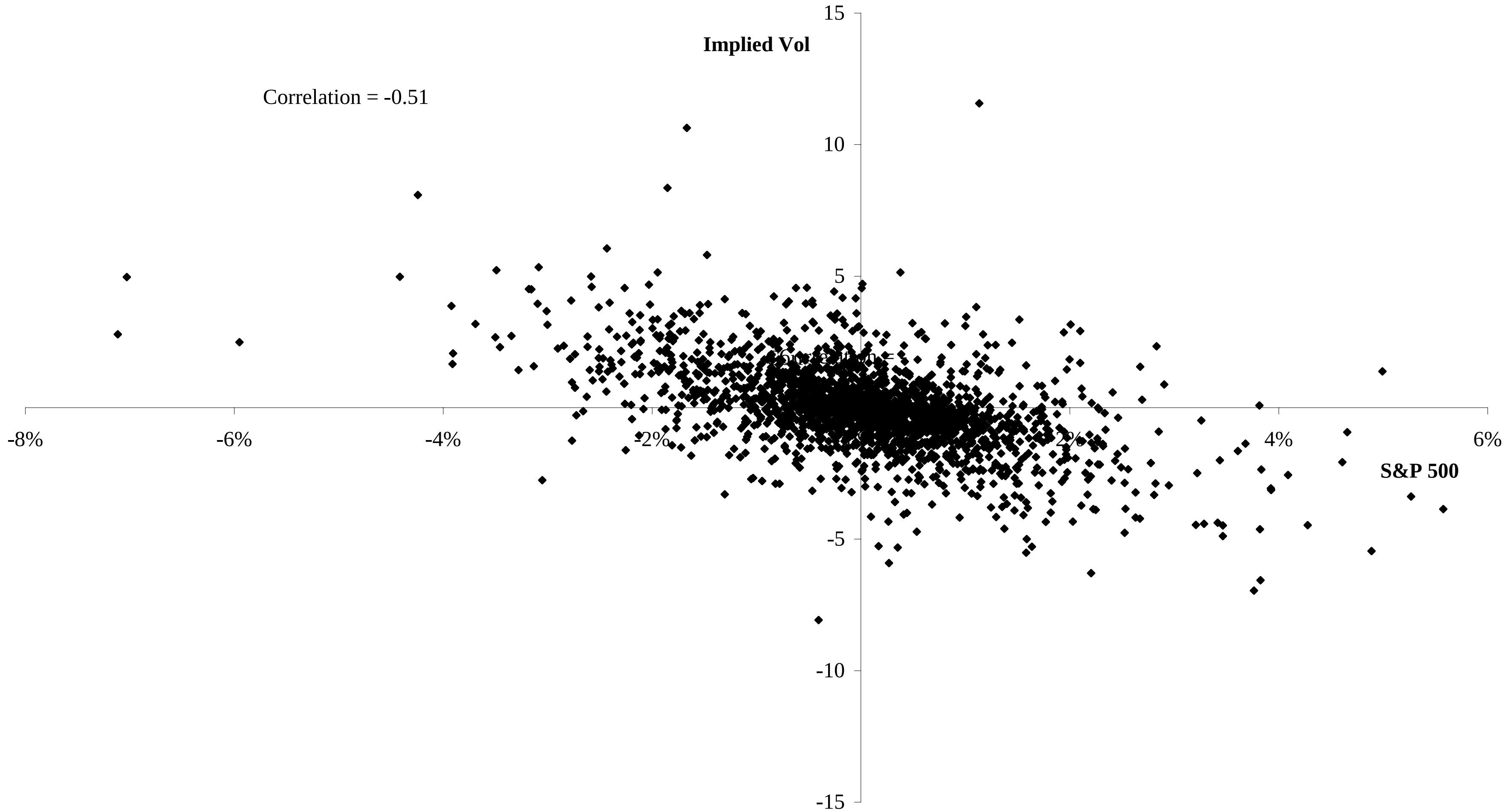
Futures Interest Rate and Its Implied Volatility, United States



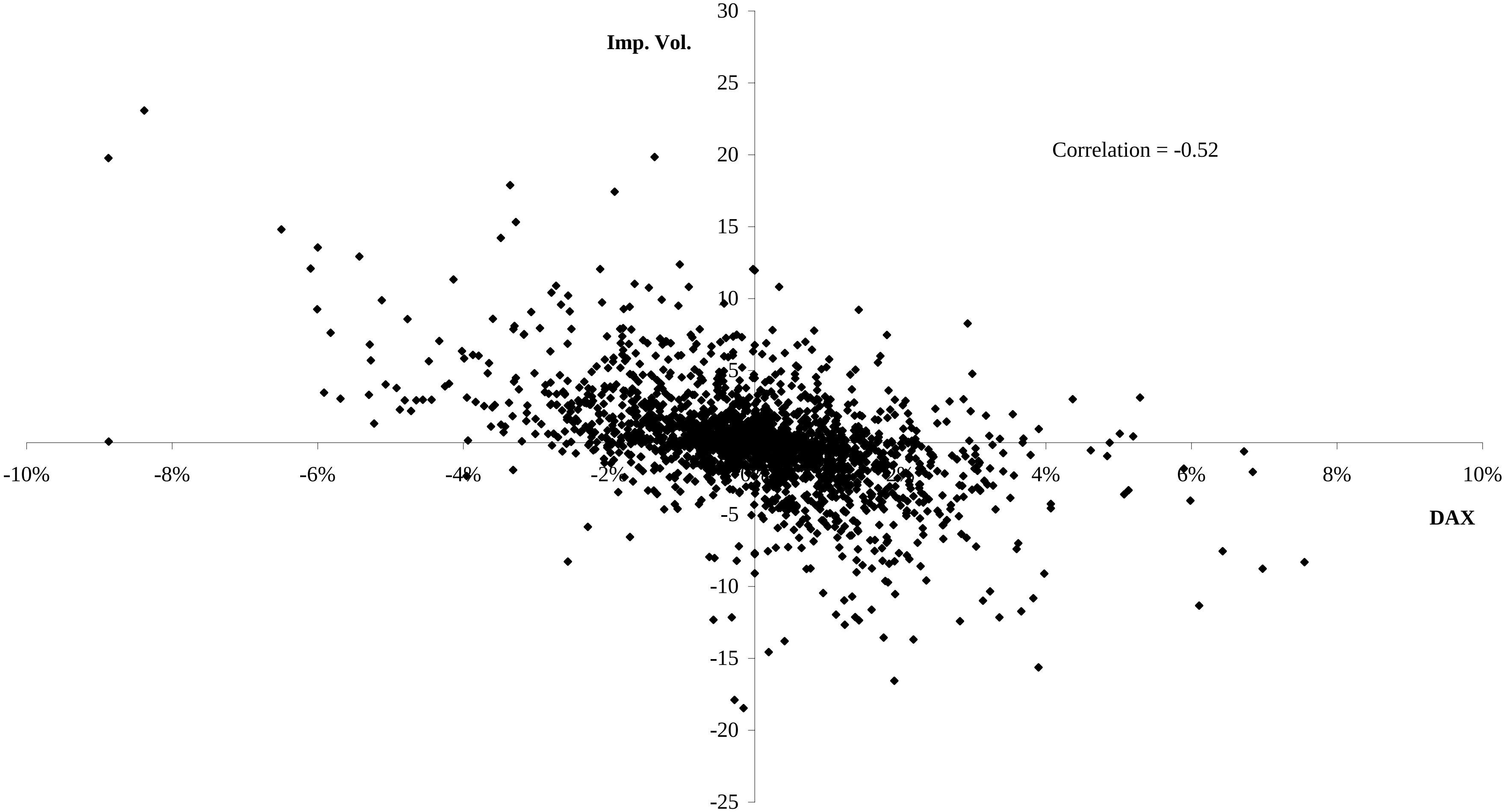
Change in the CBOE VIX vs Change in the log of the S&P500 Index



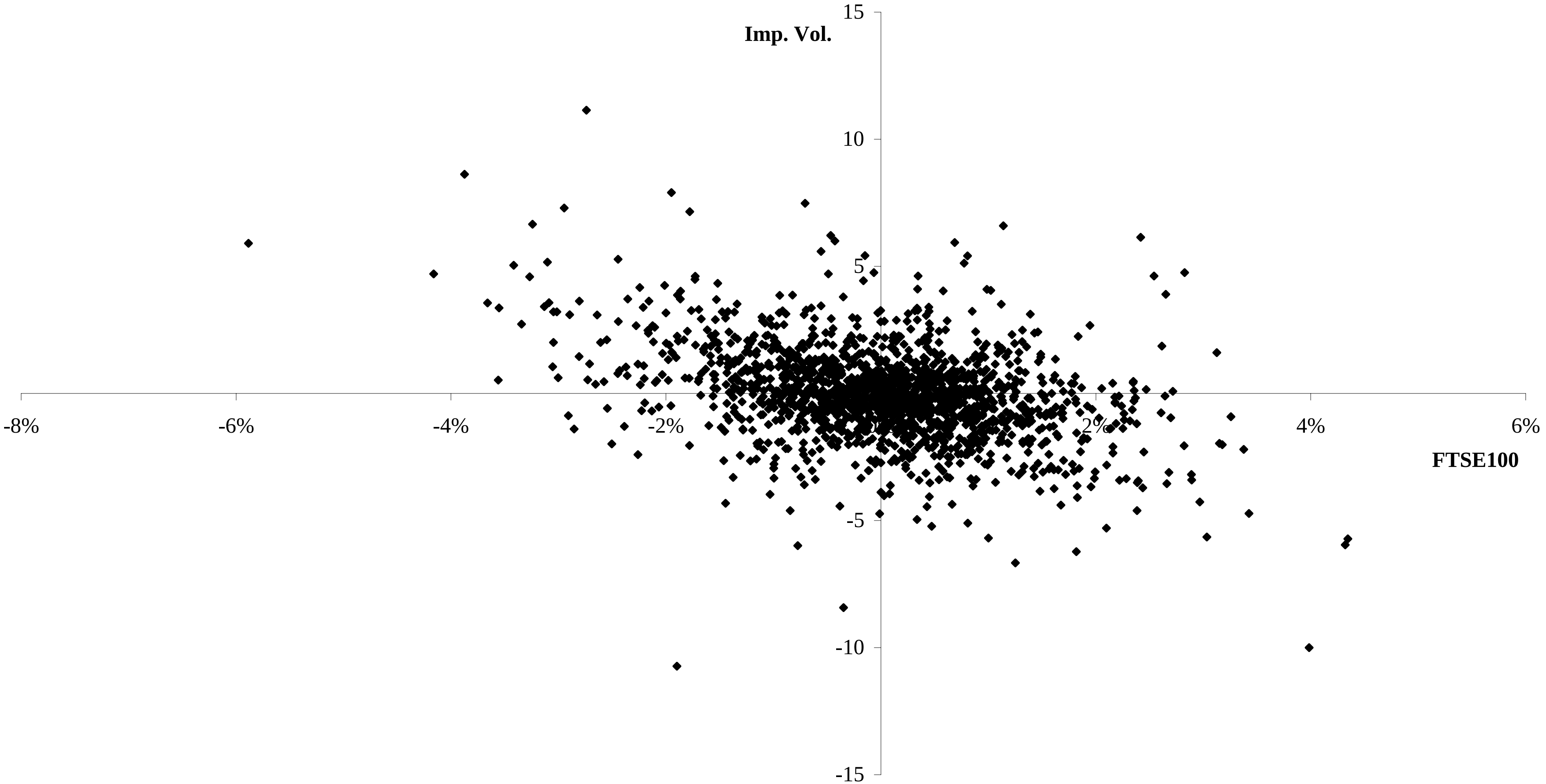
Change in Implied Vol vs Change in Log of Price Index
S&P 500, 1994-2002



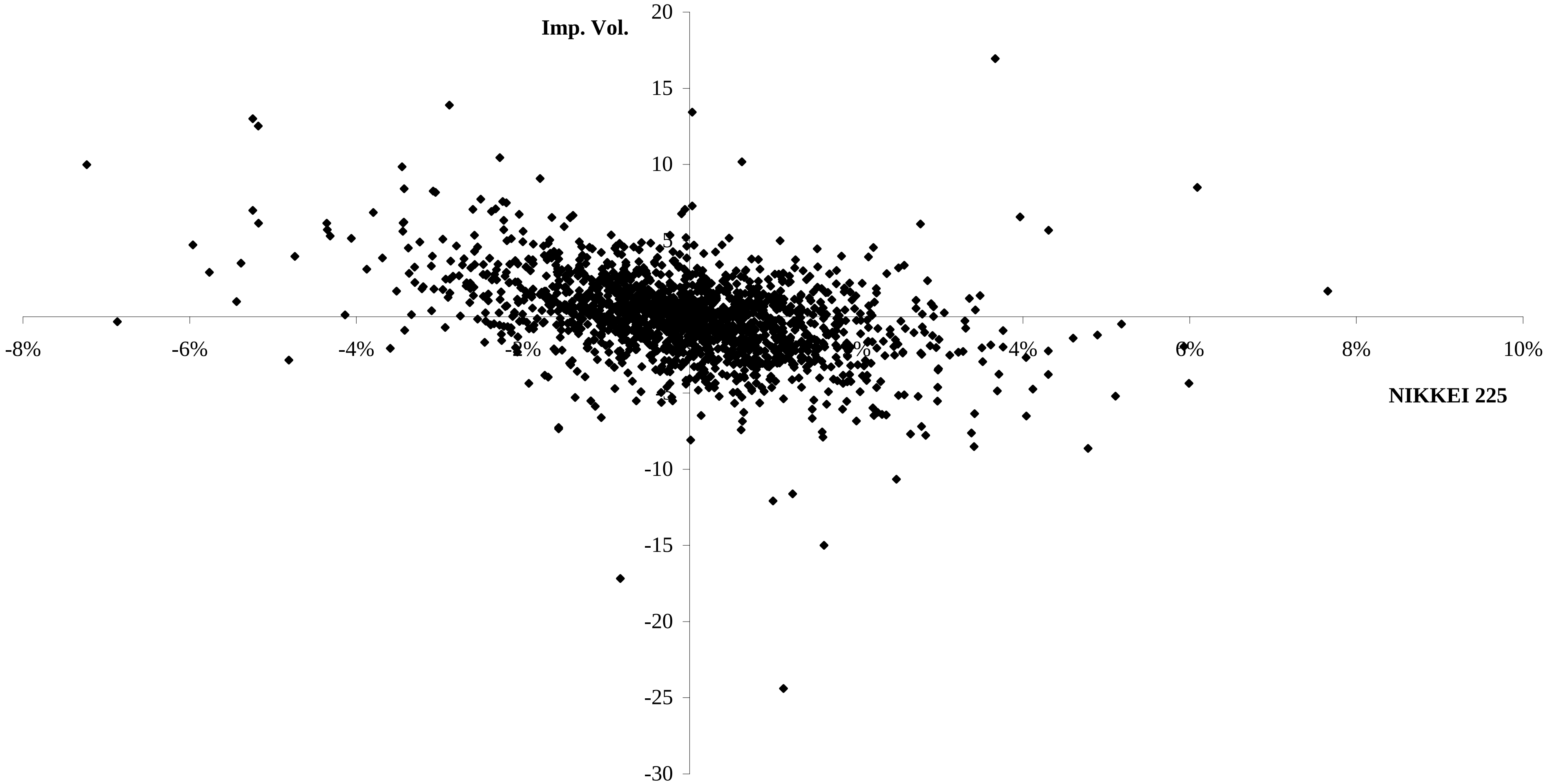
Change in Implied Vol vs. Change in Log of Price Index
DAX, 1994-2002



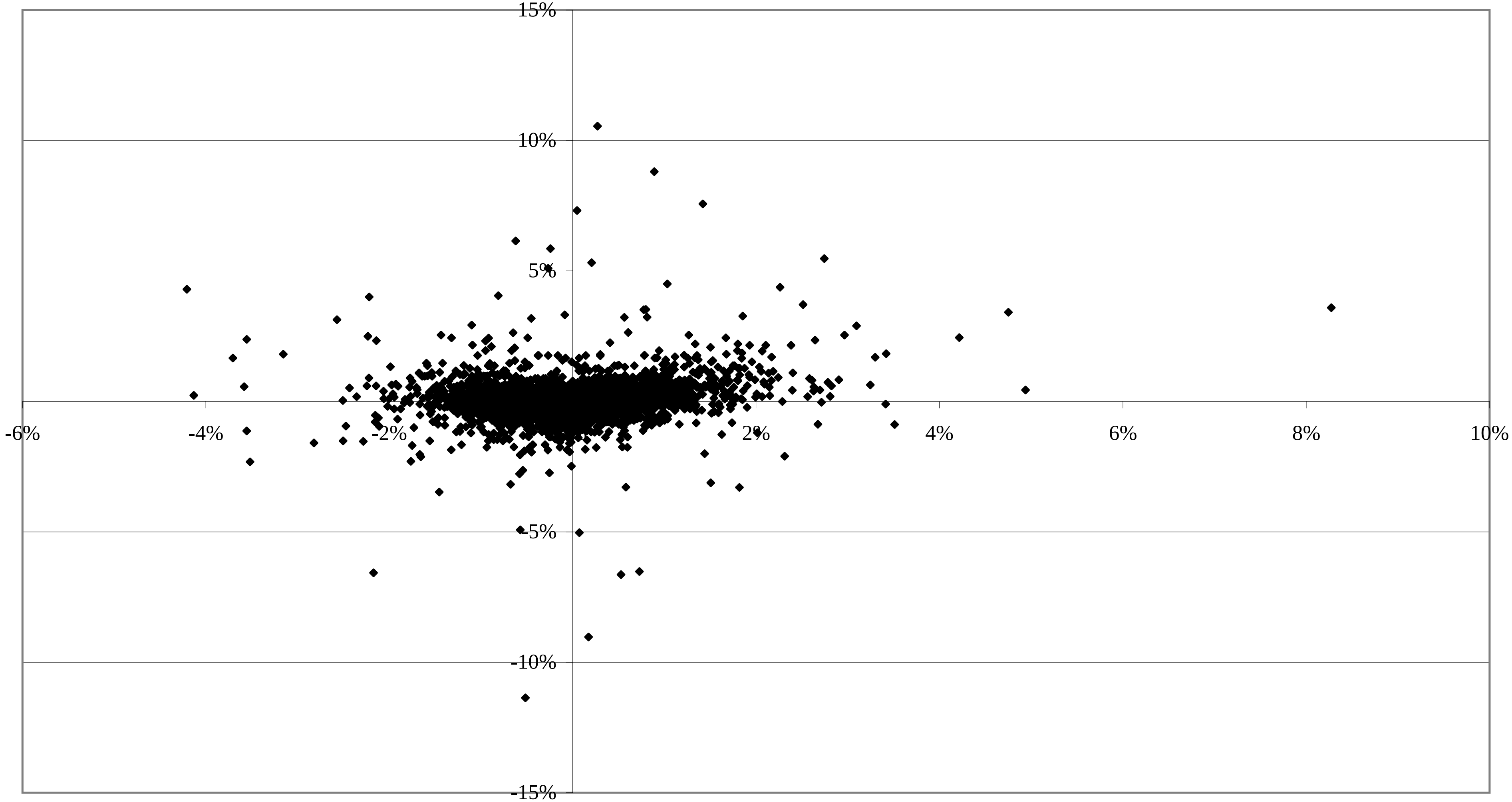
Implied Vol vs. Change in Log of Price Index
FTSE 100



Implied Vol vs. Change in Log of Price Index
NIKKEI 225



Change in Implied Vol versus Change in Futures FX Rate for USD/JPY



Change in Implied Volatility versus Change in Futures Rate, United States

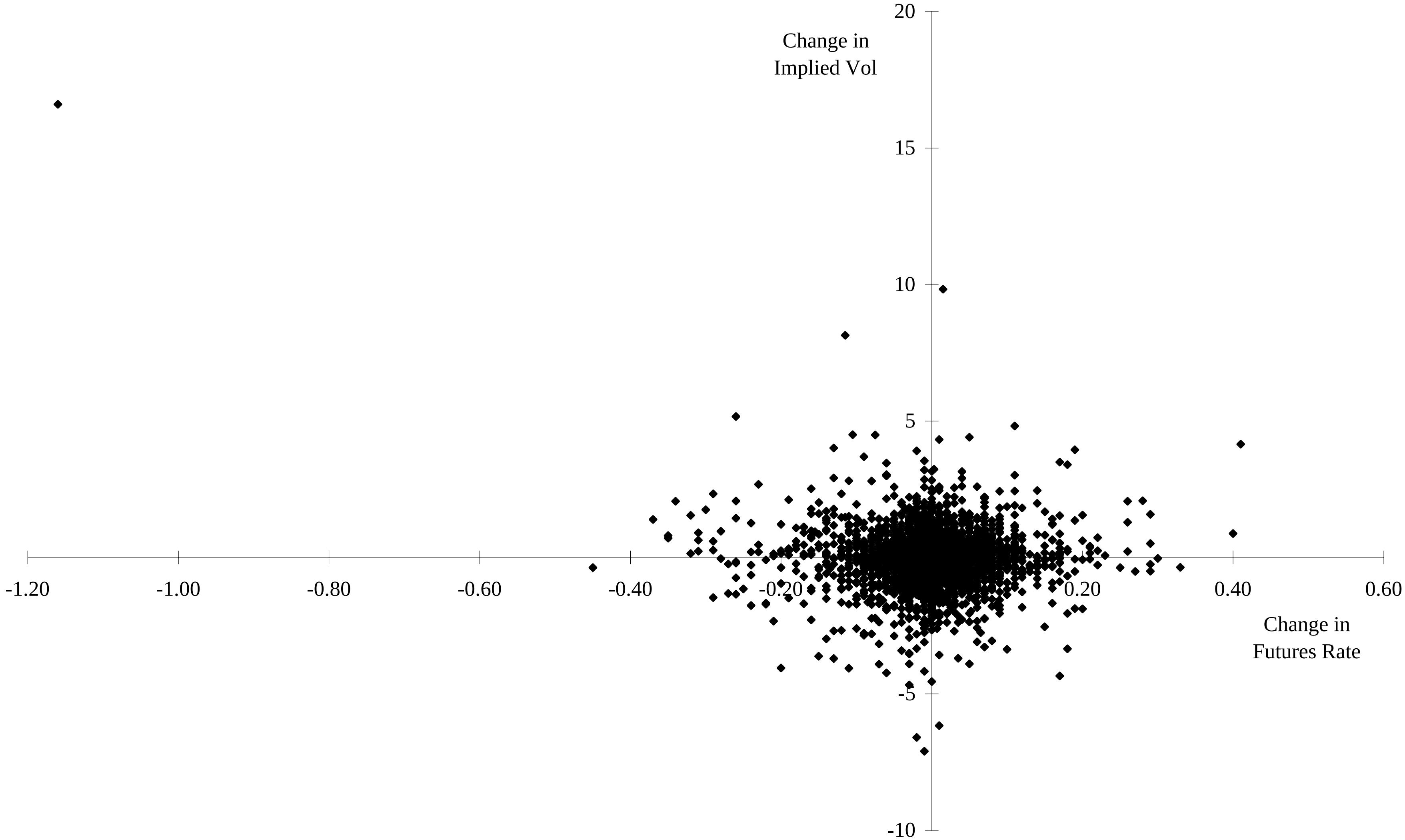


Figure 6
Change in Implied Volatility versus Change in Futures Rate, Germany

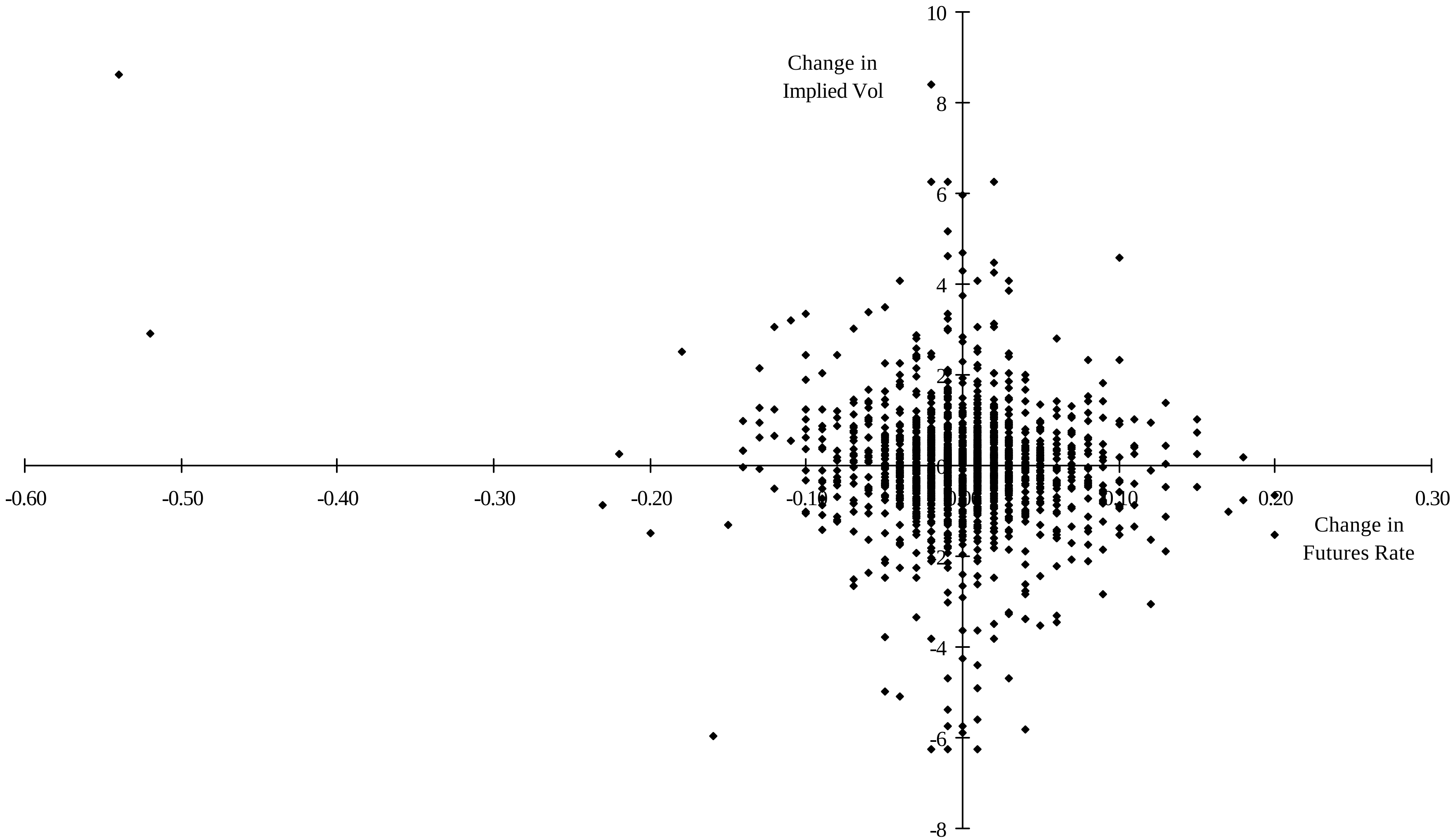
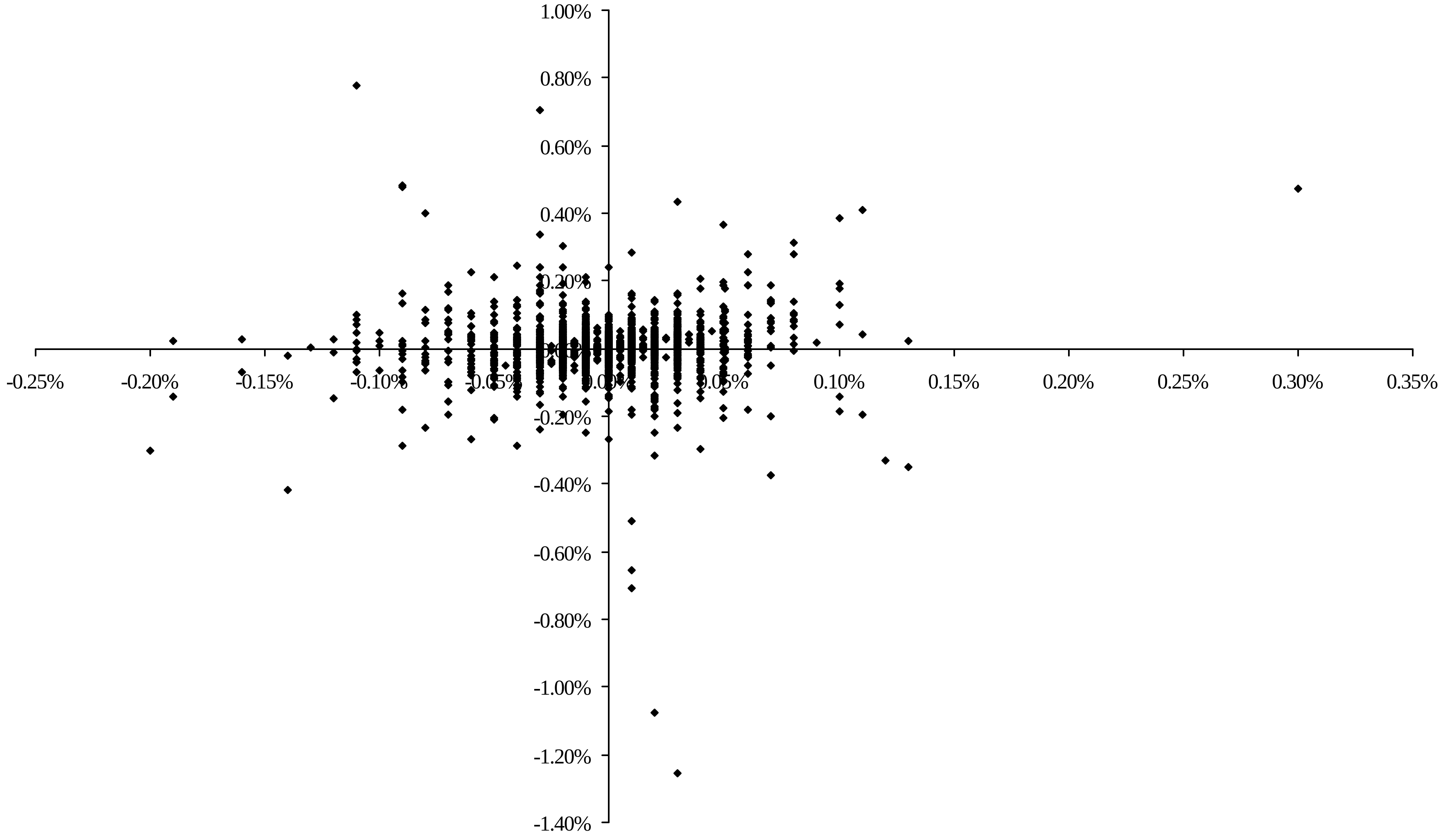
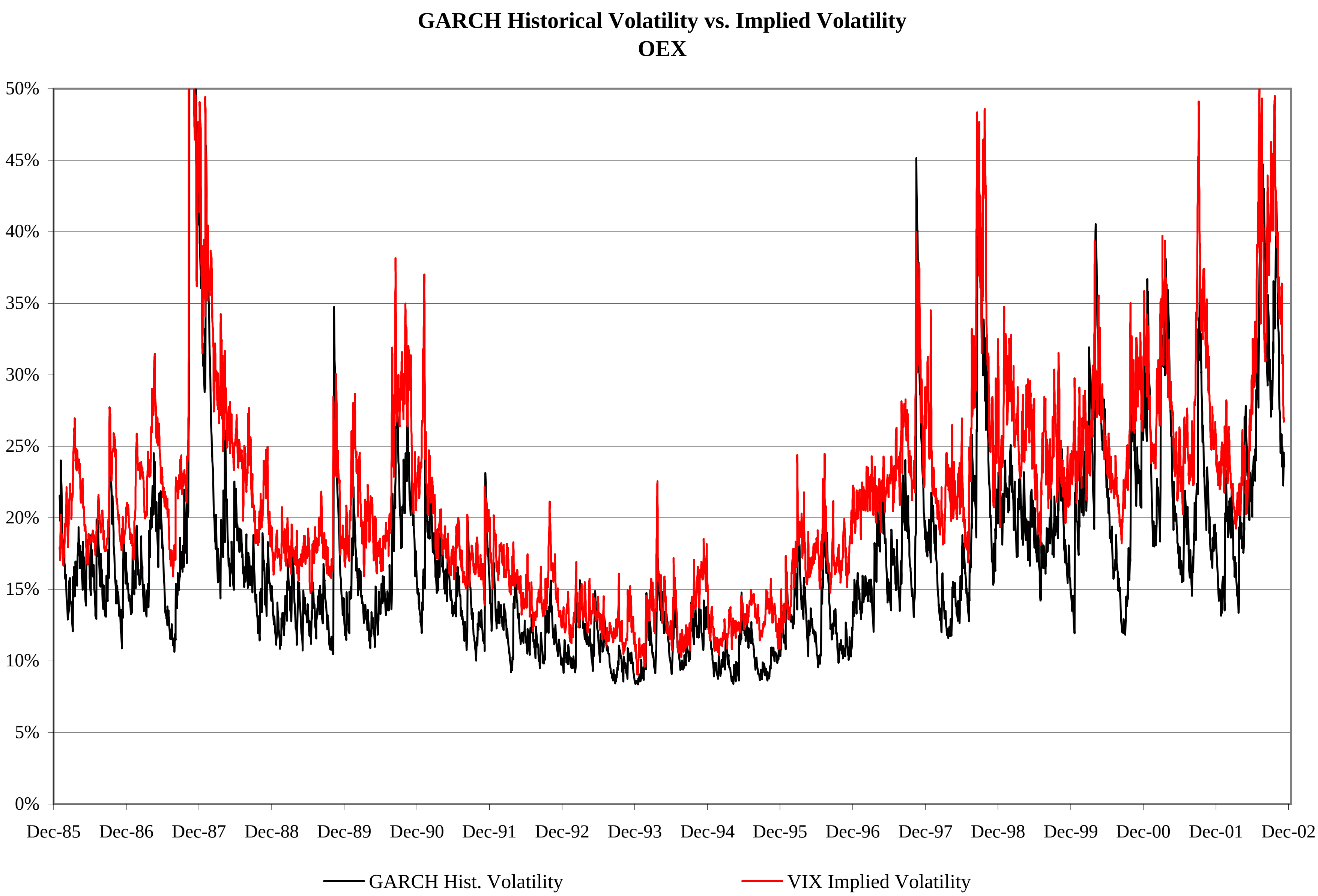
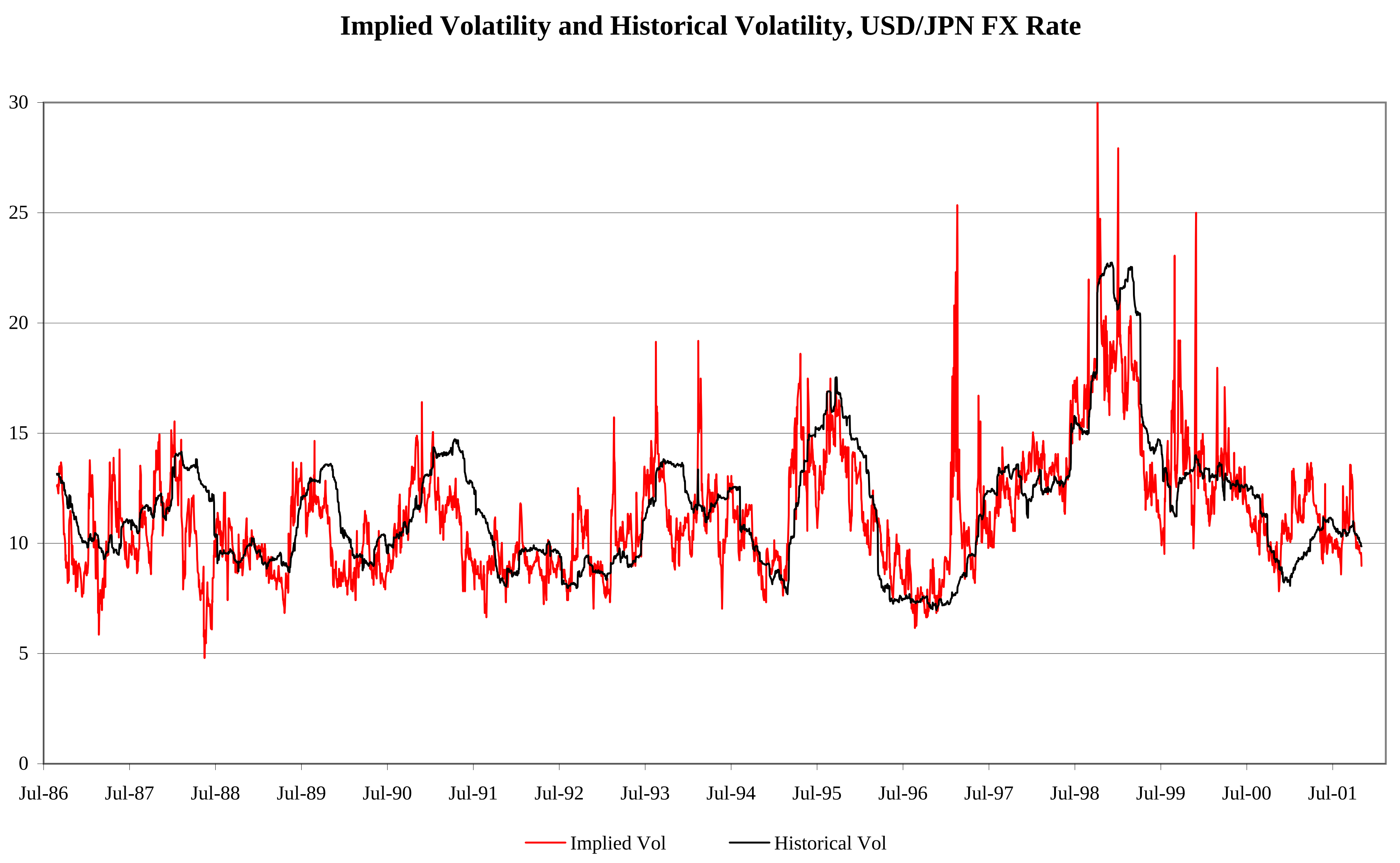


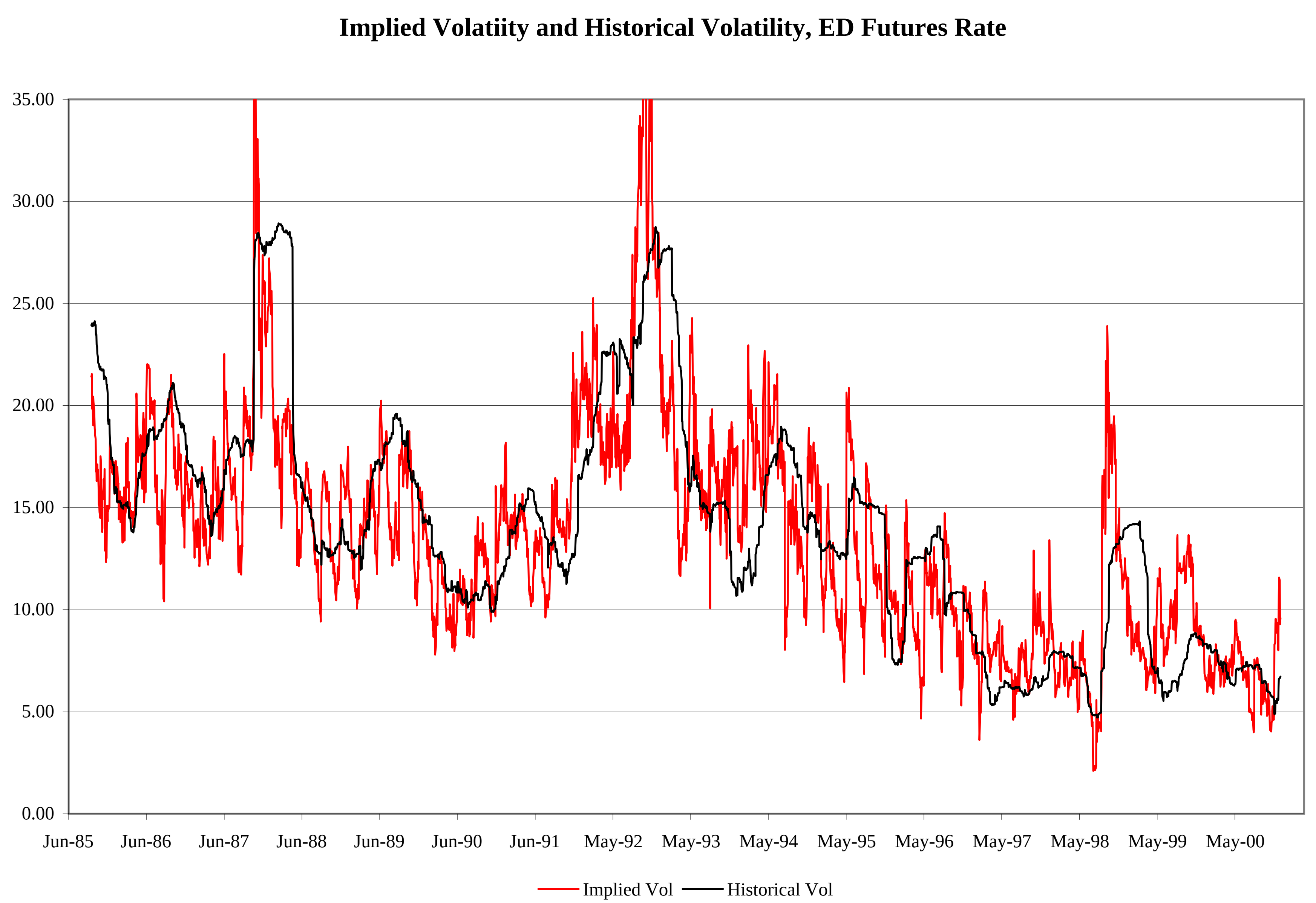
Figure 7
Change in Implied Volatility (for Levels) versus Change in Futures Rate, Japan

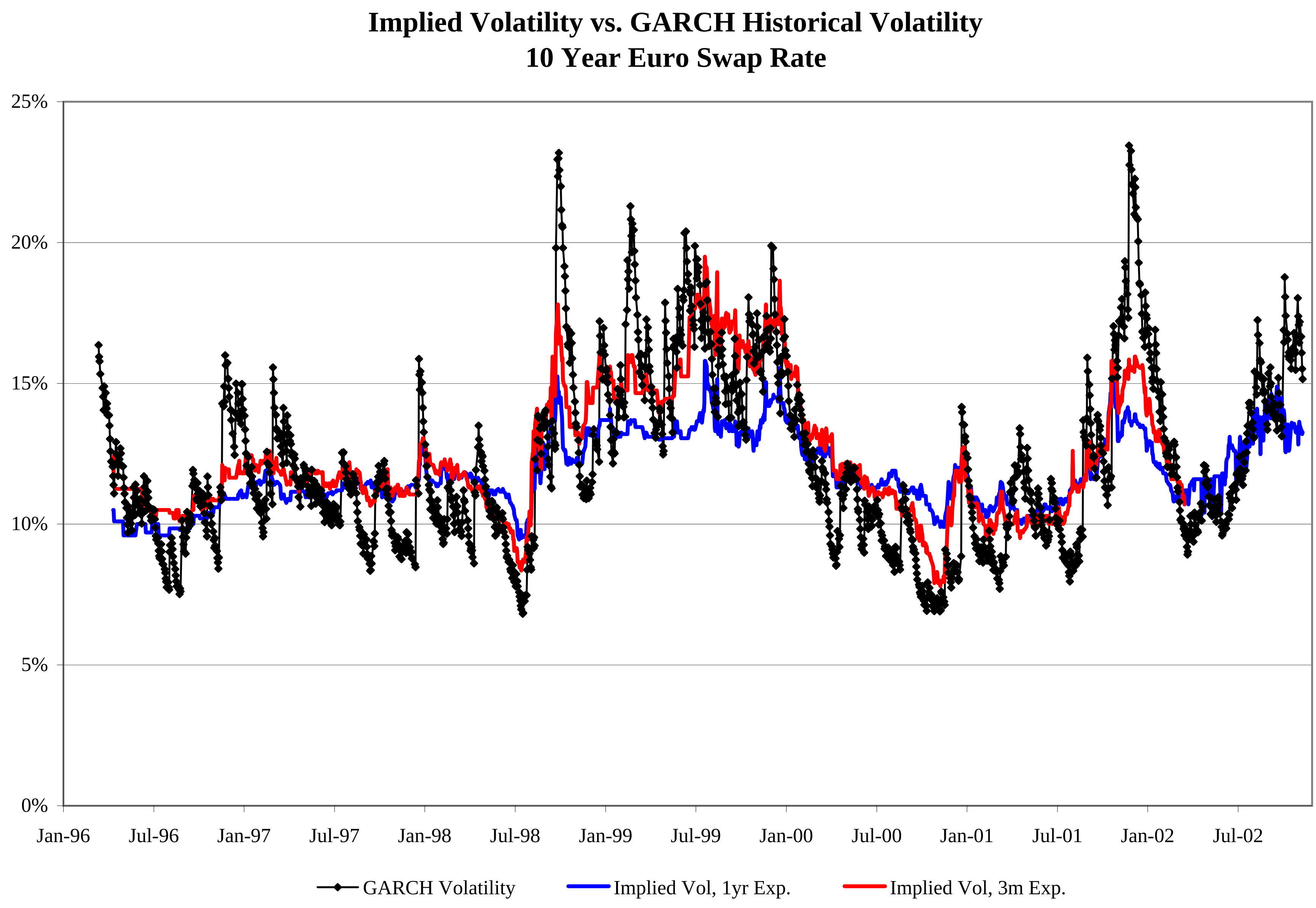




GARCH Historical Volatility vs. Implied Volatility
OEX
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
Dec-85
Dec-86
Dec-87
Dec-88
Dec-89
Dec-90
Dec-91
Dec-92
Dec-93
Dec-94
Dec-95
Dec-96
Dec-97
Dec-98
Dec-99
Dec-00
Dec-01
Dec-02
GARCH Hist. Volatility
VIX Implied Volatility


Implied Volatility and Historical Volatility, USD/JPN FX Rate
30
25
20
15
10
5
0
Jul-86
Jul-87
Jul-88
Jul-89
Jul-90
Jul-91
Jul-92
Jul-93
Jul-94
Jul-95
Jul-96
Jul-97
Jul-98
Jul-99
Jul-00
Jul-01
Implied Vol
Historical Vol


Implied Volatiity and Historical Volatility, ED Futures Rate
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
Jun-85
Jun-86
Jun-87
Jun-88
Jun-89
Jun-90
Jun-91
May-92
May-93
May-94
May-95
May-96
May-97
May-98
May-99
May-00
Implied Vol
Historical Vol


Implied Volatility vs. GARCH Historical Volatility
10 Year Euro Swap Rate
25%
20%
15%
10%
5%
0%
Jan-96
Jul-96
Jan-97
Jul-97
Jan-98
Jul-98
Jan-99
Jul-99
Jan-00
Jul-00
Jan-01
Jul-01
Jan-02
Jul-02
GARCH Volatility
Implied Vol, 1yr Exp.
Implied Vol, 3m Exp.

A Stochastic Volatility Model for Equity Prices
Duffie, Pan, Singleton Model

$$d \ln P = \left(r_d - \tfrac{1}{2} v\right) dt \;+\; \sqrt{v}\, dZ_1 \;+\; dJ_1^*$$

$$dv \;=\; \left(\kappa \bar{v} - \kappa v - \lambda v\right) dt \;+\; \sigma \sqrt{v}\, dZ_2 \;+ dJ_2^*$$

- The jump in the stock price is a Poisson jump process with a normally distributed jump size. The jump in the volatility factor is a Poisson process with a jump size that is exponentially distributed.
- There is negative correlation between the Brownian motions and the jump processes can be jumps that occur simultaneously.
- The characteristic function for the log of the stock price is exponential affine and solutions for European calls and puts require only quick numerical integrations (Fourier inversion).

A Stochastic Volatility Model for Foreign Exchange Rates

$$d \ln S = \left(r_d - r_f - \tfrac{1}{2} v\right) dt + \sqrt{v}\, dZ_1 + dJ_1^*$$

$$dv = \left(k \bar{v} - k v - l v\right) dt + s \sqrt{v}\, dZ_2 + dJ_2^*$$

- The two jump processes have a similar specification as before, but the correlation between the Brownian motions can be significantly different and the dependence between the jump processes can be different. It may be better to model the jumps as independent.

- Solutions for forward and futures FX rates. The characteristic function for the log of the FX rate is exponential affine. Solutions for European calls and puts require numerical integration only (Fourier inversion).

Term Structure of Implied Volatility

- The term structure of volatility will require additional factors

- 2 factor model for volatility, in which one factor determines the average level of volatility and one factor determines the slope of this term structure.

- Linear structure for model, $v(t) = a_1 y_1(t) + a_2 y_2(t)$

Long Dated Multi-Currency Options and Derivatives

$$d \ln S = \left(r_d(y) - r_f(y) - \tfrac{1}{2}v(y)\right)dt \;+\; \sqrt{v(y)}\, dZ_0 \;+\; dJ_0^*$$

$$dy_j \;=\; \left(\kappa_j \theta_j - \kappa_j\, y_j - \lambda_j\, y_j\right)dt \;+\; \sigma_j \sqrt{y_j}\, dZ_j \;+ dJ_j^*$$

$$j \;=\; 1, \ldots, K$$

- Short term variation is dominated by the random disturbance terms.
- As the time horizon is extended to the long term, the variability of the interest rates becomes more important.
- This requires integration with interest rate term structure modeling and calibration of interest rate and FX correlations.

Hedging Stochastic Volatility Risk

- Measuring and Hedging Kappa/Vega Risk
  - Compute kappa for all options and manage overall kappa exposure
  - Compute $\partial V/\partial \sigma$ from Black-Scholes model
  - Bucket kappa exposure by type and term

- Hedging Stochastic Volatility
  - Treat stochastic volatility as another random state variable and compute its partial
  - Compute $\partial V/\partial v$ from stochastic volatility model and translate to the partial for $\sqrt{v}$
  - In a multifactor model for volatility, compute the partials for the factors that determine volatility risk $\partial V/\partial y$

Managing Jump Risk

- Need to balance long/short positions in options
- Brute Force: simulate jumps and revalue all positions, both options and hedges
  - Compute a 99% VaR Loss
  - May need to use additional measures of tail risk
- Run scenarios which incorporate plausible jump risks, or the ones that could be most damaging
  - Equity markets down 30% and increase equity implied volatilities
  - Recompute kappa or stochastic volatility risk under each scenario

Calibrate Model to S&P 500 Options

- DPS Double Jump Model

$$d \ln P = \left(r_d - \tfrac{1}{2}v\right)dt \;+\; \sqrt{v}\, dZ_1 \;+\; dJ_1^*$$

$$dv \;=\; \left(\kappa\bar{v} - \kappa v - \lambda v\right)dt \;+\; \sigma\sqrt{v}\, dZ_2 \;+ dJ_2^*$$

- Parameters

κ = 7.0

$\bar{v}$ = 0.0258 ($\sqrt{\bar{v}}$ = 16%)

σ = 0.50

ρ = -0.7

λ = -5.0

Risk Neutral

$\bar{v}$ = 0.0904 ($\sqrt{\bar{v}}$ = 30%)

μ_J = -0.06 (-6%)

σ_J = 0.15

$\mu_{v,J}$ = 0.10 (+19% from $\sqrt{\bar{v}}$)

ρ_J = -0.6

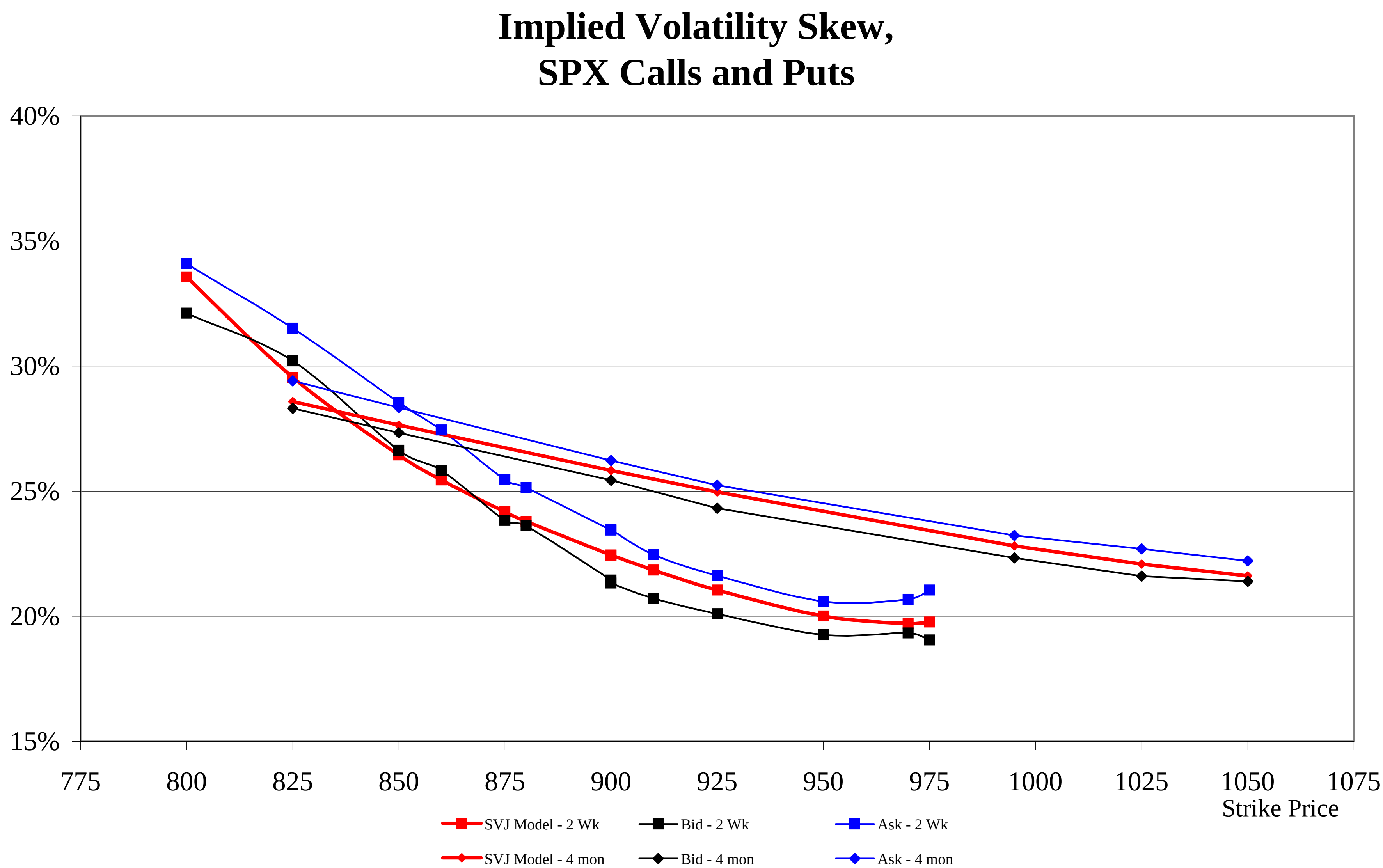
Risk Neutral

Jump Intensity = 0.40

$\sqrt{v(0)}$ = 20.6%

$P(0)$ = 903.85

2 Week ATM Implied Volatility = 22.4%

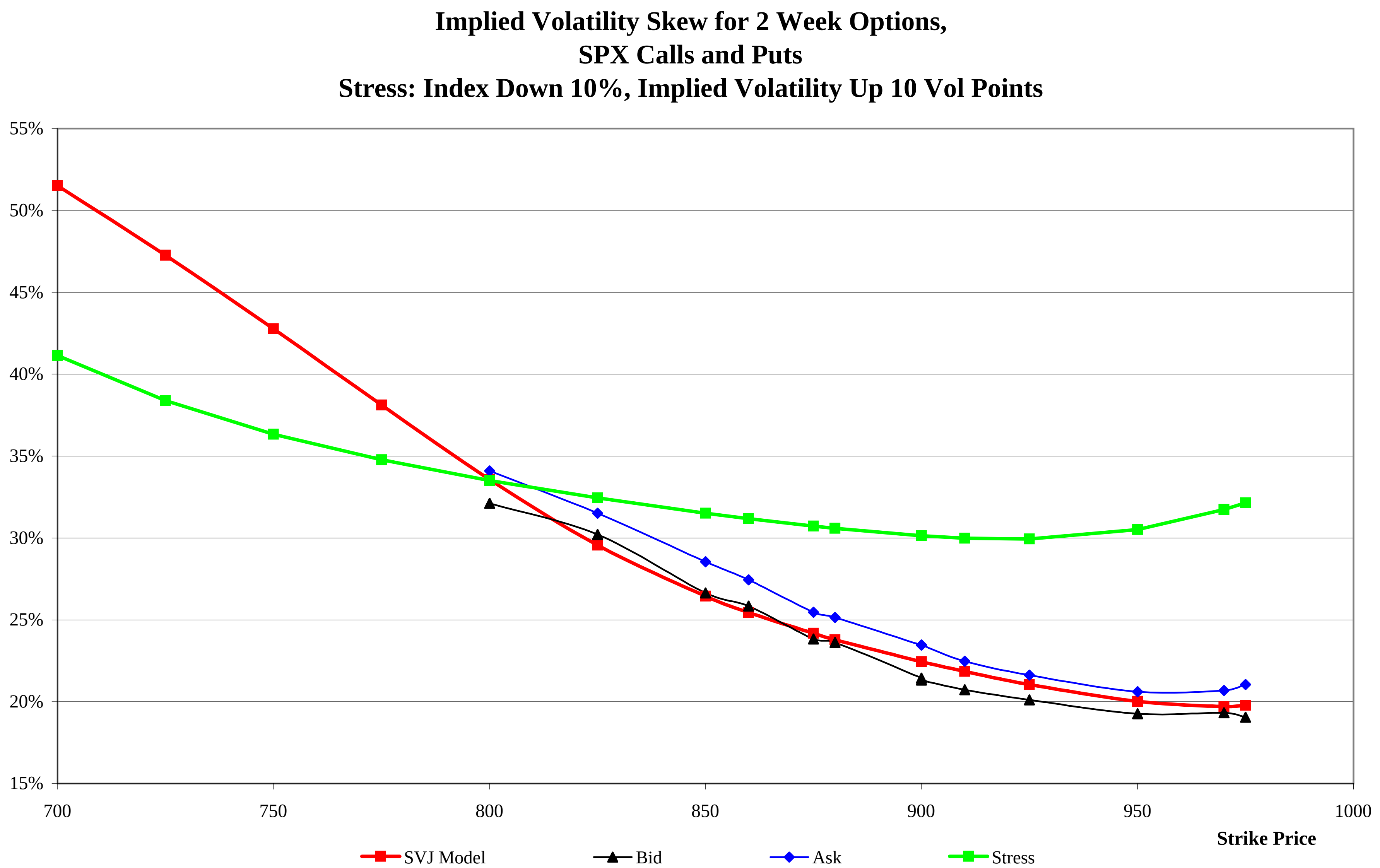


Implied Volatility Skew,
SPX Calls and Puts
40%
35%
30%
25%
20%
15%
775
800
825
850
875
900
925
950
975
1000
1025
1050
1075
Strike Price
SVJ Model - 2 Wk
Bid - 2 Wk
Ask - 2 Wk
SVJ Model - 4 mon
Bid - 4 mon
Ask - 4 mon

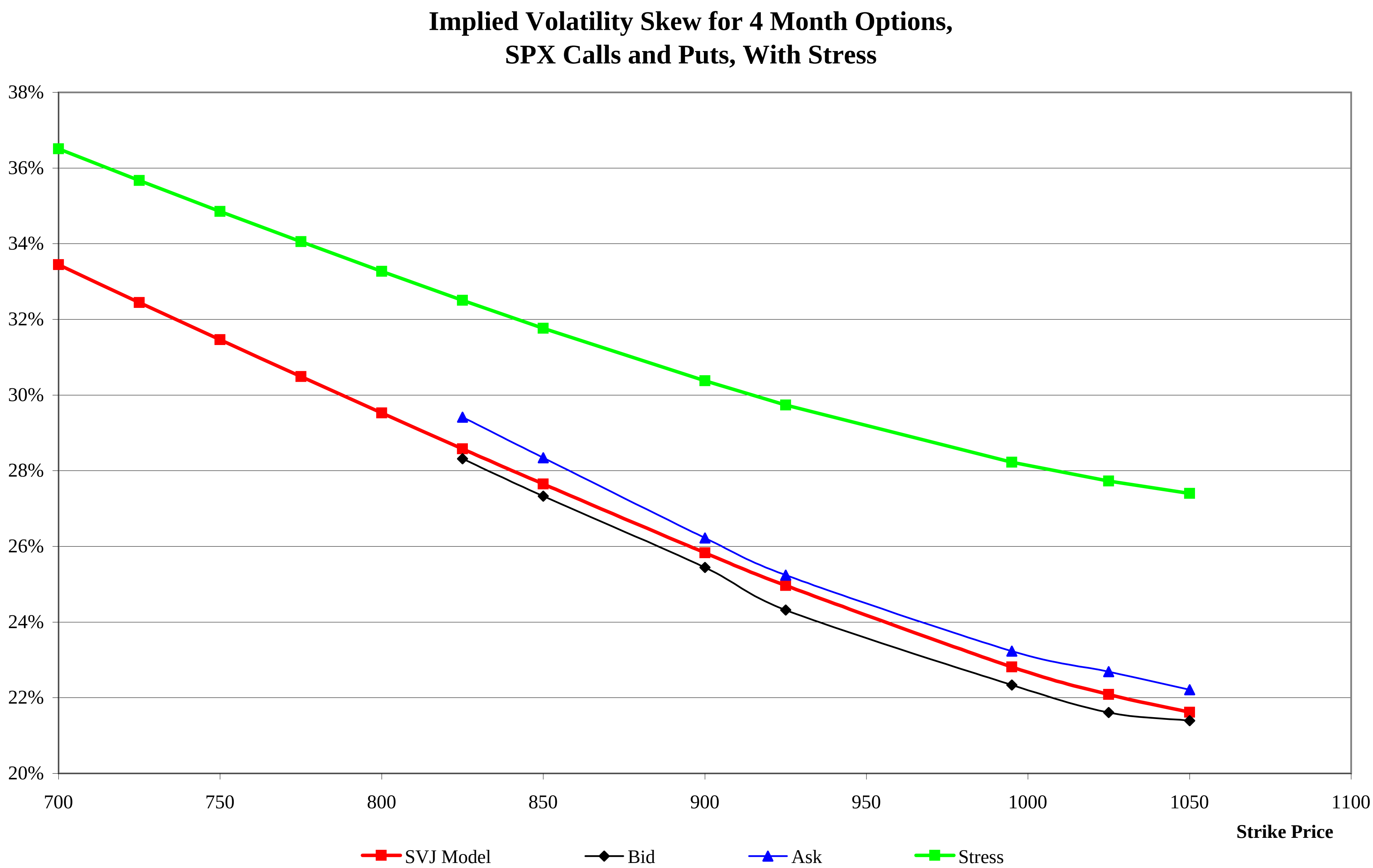
Delta and Kappa Sensitivites for SPX Index Options

Time to Expiration	Adjusted Stock Price	Instant. Volatility	Strike	SVJ Delta	B-S Delta	SVJ Kappa	B-S Kappa
2 weeks	902.76	20.63%	800	-0.0142	-0.0388	0.0480	0.1660
2 weeks	902.76	20.63%	825	-0.0349	-0.0671	0.1330	0.2544
2 weeks	902.76	20.63%	850	-0.0898	-0.1296	0.3058	0.4080
2 weeks	902.76	20.63%	860	-0.1283	-0.1709	0.4002	0.4878
2 weeks	902.76	20.63%	875	-0.2098	-0.2558	0.5541	0.6128
2 weeks	902.76	20.63%	880	-0.2438	-0.2906	0.6038	0.6513
2 weeks	902.76	20.63%	900	-0.4127	-0.4588	0.7503	0.7501
2 weeks	902.76	20.63%	900	0.5873	0.5412	0.7503	0.7501
2 weeks	902.76	20.63%	910	0.4885	0.4457	0.7690	0.7475
2 weeks	902.76	20.63%	925	0.3371	0.3035	0.7066	0.6650
2 weeks	902.76	20.63%	950	0.1314	0.1191	0.4187	0.3893
2 weeks	902.76	20.63%	970	0.0432	0.0442	0.1826	0.1900
2 weeks	902.76	20.63%	975	0.0312	0.0342	0.1393	0.1557
4 months	897.18	20.63%	825	-0.2102	-0.2731	1.6368	1.8223
4 months	897.18	20.63%	850	-0.2604	-0.3298	1.8476	1.9796
4 months	897.18	20.63%	900	0.6151	0.5383	2.1683	2.1664
4 months	897.18	20.63%	925	0.5415	0.4656	2.2368	2.1695
4 months	897.18	20.63%	995	0.3156	0.2631	1.9413	1.7990
4 months	897.18	20.63%	1025	0.2255	0.1894	1.6089	1.5065
4 months	897.18	20.63%	1050	0.1617	0.1388	1.2884	1.2433

Note: Increase instantaneous volatility to increase ATM implied volatility by 1% for SVJ Model.
B-S kappas for a +1% change in implied volatility.



Implied Volatility Skew for 2 Week Options,
SPX Calls and Puts
Stress: Index Down 10%, Implied Volatility Up 10 Vol Points
55%
50%
45%
40%
35%
30%
25%
20%
15%
700
750
800
850
900
950
1000
Strike Price
SVJ Model
Bid
Ask
Stress



Implied Volatility Skew for 4 Month Options,
SPX Calls and Puts, With Stress
38%
36%
34%
32%
30%
28%
26%
24%
22%
20%
700
750
800
850
900
950
1000
1050
1100
Strike Price
SVJ Model
Bid
Ask
Stress

Change in Option Valuations for Stress

Stock Prices Down 10%

Implied Vol Increases by 10 Vol Points

	SVJ	Black-Scholes		
Strike	**Model**	**Method 1**	**Method 2**	
800	+15.66	+22.24	+16.05	
825	+27.03	+31.73	+26.97	new ATM
850	+41.86	+44.59	+41.53	
860	+48.38	+50.45	+48.23	
875	+58.21	+59.47	+57.87	
880	+61.40	+62.44	+61.02	
900	+72.96	+73.36	+72.79	ATM
900	-17.42	-16.92	-17.48	ATM
910	-12.67	-12.37	-12.69	
925	-7.13	-7.02	-7.14	
950	-2.04	-2.06	-2.06	
970	-0.59	-0.63	-0.63	
975	-0.43	-0.47	-0.47	
825	+42.26	+53.82	+48.44	new ATM
850	+48.53	+59.61	+54.38	
900	-29.16	-19.13	-24.61	ATM
925	-23.16	-14.25	-19.14	
995	-9.73	-4.20	-5.71	
1025	-6.01	-1.74	-2.24	
1050	-3.84	-0.44	-0.44	

SVJ Model: increase instantaneous volatility so that near term ATM implied vol increases by 10 vol points

Black-Scholes Model

Method 1: +10 vol point increase for each B-S implied volatility

Method 2: increase the ATM B-S implied volatility by 10 vol points and translate the skew.

Exercise: Revalue an Option Portfolio

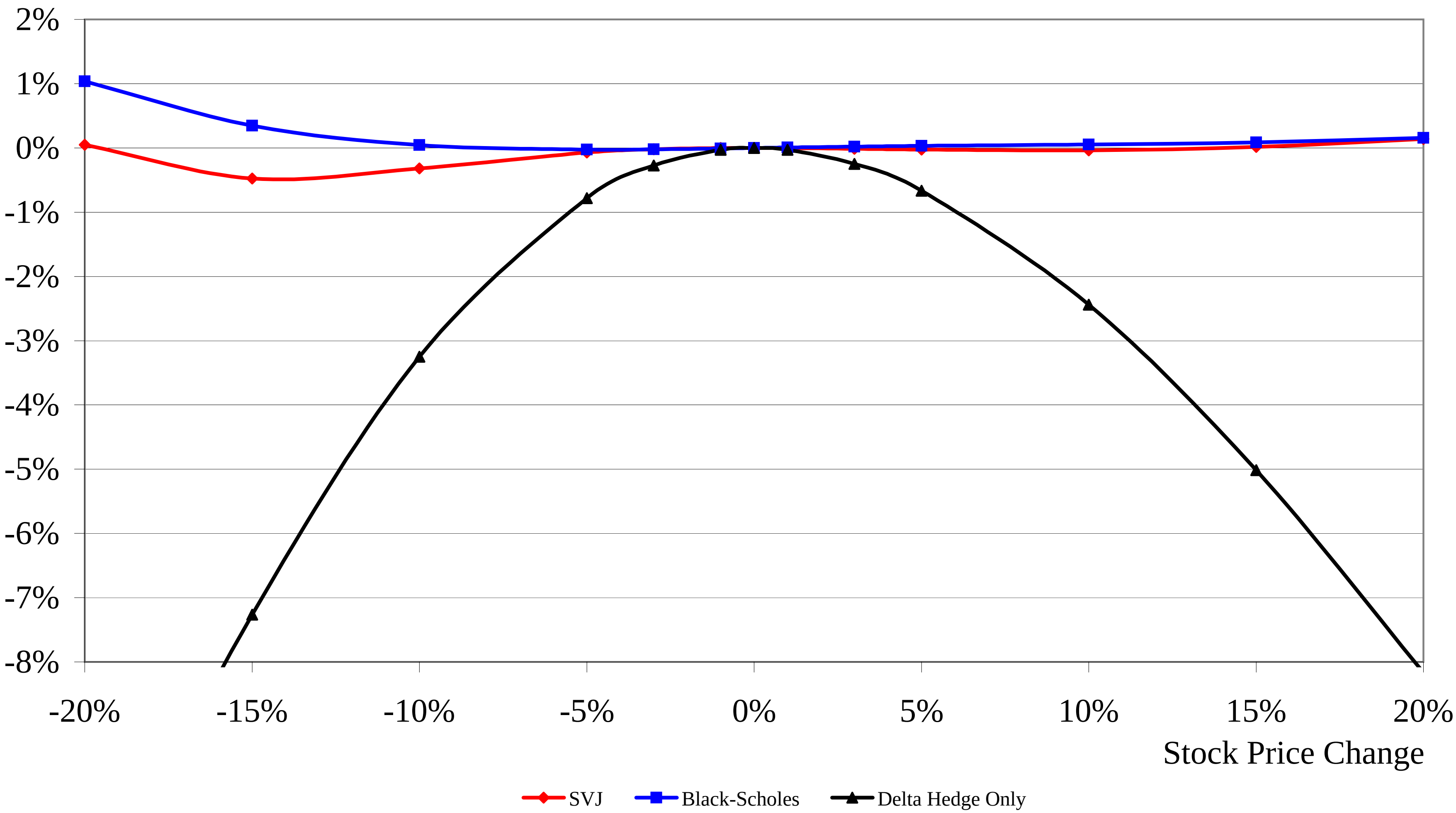
- Strategy: We believe that ATM implied volatility is too high and will decrease. Go short kappa and delta hedge. We want to hedge the gap risk.

- Option Portfolio

	Position	SVJ Delta	SVJ Kappa	B-S Delta	B-S Kappa
Sell ATM 925 calls	-100	.5415	2.24	.4656	2.17
Buy the 825 put	+95	-.2102	1.64	-.2731	1.82
Buy the 1025 call	+40	.2255	1.61	.1894	1.51
Adjust stock position to be delta neutral					

- Calculations
  - SVJ Model: Use SVJ Greeks and SVJ model to revalue portfolio, Net kappa = -3.8
  - B-S Model: Use B-S Greeks and B-S model to revalue portfolio, Net kappa = +16.3
  - Delta Hedge Only: Use SVJ Greeks for delta hedge, Net kappa = -224

Price Slides for SPX Option Portfolio



Calibrate Model to DAX Options

- DPS Double Jump Model

$$d \ln P = \left(r_d - \tfrac{1}{2}v\right)dt \;+\; \sqrt{v}\,dZ_1 \;+\; dJ_1^*$$

$$dv \;=\; \left(\kappa\bar{v} - \kappa v - \lambda v\right)dt \;+\; \sigma\sqrt{v}\,dZ_2 \;+ dJ_2^*$$

- Parameters

κ = 2.0

$\bar{v}$ = 0.052 ($\sqrt{\bar{v}}$ = 23%)

σ = 0.50

ρ = -0.78

λ = -1.5

Risk Neutral

$\bar{v}$ = 0.208 ($\sqrt{\bar{v}}$ = 46%)

μ_J = -0.09 (-9%)

σ_J = 0.25

$\mu_{v,J}$ = 0.08 (+14% from $\sqrt{\bar{v}}$)

ρ_J = -0.95

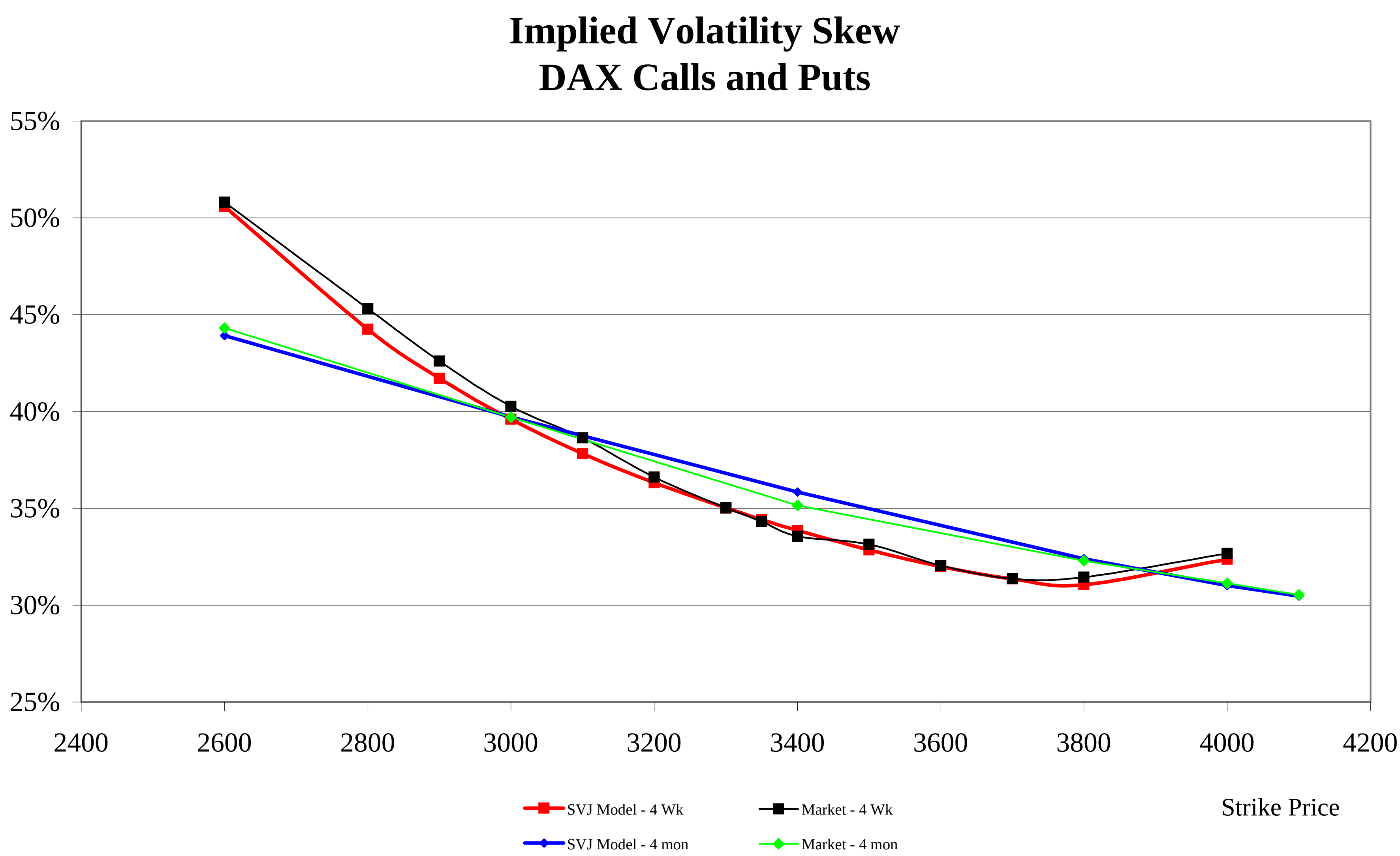
Risk Neutral

Jump Intensity = 0.40

$\sqrt{v(0)}$ = 32.2%

$P(0)$ = 3326

4 Week ATM implied volatility = 35%



Implied Volatility Skew
DAX Calls and Puts
55%
50%
45%
40%
35%
30%
25%
2400
2600
2800
3000
3200
3400
3600
3800
4000
4200
Strike Price
SVJ Model - 4 Wk
Market - 4 Wk
SVJ Model - 4 mon
Market - 4 mon

Delta and Kappa Sensitivites for DAX Index Options

Time to Expiration	Adjusted Stock Price	Instant. Volatility	Strike	SVJ Delta	B-S Delta	SVJ Kappa	B-S Kappa
4 weeks	3325.68	32.24%	2600	-0.0140	-0.0325	0.2167	0.6912
4 weeks	3325.68	32.24%	2800	-0.0404	-0.0687	0.7202	1.2471
4 weeks	3325.68	32.24%	2900	-0.0696	-0.1030	1.1848	1.6805
4 weeks	3325.68	32.24%	3000	-0.1162	-0.1546	1.7966	2.2162
4 weeks	3325.68	32.24%	3100	-0.1851	-0.2277	2.4975	2.7982
4 weeks	3325.68	32.24%	3200	-0.2785	-0.3236	3.1675	3.3165
4 weeks	3325.68	32.24%	3300	-0.3941	-0.4389	3.6451	3.6319
4 weeks	3325.68	32.24%	3300	0.6059	0.5611	3.6451	3.6319
4 weeks	3325.68	32.24%	3350	0.5421	0.4988	3.7628	3.6747
4 weeks	3325.68	32.24%	3400	0.4762	0.4355	3.7813	3.6287
4 weeks	3325.68	32.24%	3500	0.3454	0.3124	3.5068	3.2744
4 weeks	3325.68	32.24%	3600	0.2277	0.2054	2.8785	2.6505
4 weeks	3325.68	32.24%	3700	0.1346	0.1237	2.0664	1.9252
4 weeks	3325.68	32.24%	3800	0.0707	0.0697	1.2808	1.2776
4 weeks	3325.68	32.24%	4000	0.0147	0.0235	0.3066	0.5432
4 months	3324.64	32.24%	2600	-0.0923	-0.1261	3.3938	3.9764
4 months	3324.64	32.24%	3000	-0.2136	-0.2709	6.2901	6.3055
4 months	3324.64	32.24%	3000	0.7864	0.7291	6.2901	6.3055
4 months	3324.64	32.24%	3400	0.5882	0.5164	8.6177	7.5661
4 months	3324.64	32.24%	3800	0.3390	0.2822	8.0993	6.4563
4 months	3324.64	32.24%	4000	0.2211	0.1841	6.4757	5.1283
4 months	3324.64	32.24%	4100	0.1703	0.1443	5.4539	4.4035

Note: Increase instantaneous volatility to increase ATM implied volatility by 1% for SVJ Model.
B-S kappas for a +1% change in implied volatility.

Calibrate Model to S&P 500 Options
New Parameters, May 3, 2004

- DPS Double Jump Model

$$d \ln P = \left(r_d - \tfrac{1}{2} v\right) dt \; + \; \sqrt{v}\, dZ_1 \; + \; dJ_1^*$$

$$dv \; = \; \left(k\, \bar{v} - k\, v - l\, v\right) dt \; + \; s\, \sqrt{v}\, dZ_2 \; + dJ_2^*$$

- Parameters

$k \;=\; 2.5$

$\bar{v} \;=\; 0.0145 \;\; (\sqrt{\bar{v}} = 12\%)$

$s \;=\; 0.30$

$r \;=\; -0.8$

$l \;=\; -1.5$

Risk Neutral

$\bar{v} \;=\; 0.03625 \;\; (\sqrt{\bar{v}} = 19\%)$

$m_J \;=\; -0.01 \;\; (-1\%)$

$s_J \;=\; 0.10$

$m_{v,J} \;=\; 0.10$ (+22% from $\sqrt{\bar{v}}$)

$r_J \;=\; -0.6$

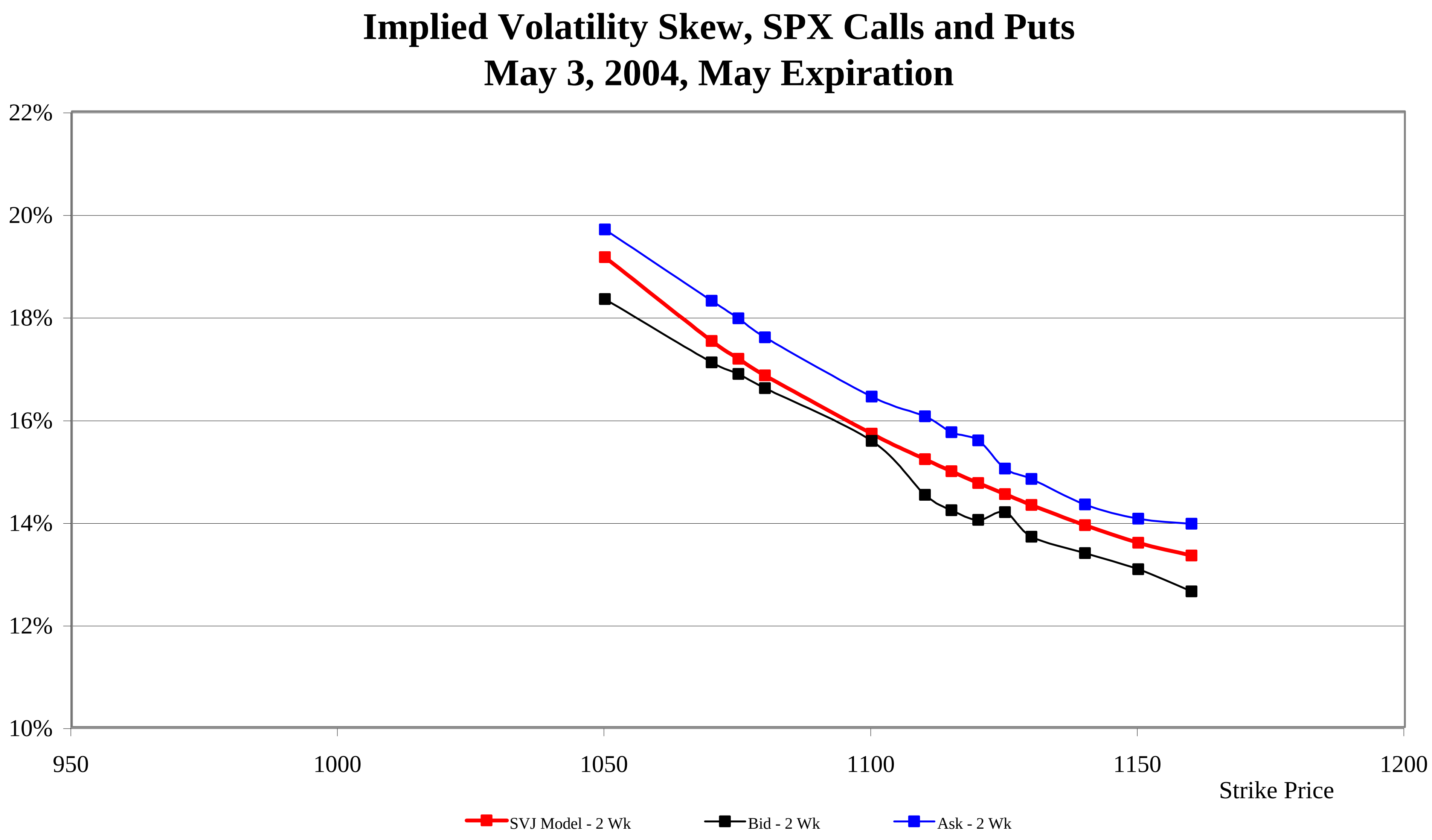
Risk Neutral

Jump Intensity = 0.35

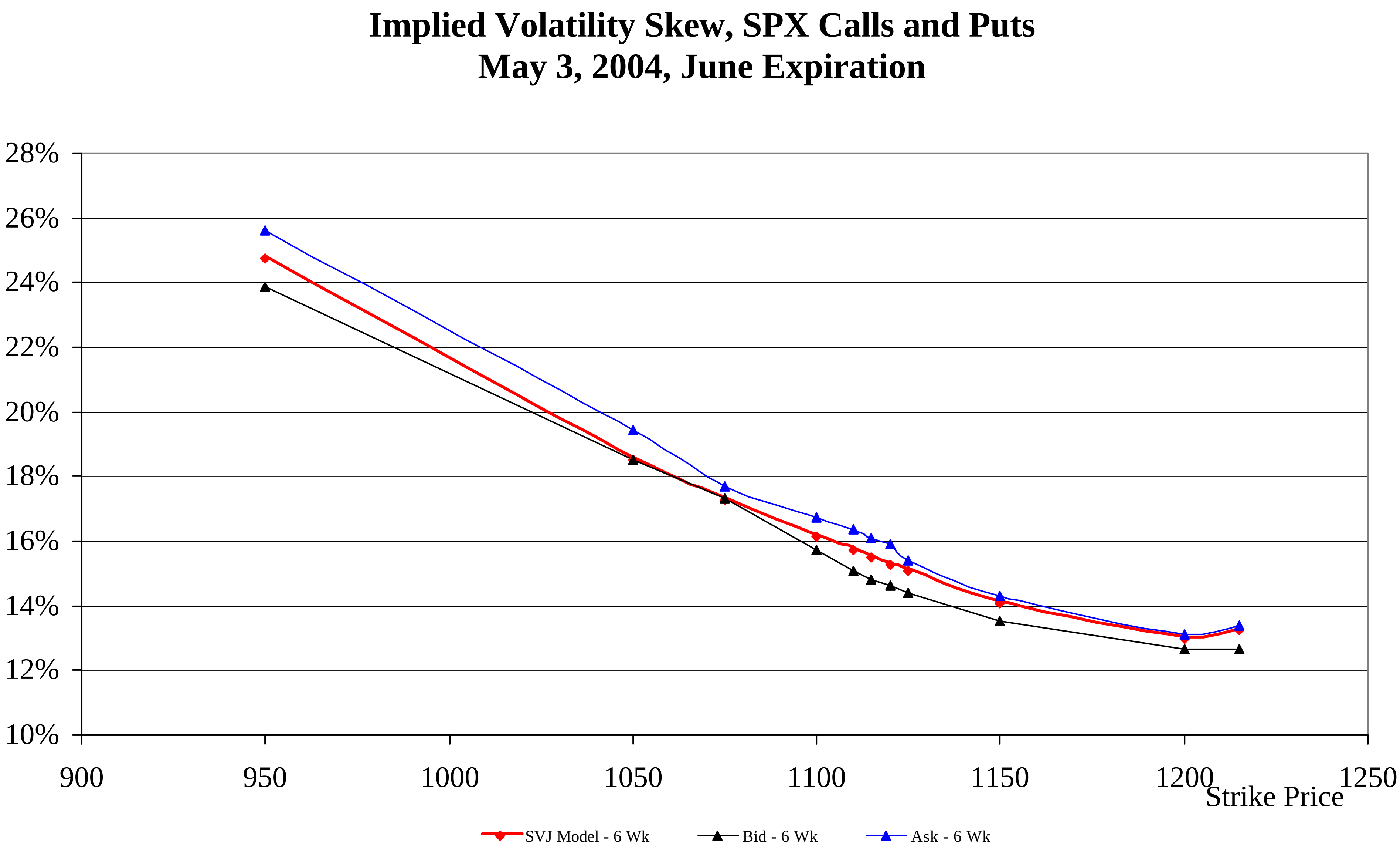
$\sqrt{v(0)} \;=\; 14.2\%$

$P(0) \;=\; 1113.96$

2 Week ATM Implied Volatility = 14.98%



Implied Volatility Skew, SPX Calls and Puts
May 3, 2004, May Expiration
22%
20%
18%
16%
14%
12%
10%
950
1000
1050
1100
1150
1200
Strike Price
SVJ Model - 2 Wk
Bid - 2 Wk
Ask - 2 Wk


Implied Volatility Skew, SPX Calls and Puts
May 3, 2004, June Expiration
28%
26%
24%
22%
20%
18%
16%
14%
12%
10%
900
950
1000
1050
1100
1150
1200
1250
Strike Price
SVJ Model - 6 Wk
Bid - 6 Wk
Ask - 6 Wk

Evaluating the Risks of Option Writing

- Net short equity options, delta hedge using spot or futures
  - This trading strategy receives the equity volatility risk premium.
  - Exposure is volatility risk and jump risk.
  - The risk premium reflects the fact that these risks occur at bad times. These risks are highly correlated with negative returns on market portfolios.

- Sell puts and delta hedge with long spot/futures positions
  - Stocks drop suddenly and implied volatilities increase.
  - Strategy is a double loser under this scenario.

- Sell calls and delta hedge with short spot/futures positions
  - Stock prices drop: gain on short call positions, but lose on hedges.
  - Delta on the calls decreases and the gain on the calls is smaller because of option gamma.
  - The increase in implied volatility reduces the gain on the call position even further.

- Long residential mortgages, short an interest rate option

- No strong evidence of a risk premium for FX volatility and interest rate volatility.

- Valuation and risk management for exotic equity and FX options.

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