3D Beamlet Transform

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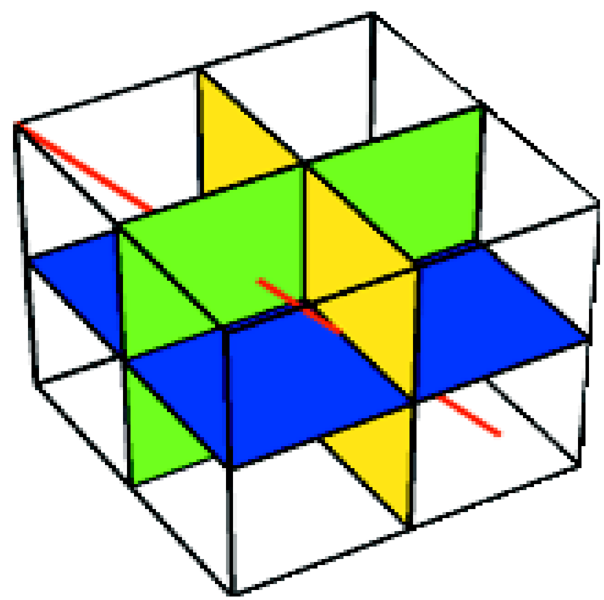


Discrete 3D Beamlet Transform

Basic ideas

- transforms a 3d digital image into segment representation.
- each coefficient is a line integral along a segment on the image.
- a strategic set of segments - the beamlet Pyramid.

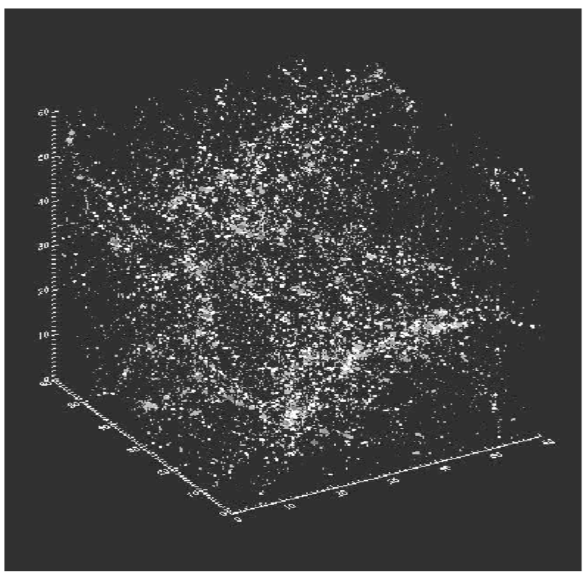
What is it good for?

- images consist of 1d relatively smooth curvical elements.
- denoising extremely noisy images.
- segmentation, feature detection and registration.
- a base for fast algorithms that optimize over curves.

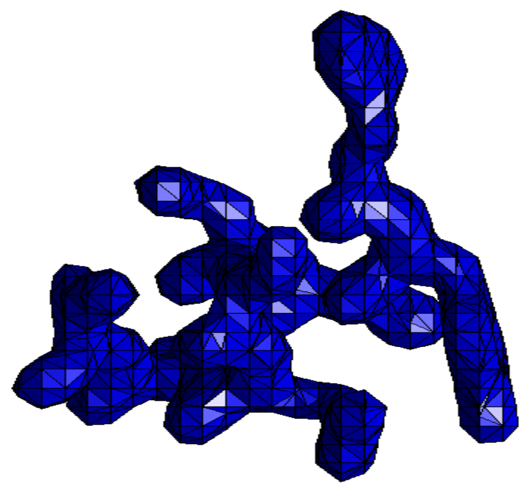
3D Applications

- Astronomy - analysing galaxy maps
- Protein Crystalography - resolution enhancement
- Night Vision - moving target detection
- Medical Imaging - segmentation

Galaxies map



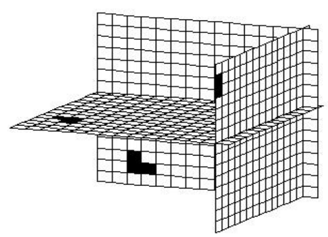
Protein electronic density map



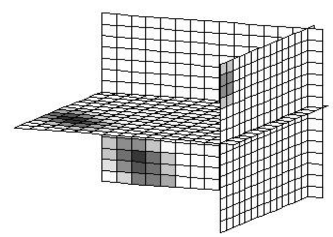
Denoising example - moving target detection (1)

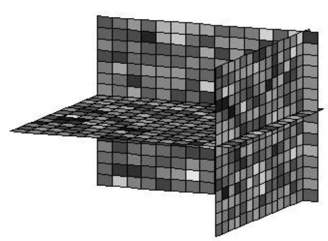
In this experiment we started with a curve in 3D, blurred it and added noise to a point when it seems taht it is totally lost inside the noise.

Original data



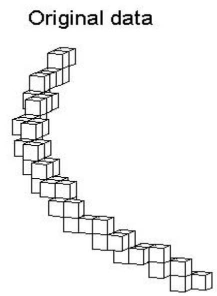
Blurred data

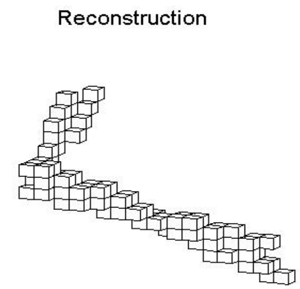


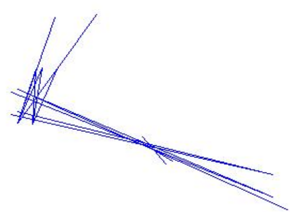


Denoising example - moving target detection (2)

We evaluated the beamlet coefficients (3M of them for a 16^3 cube) and kept the most significant 20 of them. Then we reconstructed the curve from the selected beamlets.



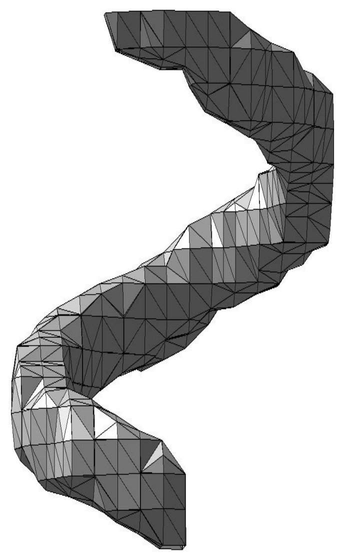




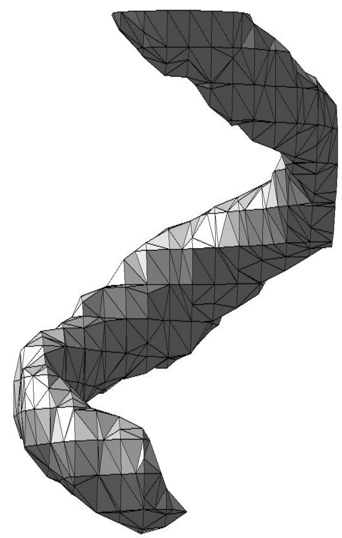
Wavelet-Beamlet compression

Here, we took a helical curve lying inside 16^3 cube. We had applied the beamlet transform followed by a wavelet transform to get 3M coefficients in the Wavelet-Beamlet domain, we kept the 150 biggest coefficients and applied inverse transfoms to get a reconstruction of the curve.

Original image



3% compression



The Beamlet Pyramid

A problem - there are much more lines than points, n^3 voxels map onto $o(n^6)$ segments when defining each segment (beam) by a pair of grid points.

Solution - Construct a startegic subset by taking only beams that correspond to pairs of boundary grid points of the image and each of it's dyadic sub-cubes.

Properties of Beamlets

- there are only $o(n^4)log(n)$ of them
- any beam can be efficiently approximated by a small chain of beamlets.
- multi level structure.
- 2 scaling relation.

Beamlet transform Algorithms

Exact evaluation of line integrals

- $o(n^5 log(n))$
- inversion by conjugate gradient method

Approximation based on the 2 scaling relation

- non-sequential memory access
- $o(n^4 log^2(n))$

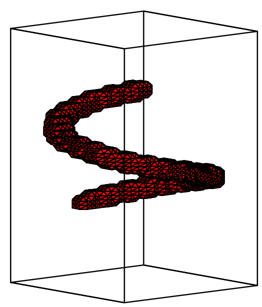
Applying 2D Slant Stack radon transform on the Sheared volume

- $o(n^4 log^2(n))$
- fast and stable inversion algorithm

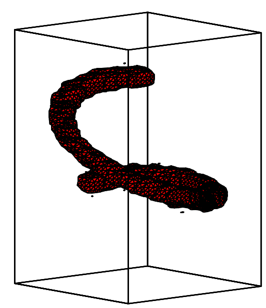
3D shearing illustration

By shearing a 3D volume using a trigonometric interpolation , each time by a different shearing angle, we can get 3D beamlet coefficients by applying fast 2D radon Transforms.

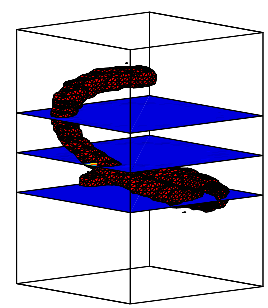
Image padded with zeros above and below



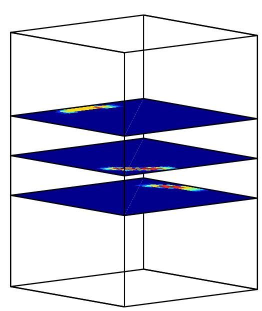
Sheared image



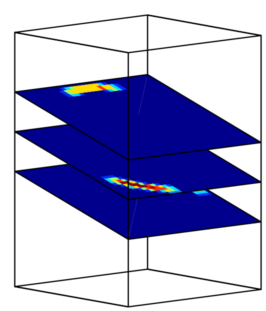
Sheared image with horizontal slices



Horizontal slices



Sheared slices



Horizontal slices of the sheared image