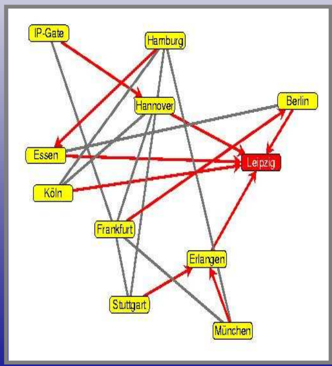
A Polyhedral Approach to IP Network Optimization

Andreas Bley
Konrad-Zuse-Zentrum Berlin (ZIB)



Overview

IP network planning problem

OSPF-Routing

Optimization model

Solution approach

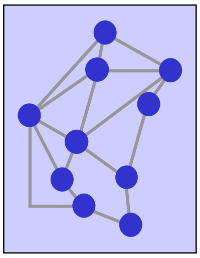
Computational results

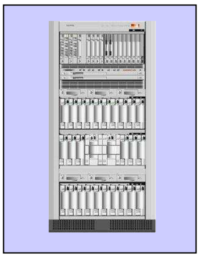
Cooperation: DFV-Verein e.V.

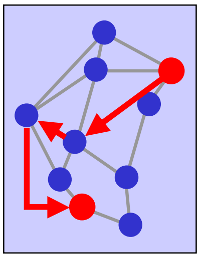
G-WiN – Gigabit Wissenschaftsnetz = Internet2 for German Universities

- *over 750 Institutions*
- *over 1.000 TByte/month in Nov-2002*

IP-Network design problem







Decisions

- network topology
- link capacities
- node hardware
- OSPF routing (weights)

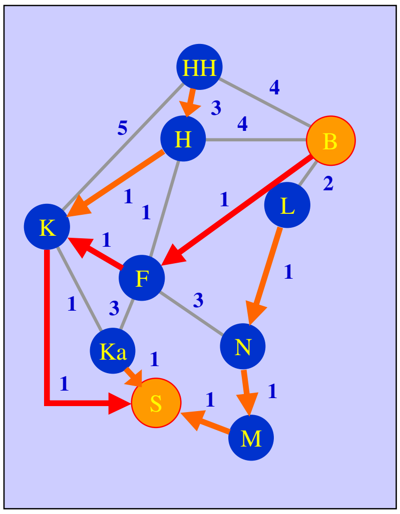
Objectives

- min link and node cost
- min max link utilization

Constraints

- hardware fits together
- survivable OSPF routing
  non-adaptive
  non-bifurcated
  normal and failure states

OSPF-Routing

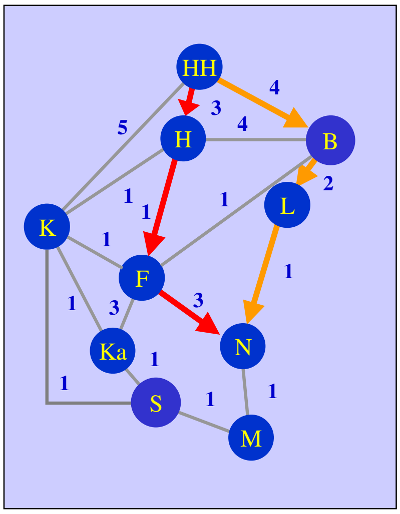


Fixed routing weights
(link lengths)

Non-bifurcated routing on shortest paths

Routing paths form Sink-Tree for each destination

OSPF-Routing



Fixed routing weights
(link lengths)

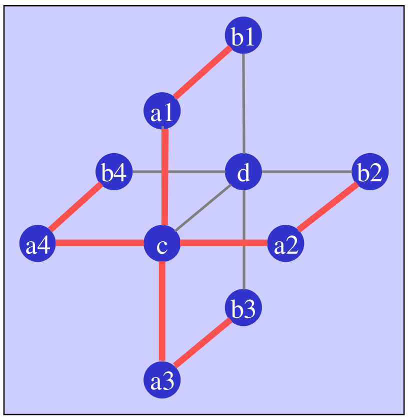
Non-bifurcated routing on shortest paths

Routing paths form Sink-Tree for each destination

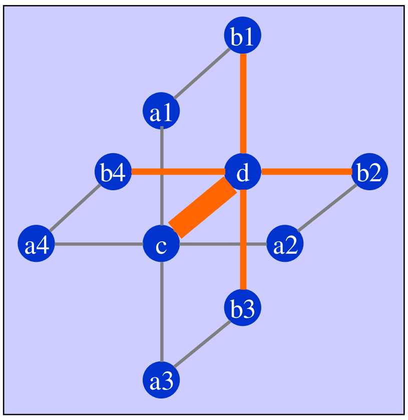
Problem: Shortest paths may not be unique!

Why unique shortest paths?

All weights **equal to 1**
Demands d(c,b1)=...=d(c,b4)=1

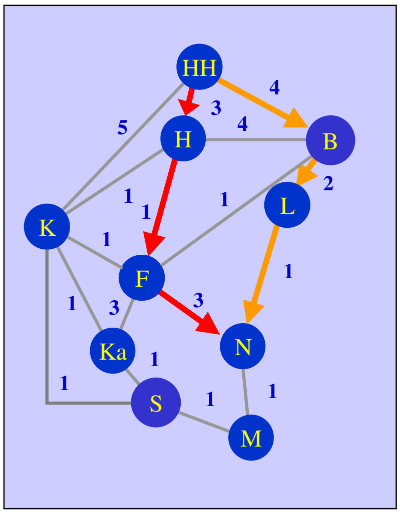


Maximum load: 1



Maximum load: 4

OSPF-Routing



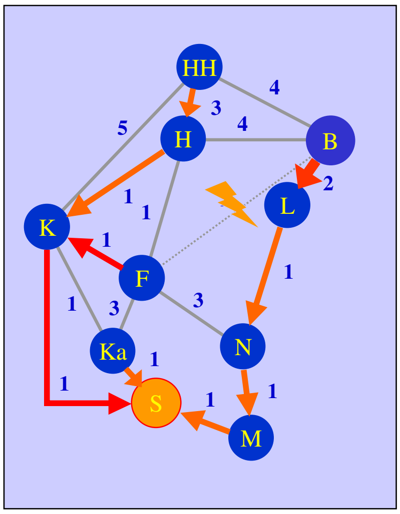
Fixed routing weights
(link lengths)

Non-bifurcated routing on shortest paths

Routing paths form Sink-Tree for each destination

Routing weights should guarantee unique shortest paths

OSPF-Routing – failure restoration



Recompute shortest paths in residual network

Unchanged routing weights

Only affected demands are rerouted

Complexity

Obs: Cost minimal network design is NP-hard
- contains disjoint paths, set partition, ...

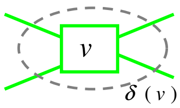
Def: Minimum congestion unsplittable shortest path routing
Given graph, edge capacities, and demands, find routing weights such that the maximum edge load (flow/capacity) is minimized.

Thm: Approximating Min-Con-USPR better than $O\left(\langle\Phi\rangle^{1/2}\right)$ is NP-hard in general.

Thm: For undirected cactus graphs, Min-Con-USPR can be approximated within factor 2.
- LP rounding

Mixed-integer programming model

Topology & Link & Node Hardware
component variables: router cards, link capacities ...
ressource constraints: # router slots, matching link interfaces, ...



$$\sum_{e\in\delta(v)} \sum_{c\in C(E)} w_e^{r,c} x_e^c + \sum_{c\in C(V)} w_v^{r,c} x_v^c \geq 0$$

OSPF Routing
unsplittable flow formulation:
binary path or arc-flow variables
demand or flow conservation

$$\sum_{P\in \mathrm{P}_{uv}^s} f^s(P) = 1$$

additional shortest path routing constraints

Capacity constraints $\displaystyle\sum_{c\in C(e)} w_e^{cap,c} x_e^c \geq \sum_{uv\in D} d_{uv}^s \sum_{P\in P_{uv}^s : e\in P} f^s(P)$

(extra variables for max link utilizations)

Shortest Path Systems

Def: Shortest Path System (SPS) $\{P_1, \dots, P_k\}$
Exist compatible routing weights l_e, s.t. P_i are unique shortest paths

OBS: SPS are **independenence systems** (but not matroids)

- Single paths are independent
- rank 1 circuits ~ conflicts between 2 paths ~ path monotonicity
- Special case: undirected SPS in cactus graphs
  All circuits have rank 1 [BenAmeur]
- Computing rank of arbitrary path set is NP-hard
- Finding compatible routing weights (testing independence) is in P

Inequalities

- Rank inequalities $\sum_{P \in C} f(P) \leq rank(C)$
- (Generalized) clique and odd hole inequalities, ...

Path monotonicity (subpath condition)

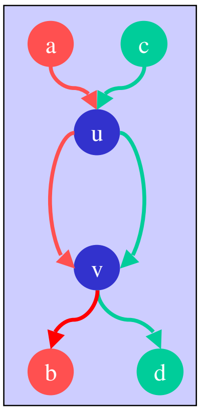
Path monotonicity

P_1, P_2 unique shortest paths w.r.t. l_e

$P_1(u,v) \subseteq P_1,\ P_2(u,v) \subseteq P_2$

$\Rightarrow\ P_1(u,v) = P_2(u,v)$

- Necessary, but not sufficient to describe SPS
- Sufficient for undirected SPS on cactus graphs
- All conflicts between 2 paths (rank 1 circuits) correspond to violated path monotnicity constraints



Rank inequalities for path-pairs of same operating state:

$$f^s(P_1) + f^s(P_2) \leq 1$$

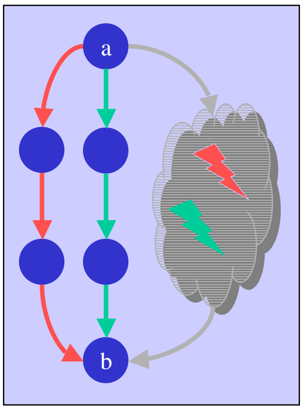
$$P_1(u,v) \neq P_2(u,v)$$

Path monotonicity (operating state coupling)

Rank inequalities for path-pairs of different operating states:
(unaffected routing paths do not change)

$$f^{s}(P_1) + f^{s}(P_2) \le 1$$

$$s_1, s_2 \in S,\ a, b \in V,$$
$$P_1, P_2 \in \mathcal{P}^{s_1}(a,b) \cap \mathcal{P}^{s_2}(a,b),$$
$$P_1 \ne P_2$$



With arc-flow variables:

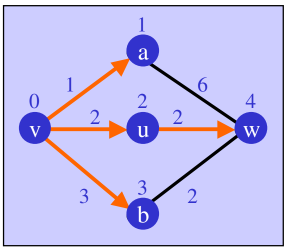
$$z_e^{a,b,s_1} \le z_e^{a,b,s_2} + [s_1 \in P_2] + [s_2 \in P_1]$$

$$[s_i \in P_j] := \begin{cases} 0 & s_i = \varnothing \\ z_{uv}^{a,b,s_j} + z_{vu}^{a,b,s_j} & s_i = uv \\ \sum_{e \in \partial^-(v)} z_e^{a,b,s_j} & s_i = v \end{cases}$$

Finding compatible routing weights

Given: Paths $P_{(v,w)}$ satisfying path monotonicity

Goal: Find $l_e \in \mathbb{N}$ such that P's are unique shortest paths



Metric Inequalities

Let $d_{(v,w)}$ be (v,w)-distance for lengths l_e.
Then $(u,v) \in P_{(v,w)}$ if and only if

$d_{(v,w)} = d_{(v,u)} + l_{(u,w)}$ and
$d_{(v,w)} < d_{(v,u')} + l_{(u',w)}$ for all $(u',w) \in \partial^-(w),\ u' \ne u$

Finding compatible routing weights

LP to compute compatible routing weights

$$\min \sum l_{(e)}$$

$$d^s_{(v,u)} + l_{(u,w)} - d^s_{(v,w)} = 0 \quad \text{if} \quad u \in P^s_{(v,w)}, u \in \partial^-(w)$$

$$d^s_{(v,u')} + l_{(u',w)} - d^s_{(v,w)} \geq 1 \quad \text{if} \quad u \notin P^s_{(v,w)}, u \in \partial^-(w)$$

$$l_{(e)} \geq 1$$

$$d^s_{(v,w)} \geq 0$$

Feasible ⇨ l_e induce shortest paths P uniquely.

Infeasible ⇨ current paths P cannot be realized as shortest paths simultaneously.

IIS of rows ⇨ min unrealizable path system C

$$\sum_{(s,P)\in C} f^s(P) \leq |C| - 1 = rank(C)$$

Solution approach

Mixed-integer programming model

Variables

- Node types, IP-Router cards, ...
- Link capacities
- Path- / Arc-flow variables

Constraints

- Technical constraints
- Link capacity constraints
- Non-bifurcated routing paths
- Shortest path routing (easy)
- Shortest path routing (hard)

Algorithms

Network design & Routing

- Cutting plane algorithm
- Branch & Cut
- Heuristics

Weight computation

Linear programming

Implementation

Separation order for arc-flow formulation

1. Subpath & (Precedence constrained) edge capacity knapacks - NOS
2. Resource constraint knapsack
3. Subpath & edge capacity knapacks - failure states
4. operating state coupling inequalities NOS – failure state
5. Shortest path IIS inequalities

Node selection and Branching

- n times best bound, then dive
- Branching order:
  - biggest fractionally routed demands first
  - components with tightest resource constraints first

Heuristics

- routing weights = duals of capacity constraints
- routing weight LP solution for fixed and near-integer routing paths

Standard `tricks´

- Only limited precision required ⇨ flow cost perturbation
- Keep formulation as small as possible (Omit variables / constraints for small demands)
- Branching order ⇨ solution times of LP

Results: integrated network design & routing

Problem	LP Opt.	Best LB	Best UB	Time (hh:mm:ss)	BB-Nodes
Atlanta nos	56.44	104.39	107.00	2:00:00	3073
G-WiN2 nos	72.41	76.00	76.00	8	21
G-WiN3 nos	240.00	240.00	240.00	6	23
G-WiN4 nos	272.00	272.00	272.00	3	3
EP98 nos	165.05	176.35	184.17	2:00:00	2114
NSF nos	229.48	324.42	475.00	2:00:00	1980
Atlanta fail	65.42	126.46	175.00	2:00:00	960
G-WiN2 fail	72.41	76.00	76.00	3:07	19
G-WiN3 fail	320.00	320.00	320.00	24:12	278
G-WiN4 fail	272.00	272.00	272.00	56	5
EP98 fail	179.54	202.25	309.23	2:00:00	623
NSF fail	318.60	389.32	735.00	2:00:00	115

Arc-flow based model, Intel P4 1.7GHz, CPLEX 7.5, Time limit 2h

Results: traffic engineering

Problem	LP Opt.	Best LB	Best UB	Time (hh:mm:ss)	BB-Nodes
Atlanta nos	0.633	0.834	0.834	7	36
G-WiN2 nos	0.482	0.576	0.576	38	775
G-WiN3 nos	0.054	0.106	0.106	8	48
G-WiN4 nos	0.030	0.057	0.057	18	50
EP98 nos	0.183	0.235	0.235	1:29	204
NSF nos	2.740	2.745	2.745	23:48	3453
Atlanta fail	1.265	1.669	1.669	14	70
G-WiN2 fail	2.610	3.059	3.059	14:17	1350
G-WiN3 fail	0.104	0.114	0.578	2:00:00	140
G-WiN4 fail	0.093	0.137	0.142	2:00:00	1284
EP98 fail	0.778	0.962	1.271	2:00:00	1406
NSF fail	0.731	0.745	0.746	2:00:00	2073

Arc-flow based model, Intel P4 1.7GHz, CPLEX 7.5, Time limit 2h