

Electric Power Network Tutorial: Basic Steady State & Dynamic Models for Control, Pricing and Optimization

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Components of my talk:

- * Evangelism: for an audience likely focused on mathematical problems in communications networks, show what makes electric power networks different, and (hopefully) what makes them interesting.
- * Start with some "fun facts" - just general nature of technology involved, (opinionated) overview of nature of industry.
- * Move on to mathematical modeling - construct underlying dynamics, associated equilibrium for steady state.

Highlight:

- state equations associated with nodes;
- role of network structure;
- peculiar coordinate system used in dynamics and steady state;
- policy/control decisions available;
- market driven control & pricing.

Old Assumptions under Regulated or State Monopoly

- * Ever increasing economies of scale for central generation plants - lowest production cost associated with central generating plants of increasing size. These economies of scale (historically) justified "natural monopoly" for generation.
- * Parallel independent transmission and distribution firms seen as highly inefficient. Long historic trend toward single interconnected synchronous grid covering large geographic area. Again, natural monopoly.

What has Changed?

- i) Public perception of significant mis-steps in generation investment for many state/regulated monopoly firms (notable in US). Examples of billions of dollars of investment in partially constructed nuclear plants abandoned (interestingly, quite commensurate with billions of dollars of waste in CA markets ...)
- ii) Significant expansion of availability of natural gas, beyond estimates of one or two decades earlier.
- iii) Significant improvements in natural gas based prime movers; combined cycle gas turbines achieving efficiencies in high 50% range, *from plants of medium size.*

iv) Strong political trends in many nations favoring market-based solutions over central planning solutions. Belief that private firms would make better investment decisions (editorial comment - evidence in US to date suggest private firms unable to make any investment decisions in transmission).

Common Features Emerging in Power Production in Many Parts of the World:

- * Assumption that distribution systems and (most of) transmission grid are still natural monopolies. Hence, these remain regulated, or state held.
- * Assumption that real time operation of transmission grid must lie in hands of centralized coordinating body, with considerable authority; oft-termed "Independent System Operator," or ISO.

- * Assumption that generation should be provided through competitive markets (though exact structure and rules for markets vary widely, and in many cases, are still evolving).
- * Many (not all) markets share a rough structure of day ahead offers made by independent generation companies, with a central market entity (ISO or Power Exchange) making initial plan of schedule/purchases.
- * Some markets allow independent arranged transactions between buyer & seller, "bilateral transaction."
Coordinated with ISO, but not with central market.

- * Emerging efforts to bring larger number of power consumers to active participation into market, possibly down to retail level (terms: "retail competition," or "customer choice")

Generation Technology Overview

- * Key observation: conversion of mechanical energy to electrical energy via rotating machines is a highly refined technology - in excess of 98% efficient in modern machines.
- * But wide variations in efficiency lie in conversion process from chemical (or nuclear) energy to rotating mechanical power.
- * Large percentage of new generation around the world (and overwhelming majority in US) within last decade has natural gas as primary fuel; large percentage as gas turbines (irony - in 1970's in US, natural gas was outlawed as a fuel for electric generation).

- * Huge market demand for these units; before economic slowdown of 2001, growing concern regarding world-wide manufacturing capability - could it keep up with demand?
- * Increasing maximum size of units (e.g., 1000 MW - so apparent economies of scale showing up in this technology as well)

New Technologies in GT Units

- * Much advance due to improved materials, and improved computer tools for modeling and optimizing combustion and fluid flow designs.
- * Better materials allow higher firing temperatures, yields higher efficiency, while limiting NO_x emissions. Potential efficiencies of 60% in combined cycle gas turbines.
- * Some manufacturers also examining steam cooling to raise efficiencies in simple cycle units (an example in partial operation: Lakeland Electric, Florida, USA site - 262 MW unit claiming 39% efficiency).

Aerospace Derivative Gas Turbines

- * "Aero" units often serve in smaller capacity (say ~5 to 50 MW) applications, where start/stop cycling for peaking may be necessary.
- * Simple cycle units, hence lower efficiency; however, some manufacturers claiming technology improvements bringing efficiency close to 40%.

Heat Recovery Steam Generators (HRSG) Typical technology of older coal fired plants

- * Historic efficiencies in coal plants topped out around 35-39%
- * Advances in gas turbine technology also driving incremental improvements in HRSG designs.
- * Significant efficiency enhancement can be gained in retrofit/upgrades of old generation steam turbines (opportunity here - in US, more than 1300 major steam generating plants over 30 yrs old!)
- * New blade designs, new coatings offer significant enhancements - efficiency improvements of up to 10% *claimed* by vendors.

Emerging Technologies

- * Tremendous interest being created by so-called "micro-turbines," and by fuel cell advances.

Some debate as to classification of microturbine, but rough characteristics:

- * power range of 10 kW to 500 kW;
- * turbomachinery based on radial flow designs, often derived from automotive turbocharger technologies;
- * usually just one rotating unit, with compressor, power turbine, and generator all on common shaft;

- * high speed operation, 10,000 to 100,000 rpm;
- * modest pressure and temperature operation, yielding fairly low emission (catalytic converters can be added);
- * very compact construction, suitable for transport by small truck;
- * generator allowed to operate at variable speed, at high frequency, into power electronic rectifier and internal DC bus, inverted to grid frequency AC.

Questions:

- * Will units achieve low maintenance, "turn-key" operation desired for target markets?
- * Cost, efficiency, and *cumulative* environmental impact of large numbers of these units?
- * Interconnection standards, and impacts on grid stability and reliability?

Fuel Cell Technologies

- * Growing experience with test installations
 - fuel cells in use in some reliability critical backup applications (e.g. Citibank, NYC), as well as US military test sites.
- * Very high capital cost remains key barrier to market acceptance.
- * Some analysts see possibility of price reductions coming from "spillover" automotive developments - major auto manufacturers placing large R&D budgets on fuel cells. However, significant difference in thermal management challenges for mobile vs. stationary power.
- * Technical applications questions for grid application - similar to microturbines -

what should be the interconnection standards? what are units' dynamic response characteristics?

Wind Power Advances

- * Old reliability problems (blade fatigue & failure) are largely eliminated in modern units.
- * Total world wide installed capacity was at about 13,400 MW in 1999 (this is approximately = capacity of world's largest single hydro facility, Itaipu, Brazil). Figures circa mid-2003 approaching 25,000 MW.
- * Wind power advocates target wind as providing 10% of electrical supply - this is presently achieved in Schleswig-Holstein region of Germany/Denmark.

Photovoltaics

- * Steady improvements in cost effectiveness & "energy payback" effectiveness of photovoltaics;
- * In older designs, energy intensive manufacturing meant that in many application areas, could be 5-7 years before energy input in manufacturing was "paid back" by energy output in operation.
- * Improved manufacturing & operational efficiency bring these figures down to 2 yr range.
- * Improved power electronics (and in large installations, solar tracking controllers) make maximum power extraction much cheaper/easier.

- * Despite advances, remains a relatively expensive technology in capital cost/kW, but extremely reliable, low maintenance.
- * Attractive for remote applications with modest power needs, where high reliability is very strong benefit (remote communication/sensing installations, lighthouses, etc.)

Power Grid Modeling for the Non-Power Engineer

What makes electric power networks different?

- * Flow of interest is power in a **sinusoidal quasi-steady state**. Hence **each flow is a two parameter quantity** ("active" and "reactive" powers, at known frequency).
- * Branches are mostly passive \Rightarrow **flow determined by boundary conditions at nodes** (controllable links do exist, but constitute less than 0.1% of all links).
- * Sinusoidal operation \Rightarrow **two parameter boundary condition at each node**.
- * **Essentially no storage at nodes**.
Hence no policy decisions to be made

relating to "service" of commodity at nodes.

What makes electric power networks different?

- * At equilibrium, nodes satisfy a power conservation law. Majority (but not all) policy/control decisions relate to choice of injection at subset of nodes, to exogenous demand at other nodes.
- * Equilibrium operating constraint #1: flows on links in capacity limits, nodal variables in limits.
- * Dominant non-equilibrium behavior (so called "swing dynamics") driven by nodal mismatch between external power injection/demand vs. absorption by network.

What makes electric power networks different?

- * Equilibrium operating constraint #2:
Operating point must be stable w.r.t. dynamics, with suitable robustness margin, *even in event of credible equipment failures.*
- * Current engineering practice often creates surrogate limits on link flows & node injections to approximate stability limits \Rightarrow link capacities function of operating point.
- * Maintaining margins against equipment failures & large demand deviations has significant dollar impact in markets.
- * Weighing robustness against rare events, with very imprecisely known probability distributions, against immediate certainty

of added operating cost. Policy conflicts are obvious.

What makes electric power networks interesting?

- * Dynamics highly nonlinear; stable equilibrium defining desired operating point never has unbounded domain of attraction.
- * Disturbances can cause state divergence from desired operating point, out of attractive domain. State divergence = blackout.
- * Timescales for dynamics relevant to stability on order of 0.1 sec. to perhaps 0.5 hr.
- * "Deregulation" has brought strong impetus for decentralized, competitive market driven operating policies.

What makes electric power networks interesting?

- * Choice of power production (and ultimately, consumption) at each node is to become a decentralized decision, based on a market clearing prices. Typical time scales: some day ahead decisions, followed by near real time 5-15 minute periodic update.
- * Competitive power marketers **strongly** interested in available link capacity.
- * Strong interest in attributing portion of flow on link to individual market players - "allocation of transmission rights." Nonlinearities and lack of exact convexity can make this questionable in true physical sense, but active attempts at financial instruments to "work around."

Nature of Basic Power Systems Model:

- * Two parameter "boundary condition" at nodes is simply a convenient representation of an instantaneous voltage that is sinusoidal:

$$v(t) = V(t) \cos(\omega t + \delta(t))$$

Voltage magnitude

relative phase angle

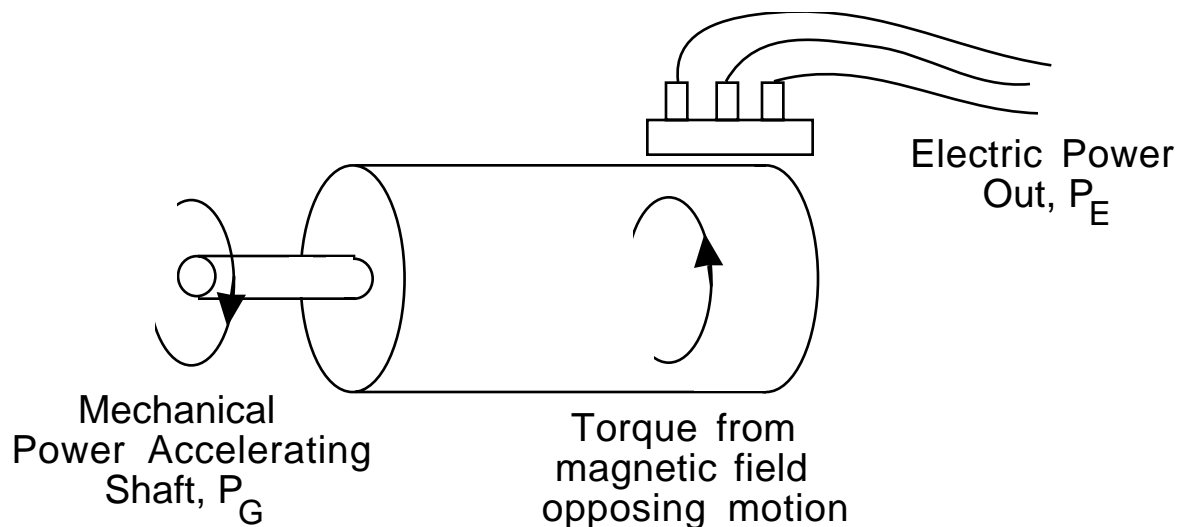
60 Hz
(377 radians/sec)

- * $V(t)$ and $\delta(t)$ controlled *indirectly*.
- * Fundamental dynamics are really just rotational Newton's law - consider nodes with generators attached.

Approximate torque as proportional to power

⇒

acceleration on generator shaft can be computed from net power



$P_G > P_E$ gen accelerates;

$P_G < P_E$ gen decelerates

* Control exercised through P_G .

Other Observations:

- 1) Physics of Synchronous generators \Rightarrow mechanical speed and electric frequency are equal. Can be used interchangeably!

$$\omega_0 + \Delta\omega \quad . = \text{normalized frequency/speed}$$

\vee
Deviation from
Synchronous speed

- 2) $\Delta\dot{\omega} := \frac{d\Delta\omega}{dt} = \frac{d\omega}{dt} = \text{acceleration}$
or rate of change of elec. freq.

$$3) \Delta\omega \text{ (interpreted as frequency deviation)} = \delta$$

Key step towards differential equations for nodal dynamics - just rotational acceleration:

$$M\Delta\dot{\omega} = (P_G - P_E)$$

\downarrow
 Normalized Rotational Inertia

\swarrow
 power conservation
 "mismatch" at the node - P_G control input from external source, P_E absorbed by network

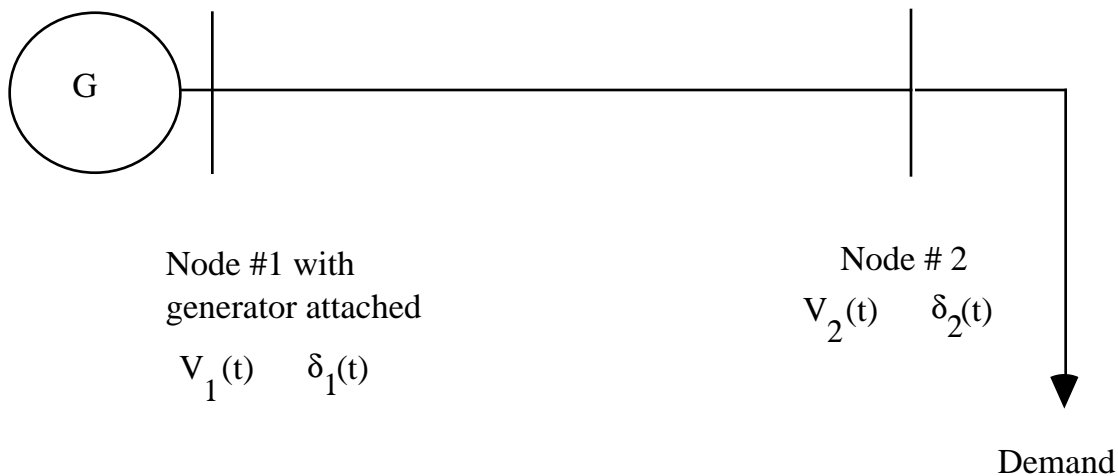
4) As noted previously, electrical power absorbed by network (flow into links) is function of $V(t)$'s and $\delta(t)$'s; simplified approximation describes as:

$$\frac{M_1 |M_2|}{X_L} \sin(\delta_1 - \delta_2)$$

where X_L is fixed, known parameter.

Because only phase differences influence flow, one can set $\delta_2 \equiv 0$ without loss of generality.

5) Simplest possible model has one generator, one link, and assumes $V(t)$'s constant. State variables associated with node #1 are "boundary" variable $\delta(t)$, and "internal" variable $\omega(t)$



Resulting dynamic equations

$$\ddot{\delta}_1 = \Delta\omega_1$$

$$\Delta\ddot{\omega}_1 = M^{-1} \left\{ -D\Delta\omega_1 + P_{G,1} - \frac{M_1 |M_2|}{X_L} \sin(\delta_1) \right\}$$

Observations:

- * Earlier statement "no energy storage at nodes" should be qualified to "no practically utilizable storage at nodes."
- * But rotating kinetic energy of generators is stored energy, and variation in this stored energy drives a fundamental mode of dynamic behavior

* Equilibrium behavior reduces to:

$$0 = \Delta\omega_1 \quad (\ddagger)$$

no deviation away from 60 Hz;

$$0 = P_{G,1} - \frac{M_1 |M_2|}{X_L} \sin(\delta_1) \quad (\ddagger\ddagger)$$

shaft mechanical power balanced by
electrical power absorbed into network.

- * Simple approximation of (11) illustrates a dominant control decision: value of P_G at each generator.
- * Physically, this is power output level of turbine/generator set. Determines rate of fuel consumption, hence dominant factor in variable operating cost.
- * In old regime, central calculations chose P_G 's, based on constrained operating cost minimization.

- * Ideally, goal of market is to create decentralized, distributed approach to this optimization problem.

- * As T.S. Eliot observed,
"Between the idea
And the reality
...
Falls the shadow"

- * In U.S. policy debate today, "the shadow" lies over:
 - (i) defining desirable level of reliability;
 - (ii) relating reliability to network constraints;
 - (iii) equilibrating market-based versus command-based mechanisms for response to constraint violations.

Steady State Behavior in Full Network Model:

The Power Flow Problem

- * At (or near) sinusoidal steady state, electrical behavior of network is linear *in the relation of voltage to current.*
- * As noted above, convenient to represent each voltage and current by complex coefficient for fundamental of its Fourier expansion; in basic circuit terms, "phasors."
- * In n node network, book-keep voltages at node as complex vector $V \in \mathbb{C}^n$, externally injected currents as $I \in \mathbb{C}^n$.
- * Excluding the *very* small percentage of controllable links, all of network's electrical behavior parameterized by a

normalized nodal ("bus") admittance matrix, $\mathbf{Y} \in \mathbb{C}^{n \times n}$.

(in simple example above, one would have

$$Y_{21} = Y_{12} = -j \frac{1}{X_L}$$

Network behavior: $I = YV$

- * Question: if this is a linear circuit, where's the challenge?
- * Answer: Physical nature of electric power production at MW levels dictates that one doesn't get to directly control V's or I's.
- * Recall single branch example above: magnitudes $|M_1| |M_2|$ controlled to fixed, setpoint values, and P_G controlled.
- * Key constraint is current balance at each node, equivalently, power balance at each node

Fundamental constraint on any market or optimization problem relating to power systems is the set of equations dictated by power balance at each node — "the power flow problem."

Structure and parameterization of problem:

let $G = \text{Re}\{\mathbf{Y}\} \in \mathbb{R}^{n \times n}$, $B = \text{Im}\{\mathbf{Y}\} \in \mathbb{R}^{n \times n}$
physically, electrical characteristics
and interconnection of transmission;

let $P \in \mathbb{R}^n$ and $Q \in \mathbb{R}^n$ be rectangular
representation of sinusoidal power;
physically, active and reactive power
absorbed/injected from generators or
loads;

potential degrees of freedom are

$$V \in \mathbb{R}^n, \delta \in \mathbb{R}^n$$

physically, normalized sinusoidal
voltage magnitudes and phase angles
at nodes.

Different physical nature of equipment connected at various nodes dictates different possible forms of constraints.

At node k with generators connected, typical characteristics allow:

i) control to setpoint of average (over each sinusoidal cycle) power delivered - hence

$P_k^0 = \text{known constant};$

ii) control to setpoint of terminal voltage magnitude - hence $V_k^0 = \text{known constant}.$

At node j having distribution substation feeding loads, typical characteristics allow very accurate short time horizon prediction of load demand:

i) hence $-P_j^0 =$ active load; treated as known constant;

ii) hence $-Q_j^0 =$ reactive load; treated as known constant.

Finally, observe that only phase angle difference of sinusoidal voltages influence flows; hence one may arbitrarily pick a reference node (say n) at which $\delta_n = 0$.

Result: from seemingly simple current balance in linear relationship $I = YV$ arises nonlinear problem as follows:

For all but one node having a generator attached (index as $i = 2, \dots, m$):

$$P_i^o = \sum_{k=1}^n V_i V_k (G_{ik} \cos(\delta_i - \delta_k) + B_{ik} \sin(\delta_i - \delta_k)) \quad (1)$$

$$V_i^o = V_i$$

"Special case" generator 1 assumed to have control system ("governor") dictating its power output in such a way that overall network wide power balance is maintained.

Hence P_1 is not specified a priori, only:

$$V_1^o = V_1$$

At nodes connected to distribution substations (index as $i = m+1, m+2, \dots n$)

$$-P_i^o = \sum_{k=1}^n V_i V_k (G_{ik} \cos(\delta_i - \delta_k) + B_{ik} \sin(\delta_i - \delta_k)) \quad (2)$$

$$-Q_i^o = \sum_{k=1}^n V_i V_k (G_{ik} \sin(\delta_i - \delta_k) - B_{ik} \cos(\delta_i - \delta_k)) \quad (3)$$

and, as noted earlier

$$0 = \delta_n$$

Observations:

- * Direct constraints on voltage magnitudes and reference phase angle are trivial.
- * Hence, fully constrained form of the power flow problem generally focuses on (1), (2) & (3), viewed as $2n - m - 1$ constraints on $(n - 1)$ δ 's, $(n - m)$ V 's.
- * Optimization formulation, for markets or otherwise, simply "opens up" degrees of freedom by allowing P_k^O 's and V_k 's at generators to be independent decision variables.
- * In centralized optimization formulation (and indirectly in market based

formulation), P_k^O 's are quantities that influence cost function - total system fuel cost per unit time.

Common formulation of markets relies on market coordinator accepting advanced offers to sell from generator owning firms - these simply become surrogates for operating cost as function of P_k^O ;

denote cost as $C_k^O(P_k^O)$ in units of \$'s/hr.

Many added complexities exist, due to intertemporal constraints on start-up & shut-down of generators, attempt to provide operating margin against credible equipment failures, additional equipment and quality of service constraints. But...

Key elements of optimization problem associated with power markets observable in following problem:

$$\text{minimize } \sum_{k=1}^m C_k(P_k^0),$$

over decision variables P_k^0 's, $k=1,2, \dots,m$
and power flow unknowns δ 's and V 's;

subject to power flow constraints (1), (2), (3).

IMPORTANT TERMINOLOGY: For Lagrange formulation of problem above, Lagrange multipliers associated with active power (P) equality constraints (1) and (2) known as "Nodal Prices" or "Locational Marginal Prices" (LMP's), in units of \$'s/hr/MW, or \$'s/MW hr.

Above denotes optimization of instantaneous operating cost. realistically, this problem may be solved and updated periodically, reflecting changing load estimates, 5-15 minute intervals.

Even without consideration of intertemporal constraints between periods, wide range of additional inequality constraints must be placed on problem to ensure operable solutions.

Additional inequalities of interest:

Voltage magnitude solutions (V's) within acceptable operating range (e.g. – 5%-8%);

Magnitude of current on each branch within acceptable limits (intertemporal issues also, as some limits are thermal heating related);

Toughest one - equilibrium solution predicted by (1), (2) (3), when reflected back into full set of differential equations, has acceptable stability properties.