A Phase Transition Model for Cascading Failure Analysis

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- Motivation - failure of single transmission line in heavily loaded electric grid often creates overloads in parallel lines.
- Equipment protection schemes then remove from service other overloaded branches.

Critical question: does effect cascade from local phenomena to “global” failure?

Motivation & Background

- Key goal #1: efficient methods for predicting "cascading" effects, identifying scenarios that lead to large area failures.

- Extension to state of the art: seek to capture influence of network dynamics in cascading overloads, including role of stochastic load variation as possible driving term.

Approach and Outcome

- Key "trick" - smooth approximation to mechanism of local branch failure.
- Exploit special structure in global dynamics, allows tractable potential function in model w/stochastic load.
- Payoff: a "phase transition" type model, identifying vulnerability to global failures.

Related Work

- Verghese and co-workers(MIT) - "Influence model." Markov chain model for failure AT NODES, with interconnection network determining "influence" of one node failure on other nodes.

Existing Work

- Thorp and co-workers (Cornell): refinements to importance sampling to allow identification of sequence random BRANCH failures leading to large area failure.
- To date, computational challenges limited overload calculation to flow values in new steady state - no dynamics.

Overview of Approach Here

- Develop o.d.e. model for branch parameter, with bi-stable equilibria: one at operational value, one at failed value.

- Drive these dynamics with coupling term that represents loading level on branch.

Overview of Approach Here

- Argue that a class of useful models can be built with special structure:

$$\dot{x} = A\nabla\vartheta(x)$$

(deterministic case)

$$d\mathbf{x} = A\nabla\vartheta(x) - \frac{1}{2}(A+A^{T})^{1/2}dW_{t}$$

(with stochastic disturbances)

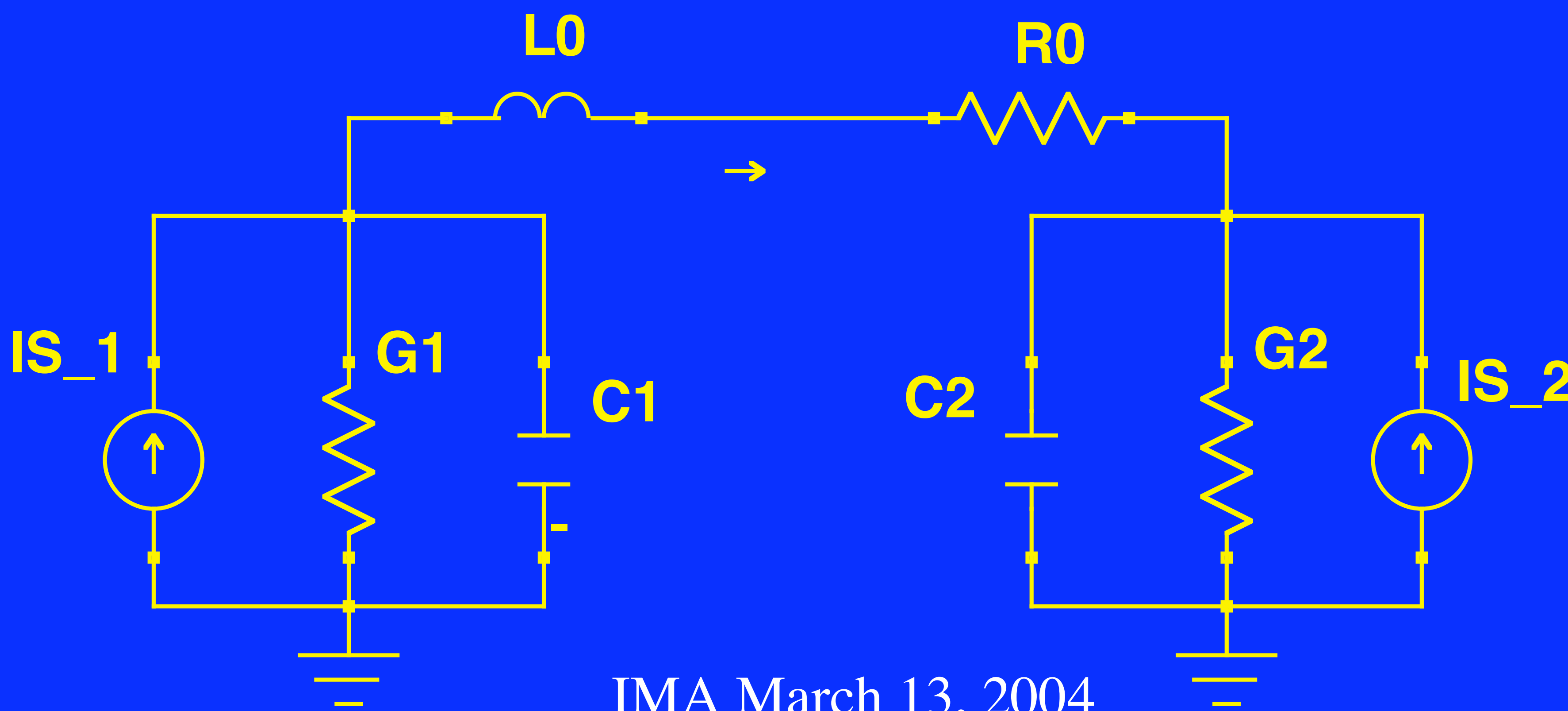
Overview of Approach Here

- Key foundation: local failure mechanism through energy storing branch element (inductance in toy circuit model to follow, and in desired power application; spring constant in finite element spring/mass/damper models, others… ?)

- Branch failure associated with element admittance approaching zero.

Motivating/Illustrative Example: Linear RLC with Inductive "Fuses"

- Simple one branch example: can we capture series branch failure in structured model?



Motivating Example: Linear RLC

- Convenient states: capacitor charges, inductor flux. Resulting model

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{\varphi}_0 \end{bmatrix} = \begin{bmatrix} -G_1 & 0 & -1 \\ 0 & -G_2 & 1 \\ 1 & -1 & -R_0 \end{bmatrix} \begin{bmatrix} q_1/C_1 \\ q_2/C_2 \\ \varphi_0/L_0 \end{bmatrix} + \begin{bmatrix} i_{s,1} \\ i_{s,2} \\ 0 \end{bmatrix}$$

Motivating Example: Linear RLC

- For fixed $i_{s,1}$, $i_{s,2}$, let q^e's φ^e denote equilibrium states. Define a "relative energy" potential function:

$$W_r(q_1,q_2,\varphi_0) = \frac{1}{2C_1}(q_1-q_1^e)^2 + \frac{1}{2C_2}(q_2-q_2^e)^2$$

$$+ \frac{1}{2L_0}(\varphi_0-\varphi_0^e)^2$$

Motivating Example: Linear RLC

- State equations then take form:

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{\varphi}_0\end{bmatrix} = \underbrace{\begin{bmatrix}-G_1 & 0 & -1\\ 0 & -G_2 & 1\\ 1 & -1 & -R_0\end{bmatrix}}_{\text{Denote as "A"}} \nabla_{q_1,q_2,\varphi_0} W_r(q_1,q_2,\varphi_0)$$

Treatment of Inductive "Fuses"

- Consider linear inductance element, letting parameter "N" represent inverse inductance; i.e., N = 1/L. Value in "normal" operation denoted N^e.

Contribution to energy becomes

$$\frac{1}{2}N\varphi^2 - N^e\varphi^e\varphi + \frac{1}{2}N^e(\varphi^e)^2$$

Treatment of Inductive "Fuses"

Observations:

$$\frac{\partial W}{\partial N} = \frac{1}{2}\varphi^2 \approx \frac{(L^e)^2}{2} i^2$$: Branch loading indicato

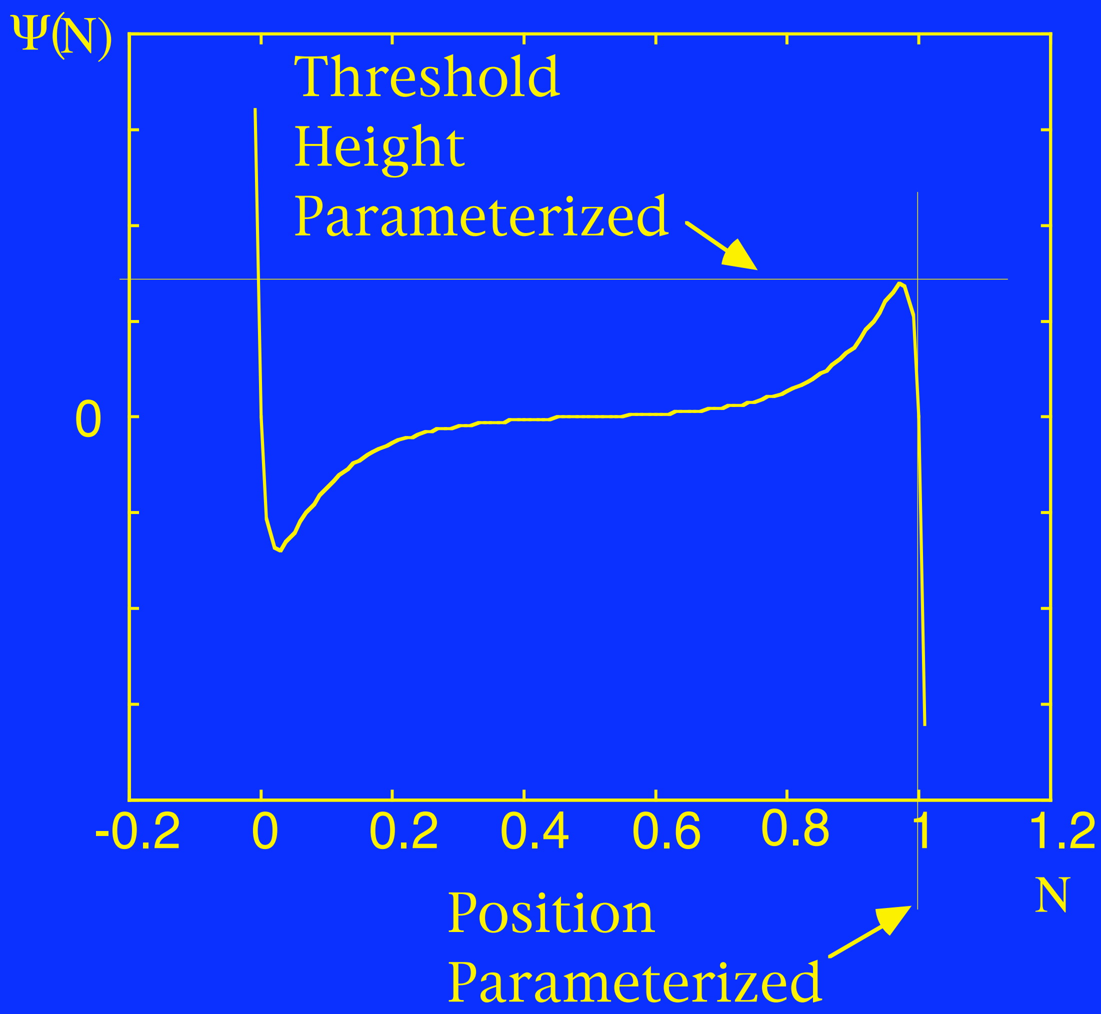
$N \rightarrow 0 \Leftrightarrow L \rightarrow \infty \Leftrightarrow$ branch impedance goes to infinity

Treatment of Inductive "Fuses"

Next - append continuous *dynamics* for N.

Key element of dN/dt driving term:
define a scalar threshold function $\Psi(N)$ to set limit at which failure is triggered, as shown on following slide…

Threshold function for "N" dynamics



Scalar function sum of weighted and shifted exponentials - hence guaranteed integratable.

"N" Dynamics

$$\dot{N} = \frac{1}{\tau}\left\{\Psi(N) - \frac{1}{2}\varphi^2\right\}$$

Behavior: N stable, in neighborhood of normal value N^e, for low flux; driven to neighborhood of zero when flux exceeds threshold (note no possible recovery back to N^e). Rate governed by τ.

Composite Dynamics Construction

Extend arguments of W_r to reflect additional dependence on N:

$$z = [q_1, q_2, \varphi_0, N]^T$$

$$W(z) = W_r^{old} + \int_{N^e}^{N} -\Psi(\eta)\,d\eta$$

Composite Dynamics

$$\dot{z} = A\nabla_z W(z)$$

$$A = \begin{bmatrix} -G_1 & 0 & -1 & 0 \\ 0 & -G_2 & 1 & 0 \\ 1 & -1 & -R_0 & 0 \\ 0 & 0 & 0 & -1/\tau \end{bmatrix}$$

Composite Dynamics Properties

- A is full rank, negative semi-definite;
- W(z(t)) therefore non-increasing along any solution trajectory;
- Equilibria only at critical points of W;
- Standard Lyapunov arguments reveal an equilibrium z^e is asymptotically stable $\Leftrightarrow$ z^e local minimum of W(z)

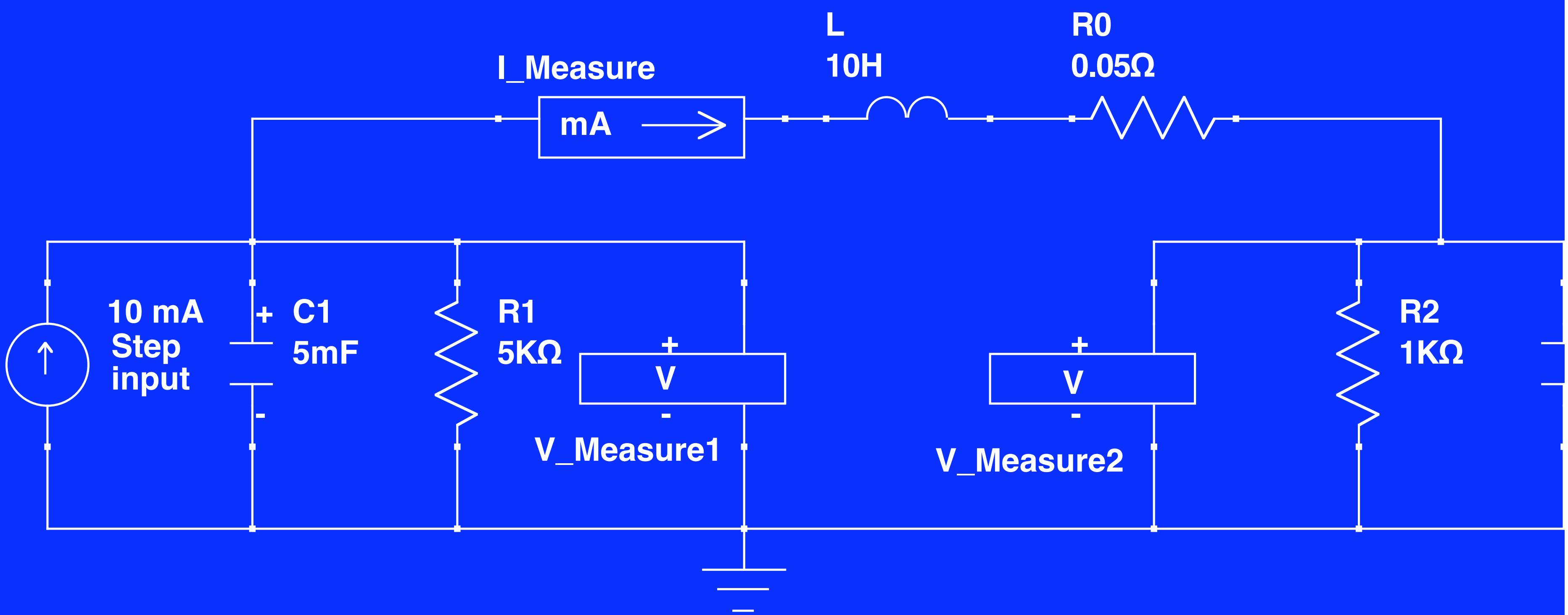
Properties in Stochastic Formulation

- Simple diagonal form of symmetric part of **A** allows introduction of uncorrelated white noise process in each state equation (see slide #8).
- W(z) plays role of Wentzell-Freidlin's quasi-potential function (hence statistical properties such as asymptotic behavior of expected time to exit a domain become calculable…)

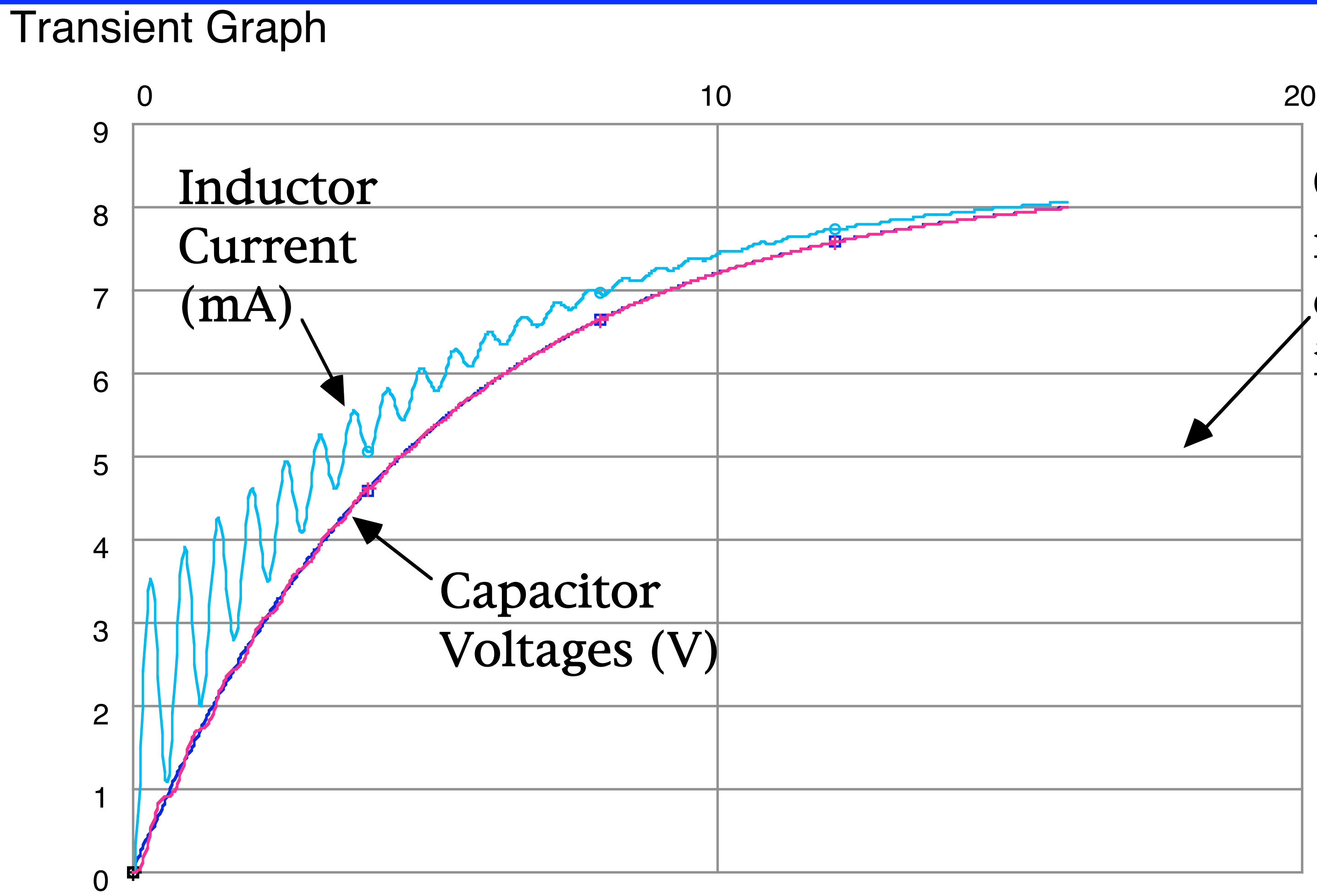
Time Domain Behavior - have we captured branch failure?

- Sequence of simulations to follow illustrate trajectories of our little 4 state toy model;
- Step input current at node 1;
- Set small threshold failure function, to illustrate triggering of inductance branch failure.

Test Circuit



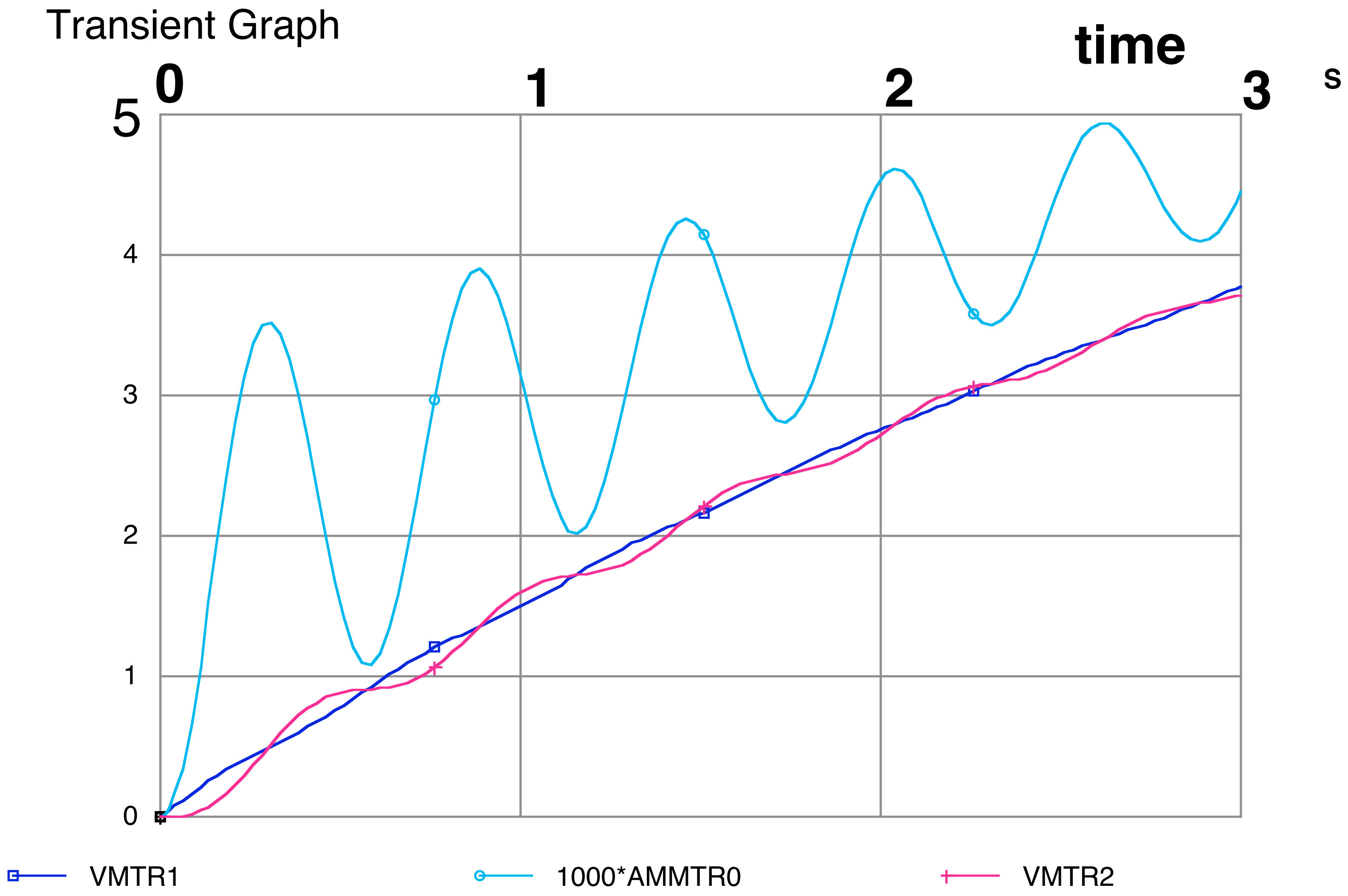
SPICE Plot-No Branch Failure



Observe for future reference: inductor current hits 5mA at its fifth local peak.

Branch Failure Threshold

- Equilibrium current after step = 8.33 mA; corresponding flux φ^e = 0.0833 Wb-turns.
- Select threshold of branch trip event to be 5 mA, or φ=0.05 Wb-turns; threshold value for $\Psi(N)$ set at $0.5\varphi^2 = 0.00125$. From SPICE plot, expect failure just before 3 seconds (0-3 sec SPICE plot follows)


Transient Graph
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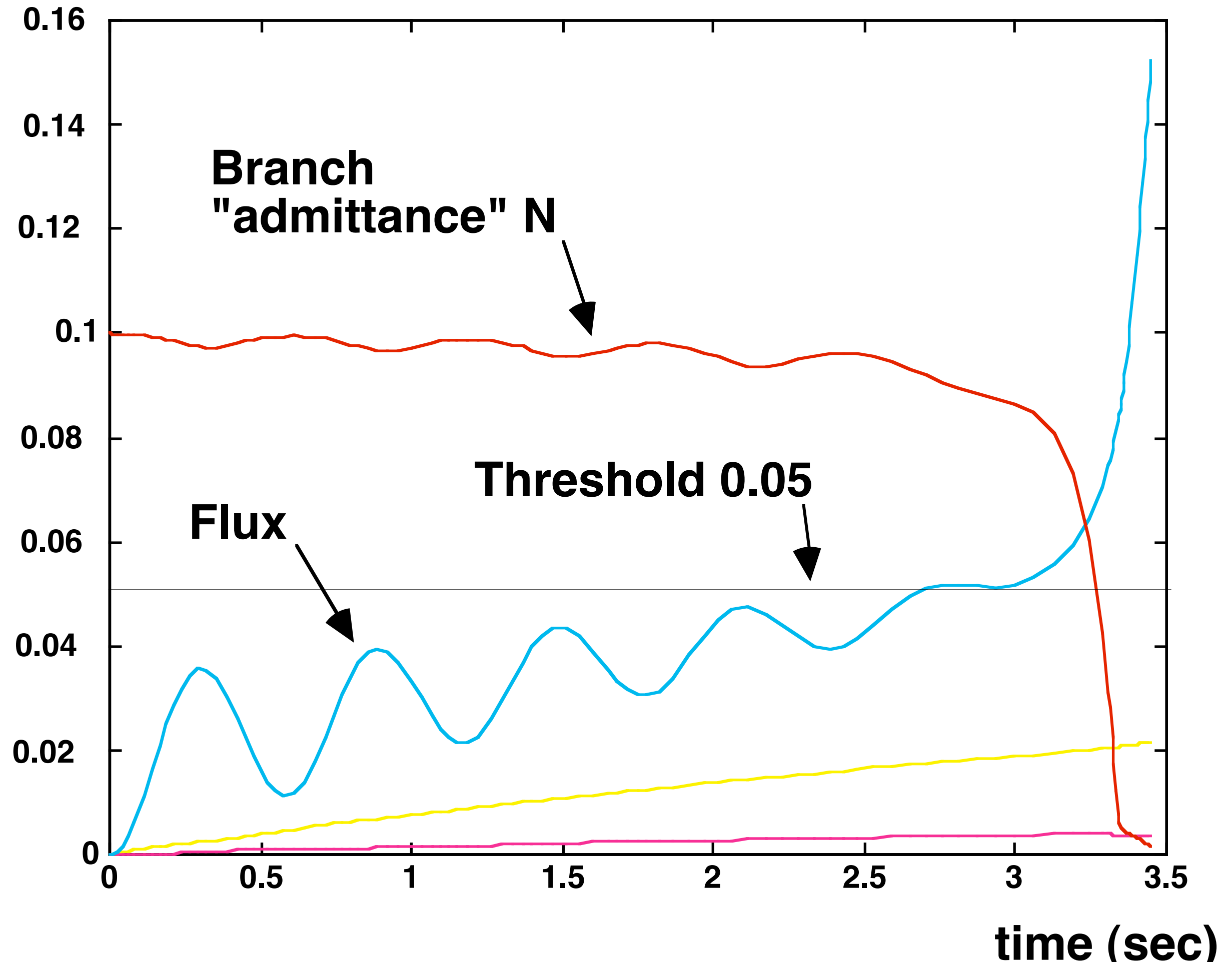
Trajectories of Branch Failure Model



Comments/Interpretations

- Correct qualitative behavior - branch parameter N rapidly decays to 0 after hitting threshold
- In pre-failure region, parameter N does “wander” slightly with flux; an undesirable artifact of in this model, but effect can be minimized.

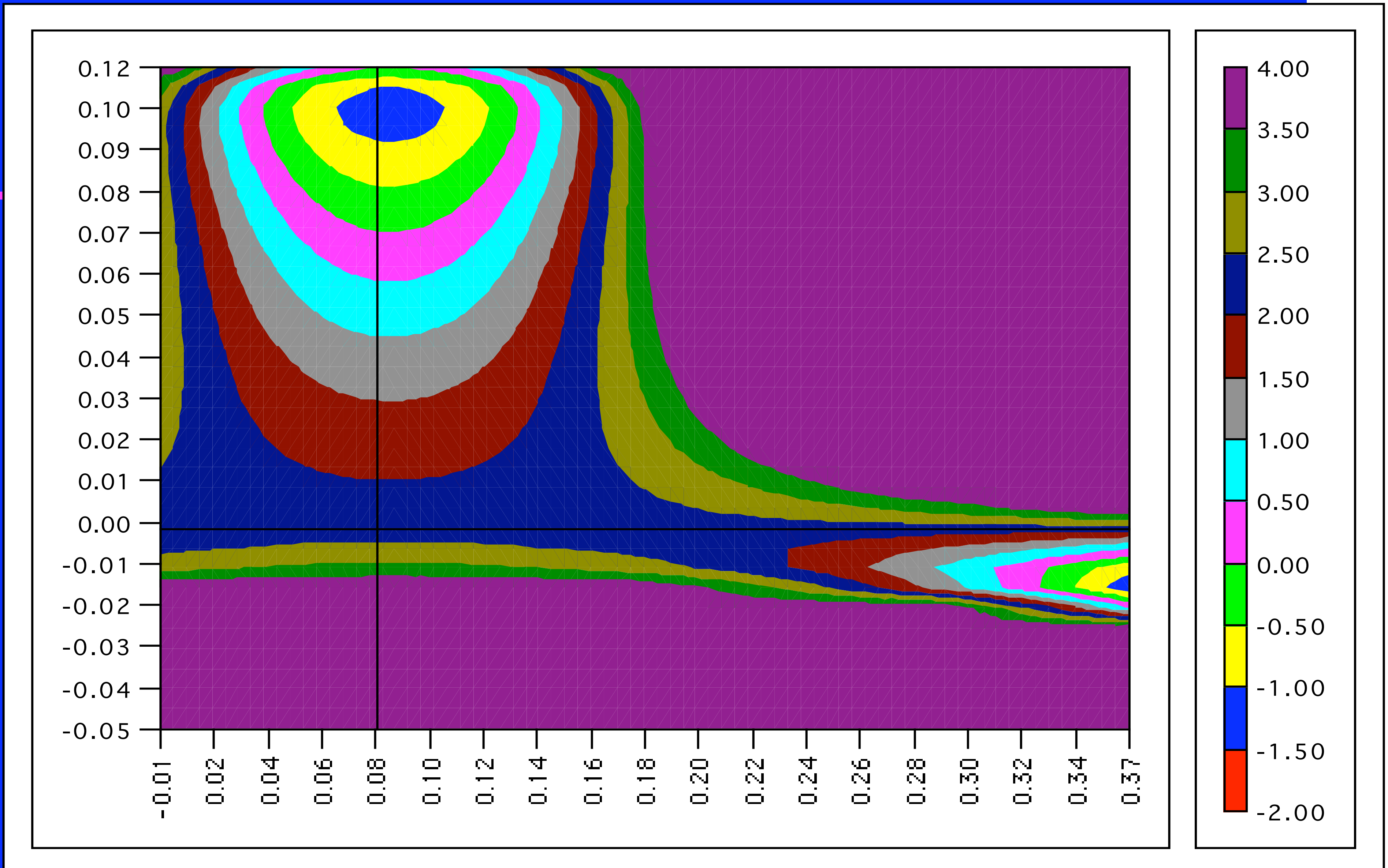
Comments/Interpretations

- Elaboration: to be friendly to numerical integration routines, slope of Ψ(N) function was made gentle in this example.
- Local variation in N with flux reduced as Ψ(N) slope is steeper (consider slide 17).
- We DO NOT advocate this model as a simulation tool - it has inherently stiff set of ode's. Payoff is all in closed form of W(z)

Use of W(z) Energy Function

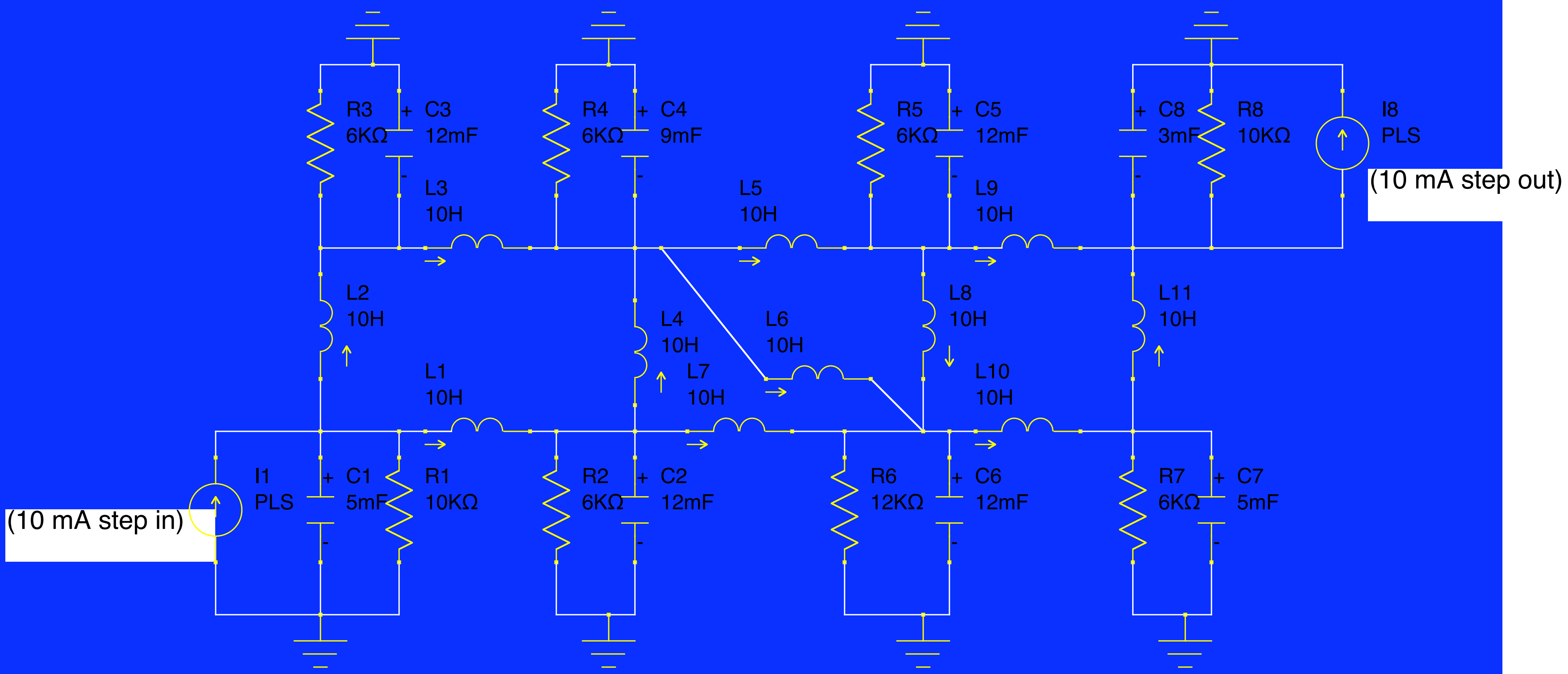
As with traditional energy/Lyapunov analysis, great power in having closed form function along which trajectories globally decay.

Contour plot to follow shows operational state creating one basin, failed state creates another. Saddle exit between the two corresponds to an unstable equilibrium.



Horizontal Axis: Flux; Vertical Axis: N (inverse inducta

Larger circuit example (from IEEE Control Systems Magazine paper



Larger circuit example

- Picks up more features in common with power problem, though underlying dynamics apart from failure mode linear.
- Even with linearity, illustrates fairly high dimensionality, with closely grouped, light damped oscillatory modes.
- Trajectories complex, predicting threshold driven failures is challenging.

Larger circuit example

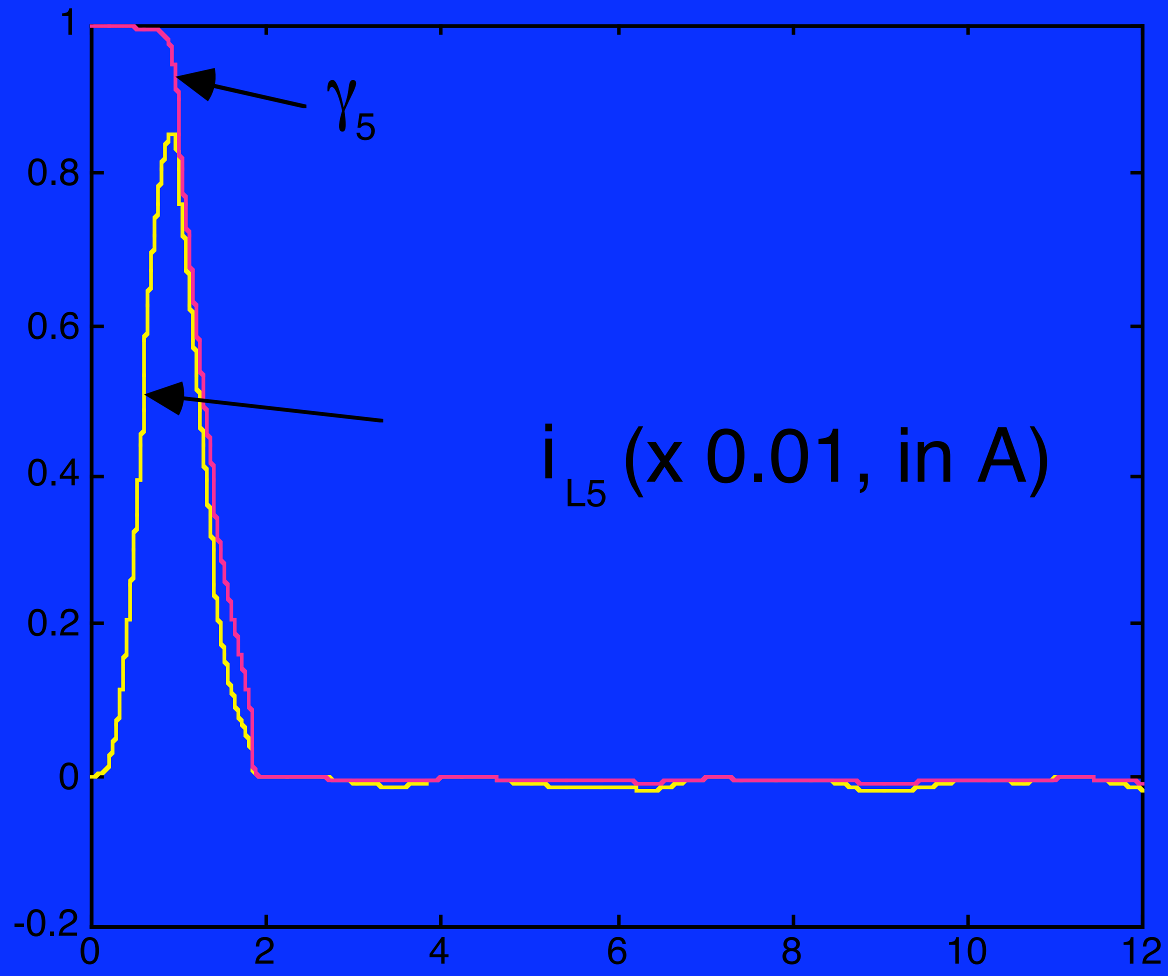
- In plots to follow, for multiple branches, associate parameter γ with each branch

$\gamma = 1$ branch in service;

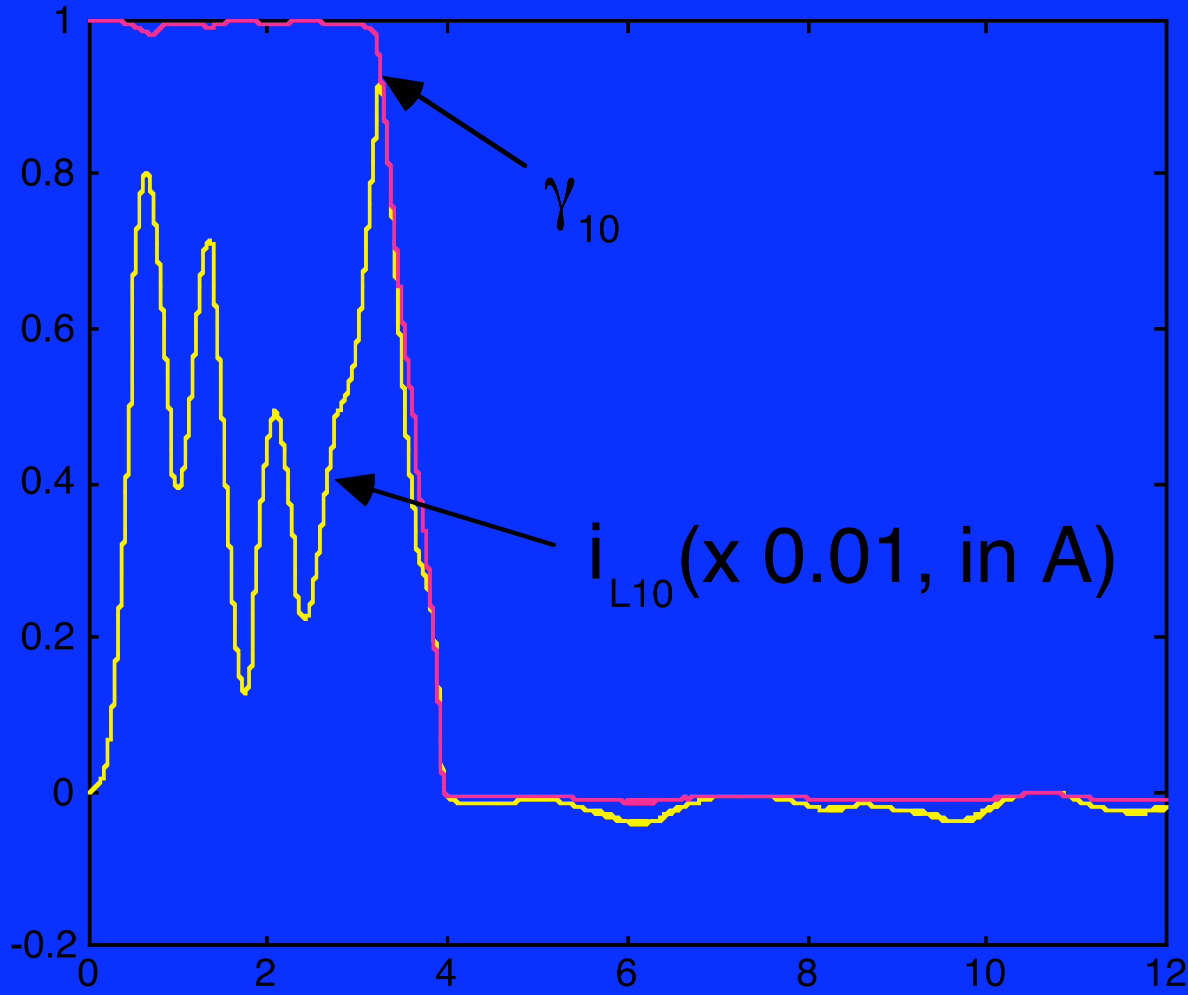
$\gamma = 0$ branch out of service.

For illustration, set failure threshold on branches #'s 5 & 10 to be 8.5 mA.

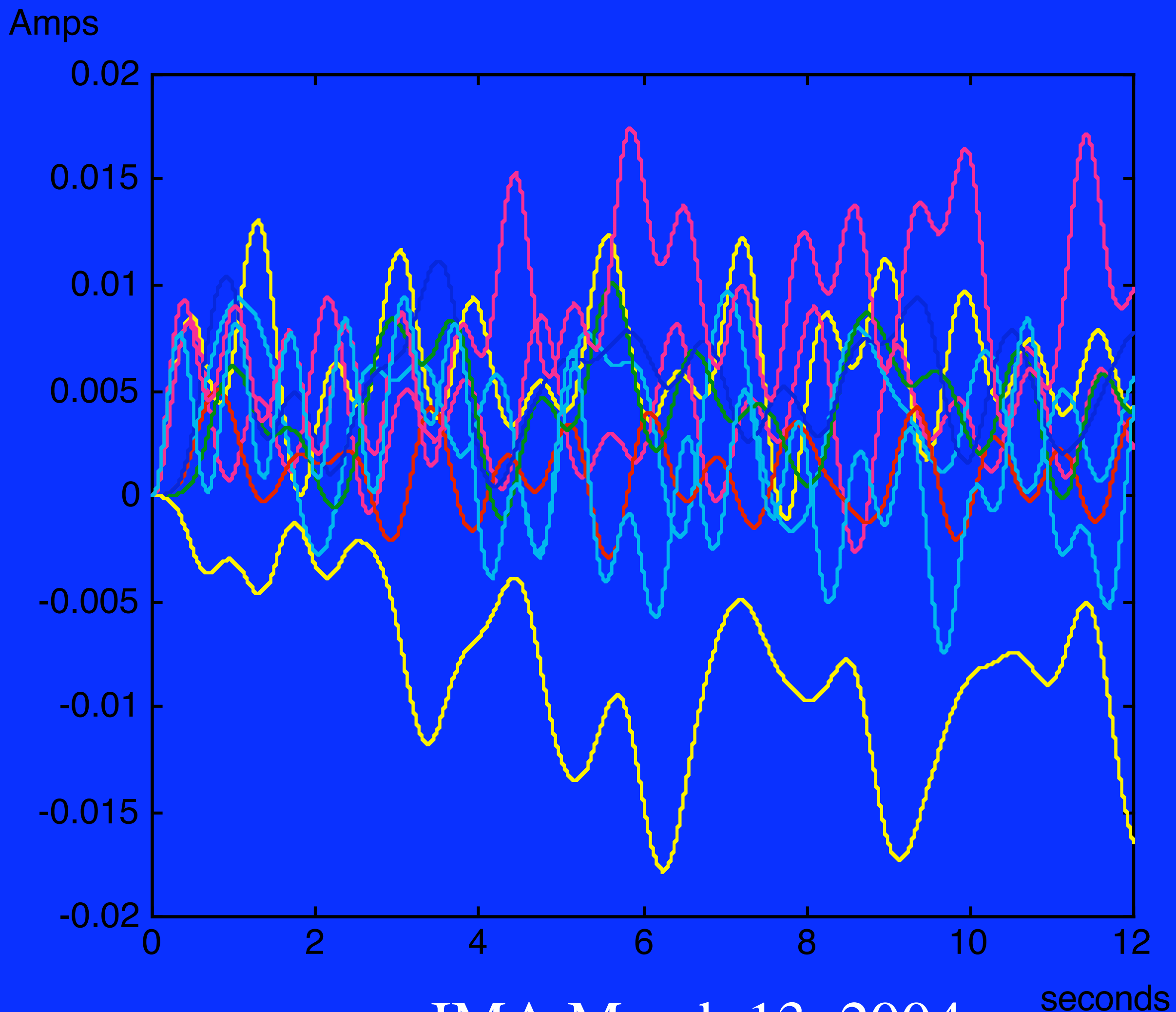
Below: Current and Gamma vs time



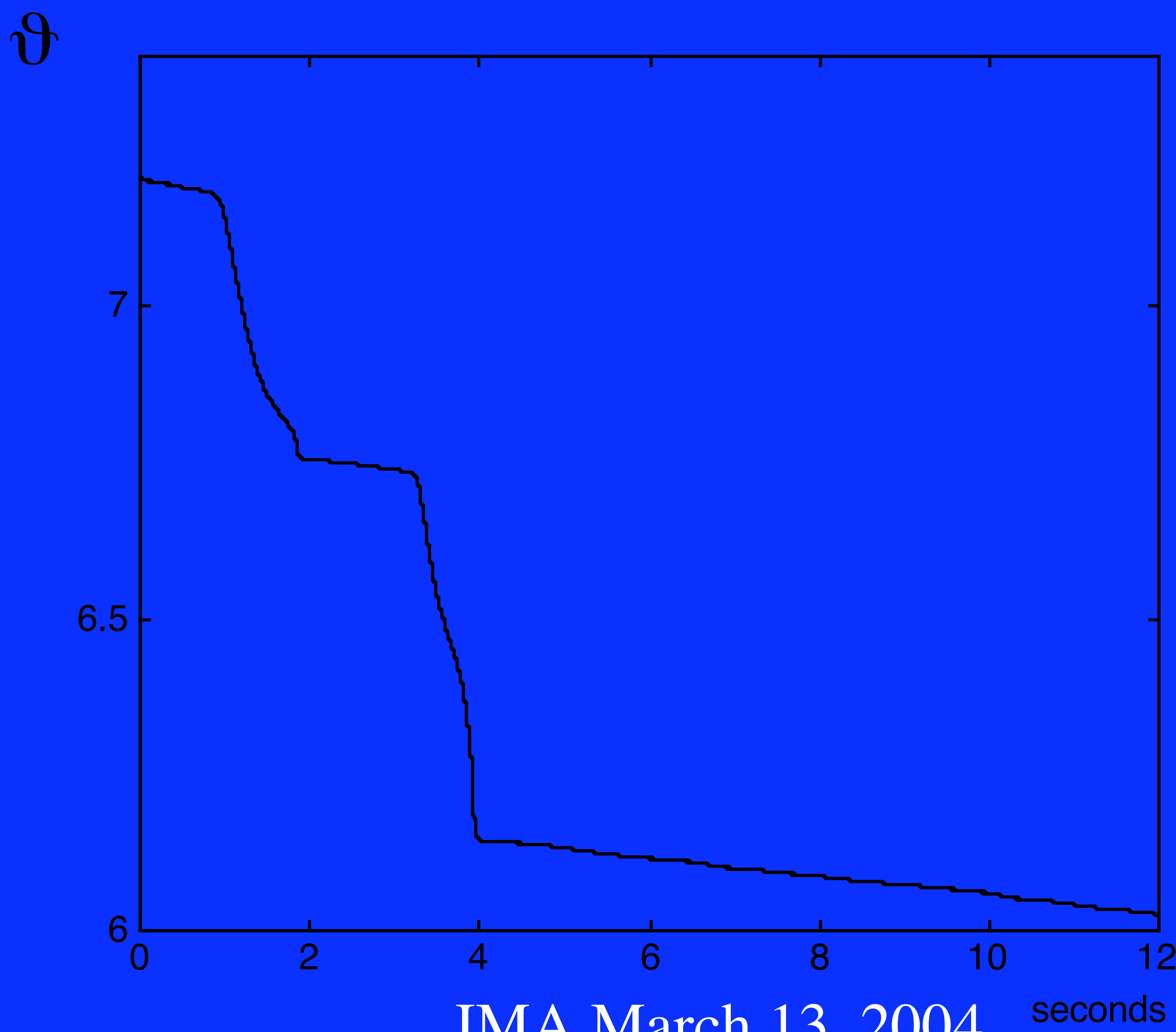
Below: Current and Gamma vs time

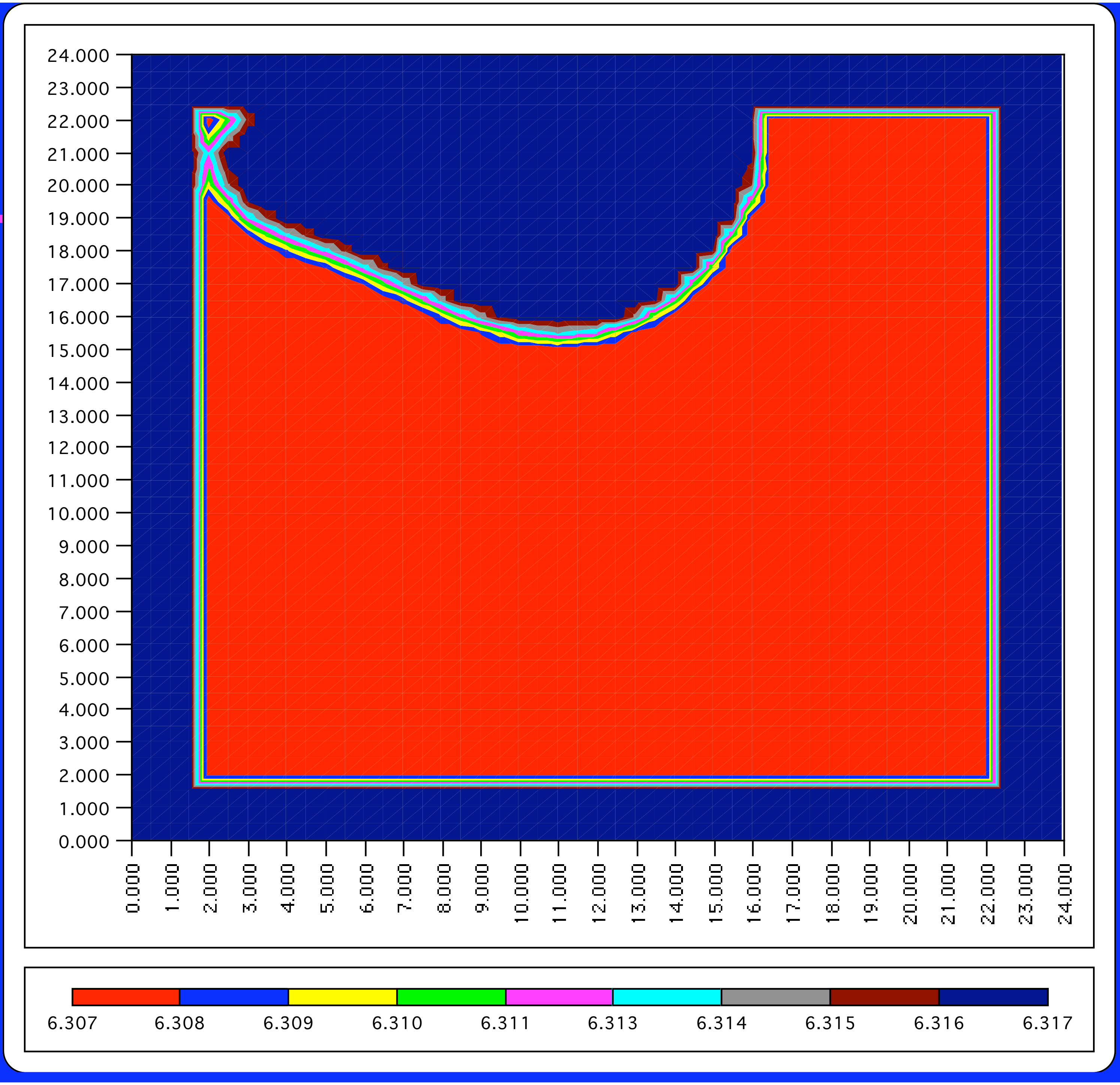


Ensemble of (remaining) branch currents in larger circuit example



Lyapunov function behavior in larger circuit example





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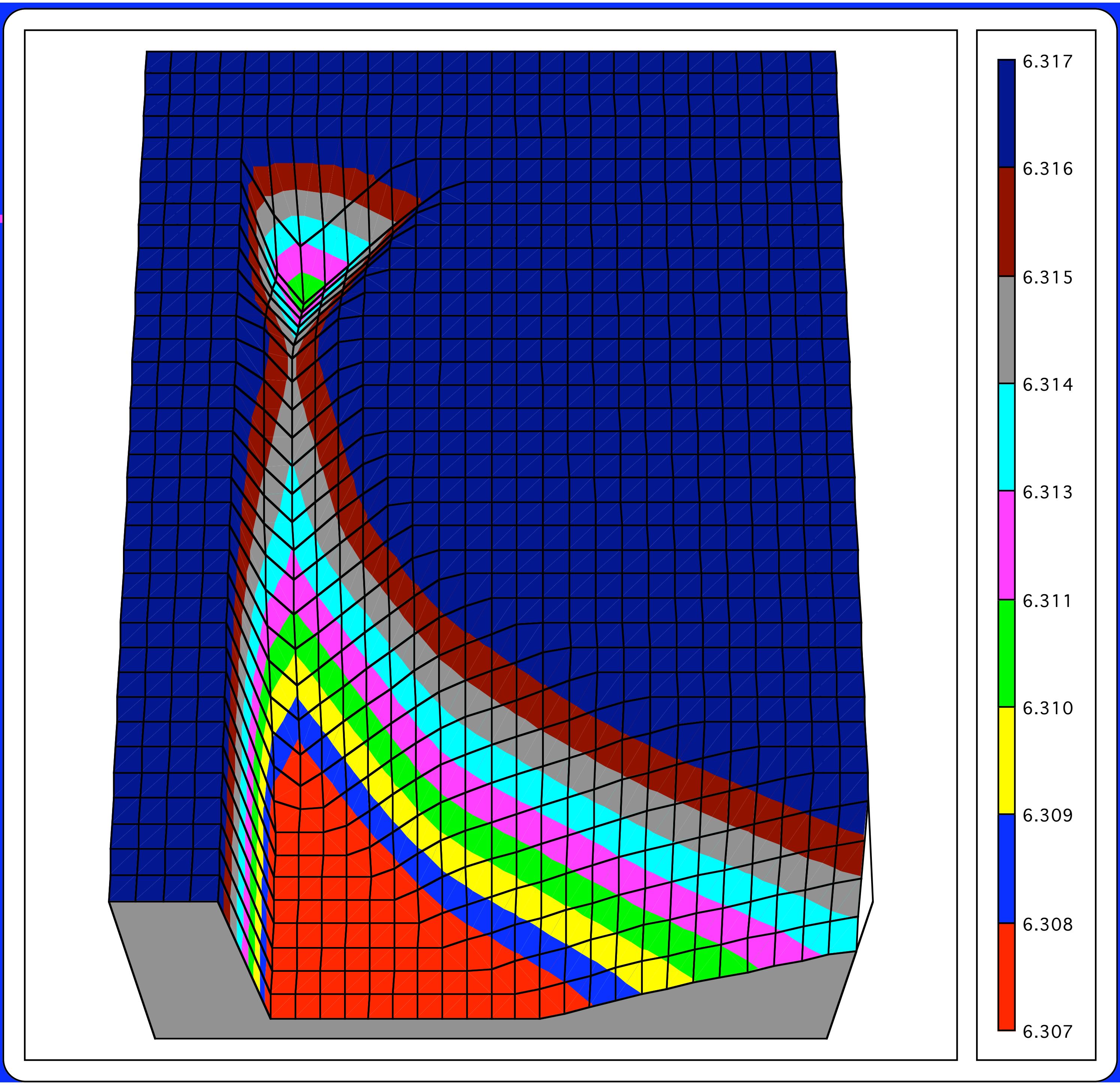
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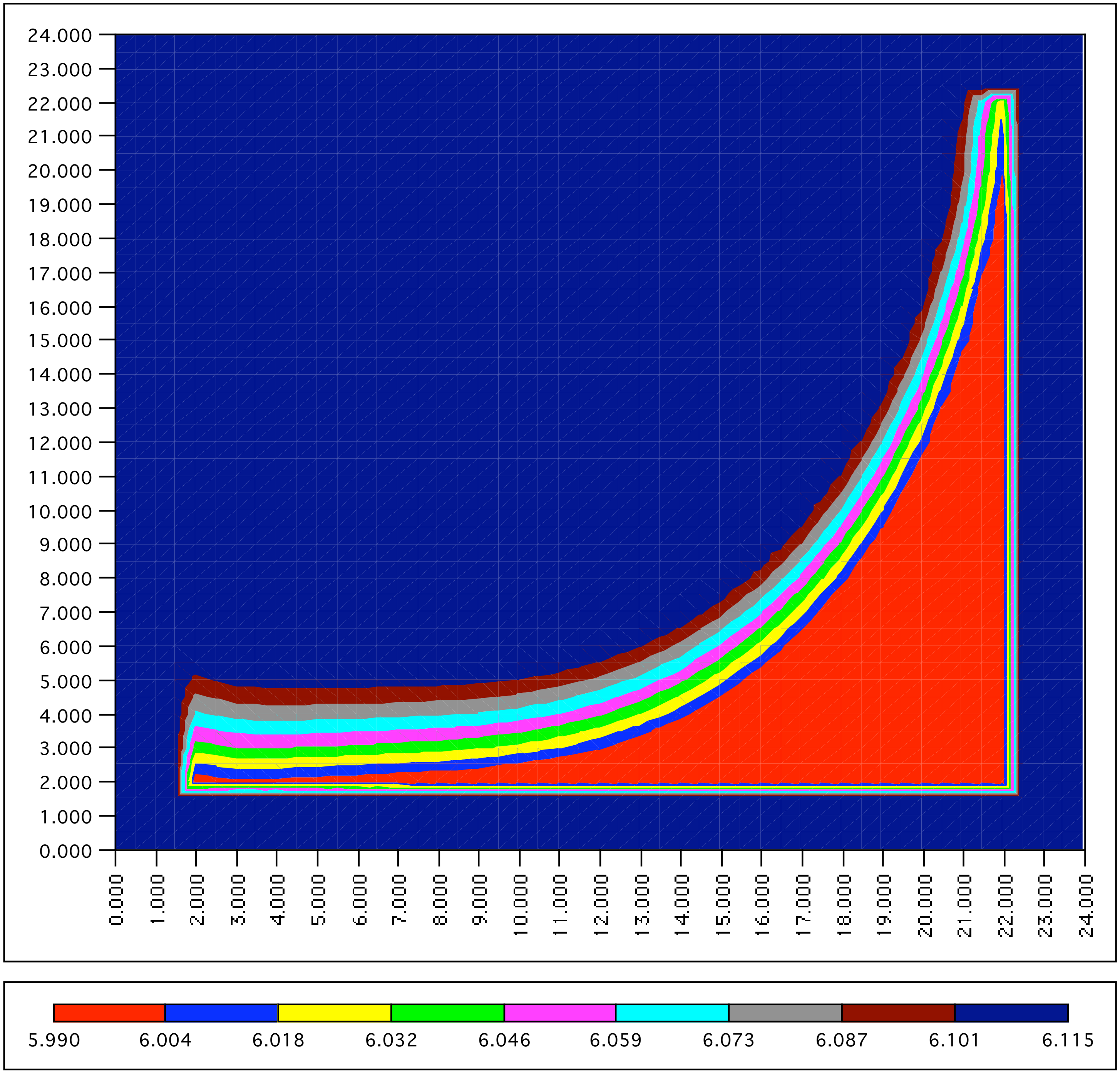
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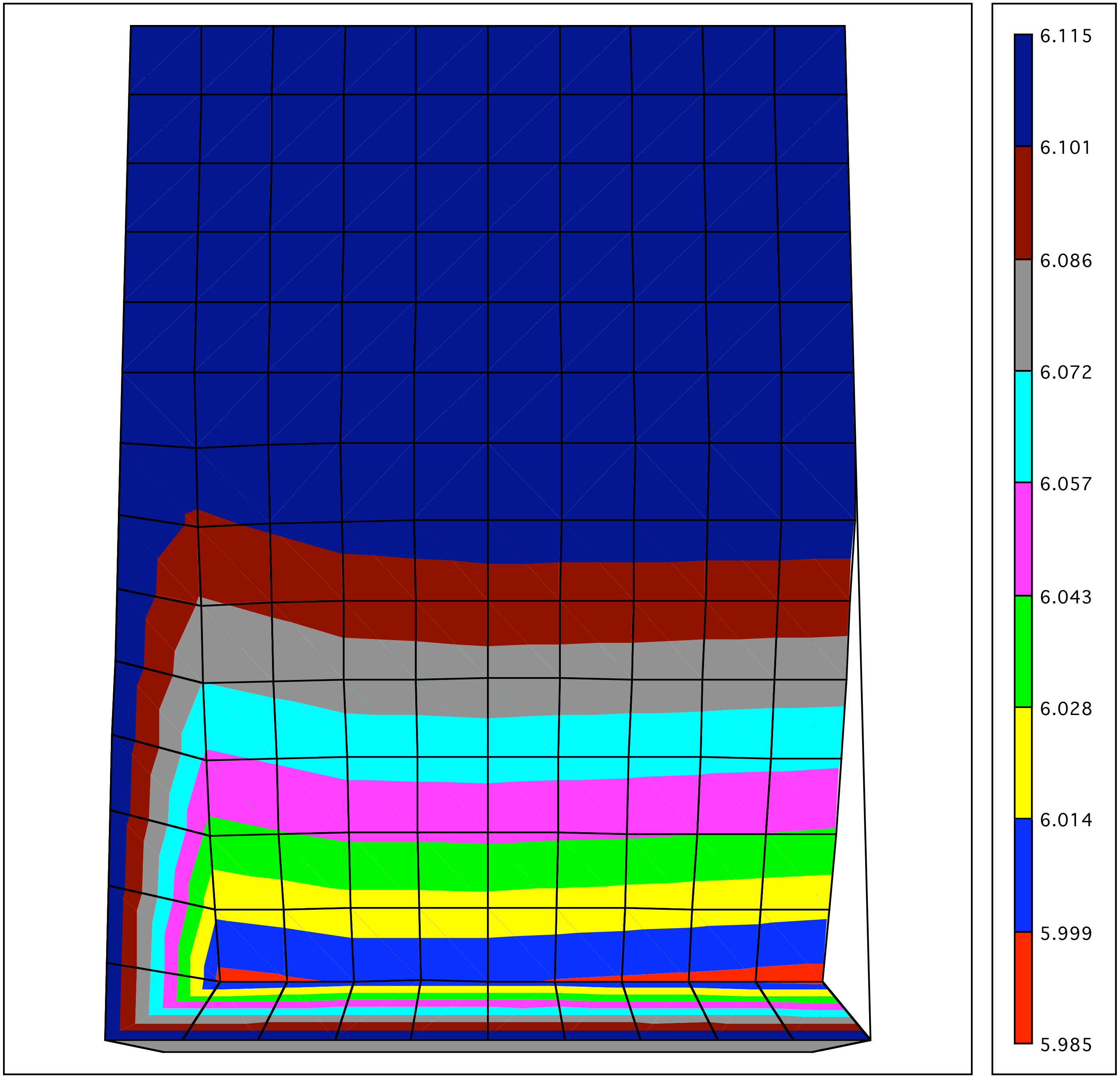
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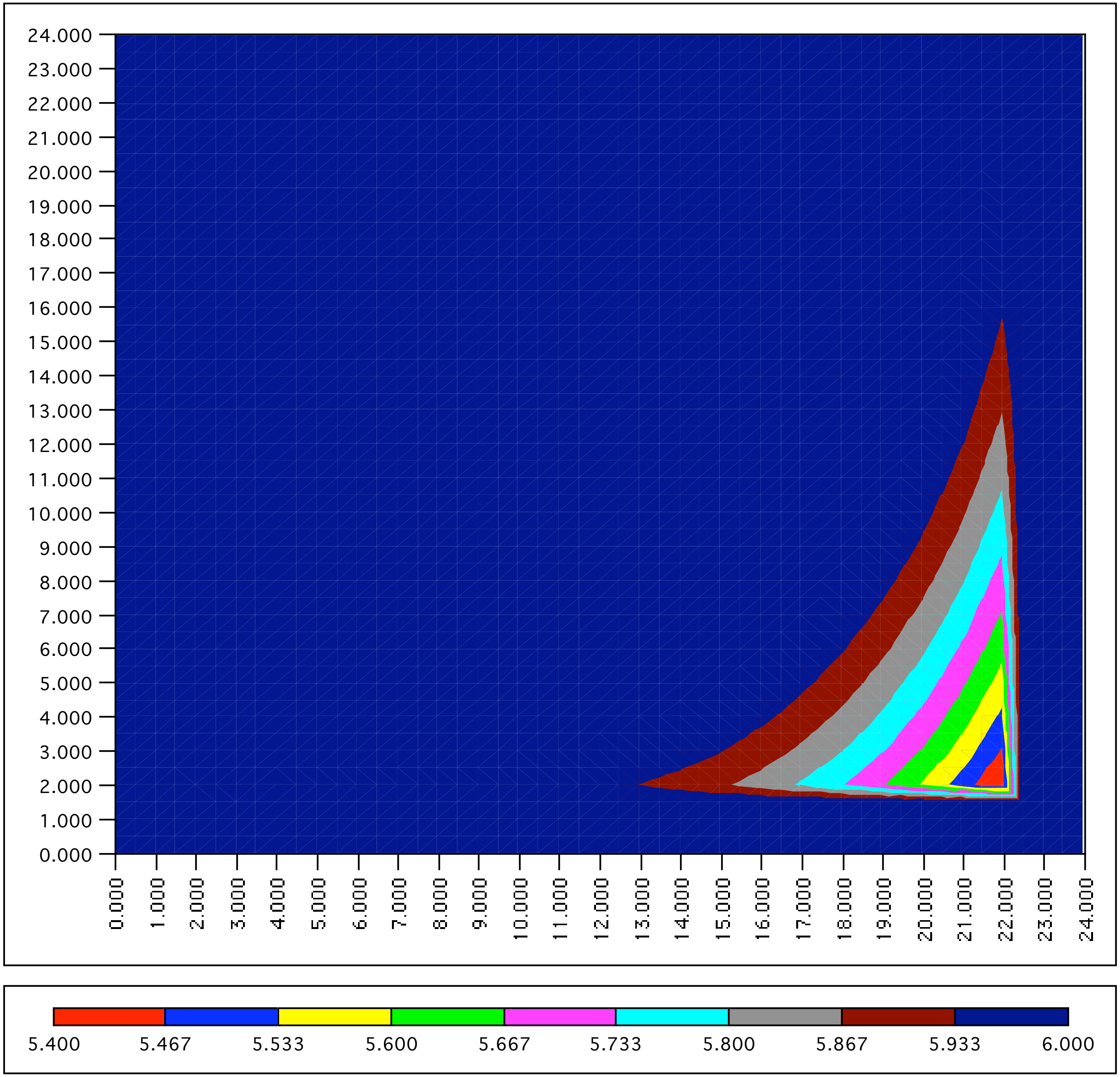

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Ongoing Work/Conclusions

- Work with former post-doc Loic Roger has progressed on same structure model for power system, including over-current relaying on branches, and over/under frequency thresholds on generators

  (but don't have any pretty pictures for those cases…)

Ongoing Work/Conclusions

- Promising avenue for introducing smooth model for branch failure, maintaining analytically tractable structure.

- Operational configurations become locally stable energy wells; "path to instability" through saddle exits.

Future Work/Conclusions

- Test of vulnerability to global failure captured in ease of escape from wells.
- Expected exit time one measure of this ease of escape/initial failure.
- Well's about equilibria for progressively degraded network configurations measure ease of capture (and hence sustainability) of these states.