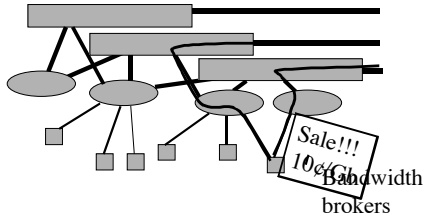
On the flow of bits and bucks in the Internet

Bruce Hajek with Ganesh Gopal, Sujay Sanghavi, and Sichao Yang
and thanks to Steven Williams (Dept. Economics, UIUC)



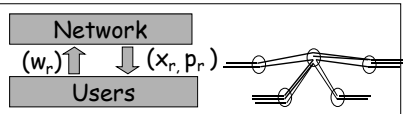
General goal, vaguely stated

Handle general network topologies
Identify resource allocation guidelines
Investigate:
- -Efficiency: Is maximum use made of the network?
- -Fairness: How is network value distributed among players?

Outline of talk

I. Strategic users (one link & flat networks)
II. Hierarchical networks
III. Future directions and implications

I. Strategic users



Consider Kelly's decomposition:

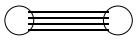
System problem: Select route flow rates (x_r) to maximize $\sum_r U_r(x_r)$ subject to: $Ax \le C$.

The bids (w_r) are also payments. Given the bid, the network selects flow rates to maximize $\sum_r w_r \log(x_r)$. Equivalently, network determines prices: $p_r = w_r/x_r$.

~~Given price p_r, user r selects bid w_r to maximize payoff $U_r(w_r/p_r) - w_r$. (Users are not strategic!)~~

What if each user r unilaterally tries to maximize payoff $U_r(x_r(w_1, w_2, \ldots, w_r, \ldots w_n)) - w_r$?

One link, one-dimensional bids, NEPs



Network

(w_r) ⇧ ⇩ □

Users

Payments equal bids*
Uniform price
Proportional allocation
Worst case efficiency 75%
(*and allowing payments not equal bids doesn't help)
(Johari and Tsitsiklis)

Network

(w_r) ⇧ ⇩ (x_r, p_r)

Users

Payments equal bids.
Non-uniform price
Worst case
efficiency 87%
(Sujay Sanghavi and H.)

Network

(b_r) ⇧ ⇩ (x_r, p_r)

Users

Payments not equal bids
Non-uniform price
Worst case
efficiency 100-%
(Sichao Yang and H.)

Network

(w_r) ⇧ ⇩ (x_r, p_r)

Users

Bids equal payments
Non-uniform price--give volume discount
volume buyer. For N=2, optimal scale invariant mechanism:

$$x_1 = \frac{w_1}{2w_2} C \text{ and } x_2 = C - x_1 \quad (\text{if } w_1 \le w_2)$$

Yields minimum efficiency ratio 87.5% for 2 buyers. (The number for arbitrarily many buyers appears to be 87.37%.)
(Sujay Sanghavi and H.)

Non-uniform price, payments are functions of bids:

Network

(b_r) ⇧ ⇩ (x_r, p_r)

Users

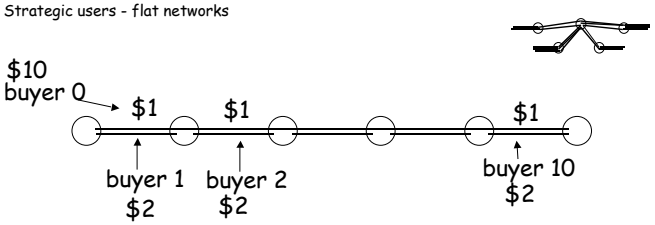
Consider payments of the form: $m_i = B_{-i}(\Phi(B)-\Phi(B_{-i}))$.

Example: $m_1 = b_2 \log((b_1+b_2)/b_2)$ $m_2 = b_1 \log((b_1+b_2)/b)$

(Suppose: Φ is strictly increasing, continuously differentiable
$u^2\Phi'(u)$ is a strictly increasing map of R_+ onto R_+.
At least two buyers i have $U_i(0)$=+infty.

The NEP is unique and efficient; prices are less than □,
For $\Phi(u)=u^{-1+\varepsilon}$, prices converge to □ as $\varepsilon \to 0$

(Sichao Yang and H.)



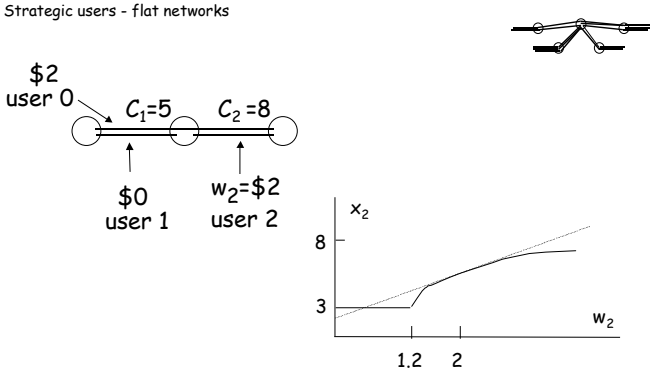
What happens if buyer 1 increases her bid to $3?

Itemized bid model: Capacity to buyer 1 increases by 0.083, to 3/4
Capacity to buyer 0 decreases by 1/12, to 1/4.
Buyer 0 has excess capacity on all other links.

Sum bid model: Capacity to buyer 1 increases by 0.011, to 21/31
Capacity to buyer 0 decreases by 0.011 to 10/31

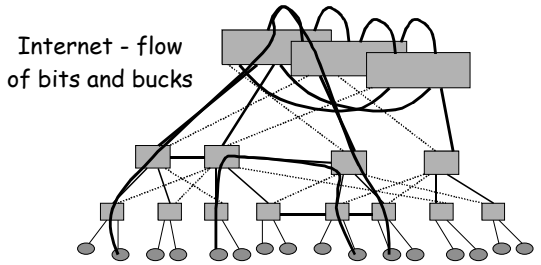
Sum model attributes more market power to multilink buyer, can lead to efficiency ratio arbitrarily close to zero. (H. and Sichao Yang)

Strategic users - flat networks



No NEP exists for the network shown under sum bid model.
The bid vector shown uniquely satisfies the first order necessary conditions, but is not an NEP (H. and Sichao Yang)

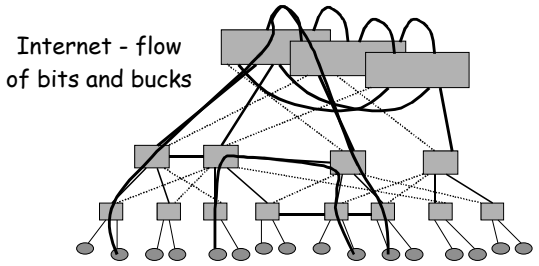
II. Hierarchical networks



Peering relationships bring mutual benefits

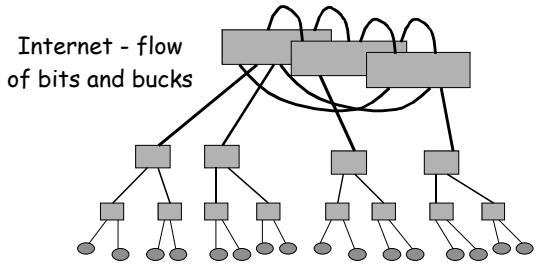
Actual or potential secondary connections bring market competition

Hierarchical networks



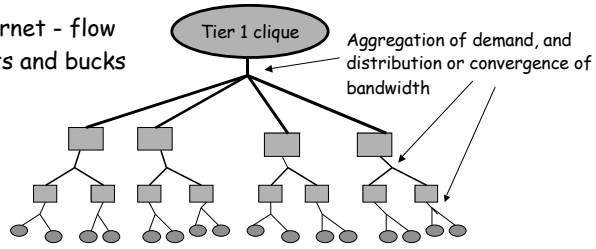
Roughly speaking, if all users have access to multiple providers (lots of dotted lines) then operators are forced to operate at cost, or at cost of second best competitor. That is the "commodity Internet."

Hierarchical networks

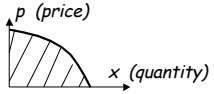


At the opposite extreme, if no dotted lines are present, network agents can participate in monopoly pricing. By understanding extremes we aim to better understand flow of bits and bucks in the Internet.

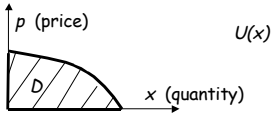
Internet - flow of bits and bucks



A wide variety of tree networks can be built up once several ways to aggregate demand are defined.



Four equivalent ways to specify the demand of a buyer, in one-to-one correspondence with each other:



- (*Value function*) *U* is a nondecreasing, concave, right-continuous function with values in [-infinity, +infinity).
- *(Demand function) x(p)* is nondecreasing, left-continuous, continuous at zero, with values in [0,+infinity].
- *(Inverse demand function) p(x)* is nondecreasing, left-continuous, continuous at zero, with values in [0,+infinity].
- *(Demand set) D*={0} or *D* is a coordinate convex, and equal to the closure of its interior.

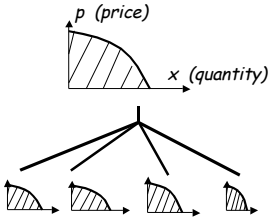
5 ways to aggregate demand

1. Sum of quantities demand

$$x(p) = x_1(p) + \cdots + x_n(p)$$

2. Vikrey-Clark-Groves allocation

$$p(C) = \frac{1}{C} \sum_i \sum_{j \neq i} \int_{C_j}^{C_{ji}} p_j(x)\,dx$$



3. Proportional allocation of given quantity (as in first part of talk)

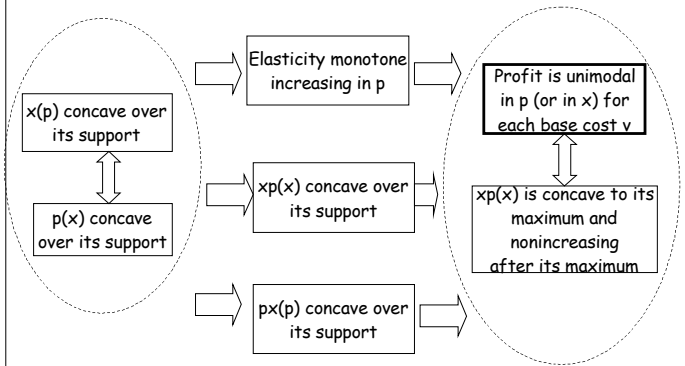
4. Broadcast with discriminatory pricing

$$p(x) = p_1(x) + \cdots + p_n(x)$$

5. Broadcast with shared payment

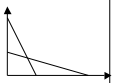
$$x(p) = \min\{x_1(p), \cdots, x_n(p)\}$$

Question: When is the profit of seller a unimodal function of price or quantity sold?



Question: Is unimodality of profit preserved under aggregation of demand?

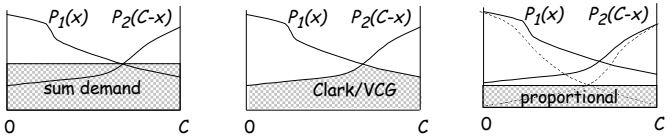
Typically only if the demands of the users are similar in shape.



Example: Selling capacity to a voice user and a file transfer user.
A high price might extract much profit from voice user.
A low price might extract much profit from file transfer user.
Intermediate prices may lead to less profit
=> hill climbing profit maximization won't work

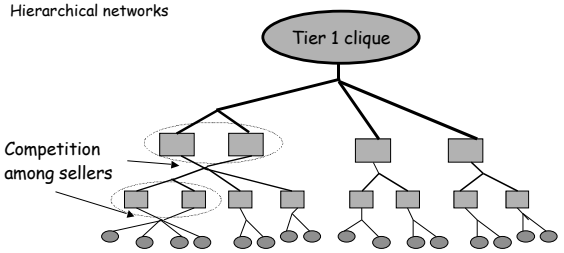
Related notion: there may be powerful incentives for price discrimination (see A. Oldlyzko's website/talk)

Question: How do aggregation methods compare regarding revenue to seller?



Note: Differences diminish with large numbers of buyers. For example:

revenue for n buyers for Clark/VCG $\geq$ revenue for n-1 buyers for sum demand (omit strongest buyer)



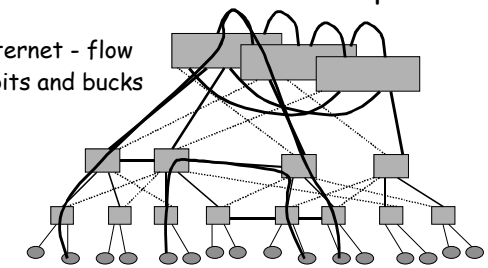
Competition of selling agents can be modeled using:
1. Bertrand-Nash equilibrium (selling agents set prices)
2. Cournot equilibrium (sellers sell set capacities, bought at fixed price)
3. Cournot equilibrium (sellers sell set capacity, bought through competition)

Summary of second part of talk

-Equilibrium pricing for tree networks can be defined bottom up.
-Many ways exist to aggregate user demand.
-Aggregation of user demand often does not preserve unimodality of profit functions of agents facing the demand.
-If all users/agents have alternate connections with suitable bandwidth, then agents sell at cost.

III. Future Directions and Implications

Internet - flow of bits and bucks



Internet pricing model is vastly different from traditional TelCo Voice. Voice/telco revenue streams are the primary source of funding of governments in many small countries. Understanding of Internet market pressures is an important ingredient in the study of socio-economic status of developing countries, as well as making predictions about the future of the Internet. (Thanks to Bill Woodcock for insights.)

For more information

Available at www.uiuc.edu/~b-hajek:

H. and Ganesh Gopal, "A Framework for Studying Demand in Hierarchical Networks (Preliminary draft)," *Workshop on Control and Pricing in Communication and Power Networks*,} Institute for Mathematics and Its Applications, March 8-13, 2004.

H. And Sichao Yang, "Strategic buyers in a sum bid game for flat networks"

Sichao Yang and H. , "An efficient mechanism for allocation of a divisible good" (Monday, IMA poster)

Sujay Sanghavi and H., "Optimal allocation of a divisible good to strategic buyers," (Monday, IMA poster)

Thanks!