Data Assimilation, Ensemble Methods, Multiscale, Inverse problems

NWP

A longstanding problem is in numerical weather prediction (NWP) where the fields are non Gaussian and of fine resolution (state vector length 10^6). Evolution or forecast equations are nonlinear. The data is paradoxically both large (10^5) but sparse relative to the atmospheric state.

Homogenization/multiscale phenomenon/upscaling

Goal: Create a macroscopic model (PDE's, constitutive relations, parameter values) that accounts for physical processes that are too small in scale to be represented explicitly in practical numerical simulations.

Tools:

- Homogenization theory (confined to relatively simple processes with idealized assumptions)
- Multiscale spatial correlations of random variables (e.g. heterogeneous media, stochastic models for turbulence)
- Moment closure schemes (as an alternative to Monte Carlo simulation) to obtain statistical distributions.
- Multigrid methods where coarse grid operators correspond to physical homogenizations of discrete fine-grid operators

Approximations to optimal nonlinear estimators

Goal: Propagate the distribution of states forward and backward in time. (Instead of just the mean state and covariances.) Such a probabilistic forecast or analysis should give a valid representation of the uncertainty.

Tools:

- Kushner-Stratonovich equation for probability distributions conditioned on past measurements and the Pardoux equation for distributions conditioned on future measurements can be approximated by a projection on a finite set of moments. This kind of Rayleigh Ritz approximation permits the input of relevant physics.
- Particle filters and Monte Carlo samples give a representation of the distribution that is easy to manipulate. But there are issues of how a small number of samples (e.g. 10^3) can approximate a high dimensional system.

Level set methods

Goal: Model discontinuous phenomenon or phases. (e.g. Atmospheric front.)

Tools: The feature is represented as the zero level set (contour at zero) of a higher dimensional surface. The evolution is based on a higher dimensional system that propagates the full surface (although only the level-set is physically meaningful). Validity is asymptotically valid for sharp fronts so these kinds of methods apply when a front is small relative to the lateral dimensions.