Upscaling and Gridding of Geologically Complex Systems

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Acknowledgments

• Flow-based gridding and permeability upscaling:
  - Xian-Huan Wen (ChevronTexaco EPTC)
  - Michael G. Edwards (University of Wales, Swansea)

• Subgrid modeling for transport:
  - Yalchini Efendiev (Texas A&M University)
  - Seong H. Lee (ChevronTexaco EPTC)
Present and Future Simulation Models

• Current models:
  – Detailed heterogeneity, simple reservoir geometry
  – Structured grids, diagonal $k$ fields (≈layered geology)
  – Fine grid $\sim O(10^7)$ cells, coarse grid $\sim O(10^5)$ cells

• “Next generation” simulation procedures:
  – Detailed heterogeneity, complex reservoir geometry
  – General grids, full tensor $k$ fields
  – Many geological scenarios for uncertainty assessment
Current and Future Reservoir Models

Current gridding/upscaling tools designed for ~layered heterogeneity, $k$ and wells aligned with $(x,y,z)$

Future tools must handle geometrically complex heterogeneity, full tensor $k$, complex wells

(outcrop photo from Jef Caers)
Channel Systems

- Considerable geometric complexity
- High and abrupt contrast in properties
- Difficult to upscale accurately
Fine Scale Equations for Two Phase Viscous Dominated Flow

- Phase velocity equations \(( j = \text{oil, water})\) with \(g = p_c = 0\)

\[
v_j = -\frac{k_{rj}(S)}{\mu_j} k \cdot \nabla p , \quad v = v_w + v_o
\]

- Combine conservation of mass and Darcy equations: pressure and saturation equations

\[
\nabla \cdot (\lambda(S) k \cdot \nabla p) = -q , \quad \phi \frac{\partial S}{\partial t} + v \cdot \nabla f(S) = q_w
\]

\[
f(S) = \frac{k_{rw}/\mu_w}{k_{rw}/\mu_w + k_{ro}/\mu_o} , \quad \lambda(S) = \frac{k_{rw}}{\mu_w} + \frac{k_{ro}}{\mu_o}
\]
Overview of Upscaling Techniques

• Moderate levels of coarsening (single phase flow parameters and grid) - $O(10^1-10^2)$ scale up
  - Flow-based gridding
  - Calculation of grid block permeability $k^*$
  - Near-well scale up (calculation of $T_w^*$, $WI$)

• High levels of coarsening (two phase flow parameters)
  - Subgrid modeling based on volume averaging and use of higher moments
Current Gridding and Permeability Upscaling Procedures

- $k^*$ computed by solving pressure equation over local region with periodic boundary conditions

- Same reservoir flow model except $k^*$ replaces $k$
Limitations of Cartesian Grids for Geologically Complex Systems

- Coarse Cartesian grid unable to align with dominant features or flow directions

- Require flow-aligned grids for greater flexibility
Limitations of Current Upscaling Methods for Geologically Complex Systems

• Purely local $k^*$ calculation may not adequately maintain effects of permeability continuity
Proposed Approach: Flow-Based Gridding with Border Region Upscaling

- Non-Cartesian flow-based grids
- Border regions for $k^*$ calculations
- Full tensor permeability in flow simulation
Flow-Based Grid Generation

- Solve fine grid single phase flow problem
- Generate streamlines and isopotentials and construct initial grid
- Apply Laplacian smoothing algorithm until grid quality measures satisfied:

\[ x_{i,j}^{k+1} = (1 - \omega)x_{i,j}^k + \frac{1}{4} \omega \left( x_{i-1,j}^k + x_{i+1,j}^k + x_{i,j-1}^k + x_{i,j+1}^k \right) \]
Improved Upscaling Method: Use of Border Region in $k^*$ Calculation

- Local problem solved over extended region
- Periodic boundary conditions imposed on outer region, $k^*$ computed over shaded region via $\langle u \rangle = -k^* \cdot \langle \nabla \rho \rangle$
- Solve least squares problem to determine symmetric, positive definite $k^*$

- Border regions applied previously by Gomez-Hernandez & Journel, 1990; in MsFEM context by Efendiev & Hou, 1999
Discretization with Tensor Permeability and/or Nonorthogonal Grids

- Full tensor $k^*$ affects finite volume stencil in the same way as nonorthogonality
  - 5 point $\rightarrow$ 9 point stencil in 2D; 7 $\rightarrow$ 27 in 3D

- Cross terms most important for locally oriented features

- Applying flux-continuous finite volume method (Aavatsmark et al., 1996; Lee et al., 1998; Edwards & Rogers, 1998)
Heterogeneous 3D Permeability Field

- Upper layers more continuous; lower layers channelized
- For each fine grid layer, compute global permeability $K_g$ for fine ($220 \times 60$) and Cartesian upscaled ($22 \times 20$) models with and without border regions
- Compute average relative error $< E >$ for $r = 0$ and $r = 1$

Brent Sequence

Fine Model:
$220 \times 60 \times 85$

(problem description from Christie and Blunt, 2001)
Global Equivalent Permeabilities: Effect of Border Regions (Cartesian Grids)

$r = 0$ (no border region)

$< E > = 46\%$

$r = 1$

$< E > = 27\%$

- continuous k
- channelized k
Global Equivalent Permeabilities: Use of Border Regions and Grid Refinement

\[ r = 0 \quad (220 \times 60) \]

\[ \langle E \rangle = 34\% \]

\[ r = 1 \quad (220 \times 60) \]

\[ \langle E \rangle = 16\% \]
Border Region with Flow-Based Grid

Extended local region and flow-based grid

- $k^*$ computed over fine cells in shaded region
Flow-Based Grid for Oriented System

100 x 100 Fine Model

\[ k_2 = 0.1 k_1 \]
\[ \theta = 30^\circ \]

Streamlines in Fine Model

Smoothing

20 x 20 FBG
Flow Results for Oriented System

- Flow left to right, unit mobility ratio

<table>
<thead>
<tr>
<th>Model</th>
<th>$Q_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine</td>
<td>0.64</td>
</tr>
<tr>
<td>Uniform</td>
<td>0.50</td>
</tr>
<tr>
<td>FBG</td>
<td>0.61</td>
</tr>
</tbody>
</table>
Oriented System - Varying Well Locations

\[ Q_z = 0.120: \text{Fine} \]
\[ 0.114: \text{FBG, } \sim 5\% \text{ Error} \]
\[ 0.110: \text{Uni., } \sim 8\% \text{ Error} \]

\[ Q_d = 0.187: \text{Fine} \]
\[ 0.168: \text{FBG, } \sim 10\% \text{ Error} \]
\[ 0.145: \text{Uni., } \sim 22\% \text{ Error} \]
Channel System - k Field and Grid

220 x 60 Fine Model

Streamlines in Fine Model

55 x 30 Coarse FBG

(model from Christie & Blunt, 2001)
Flow Results for Channel System

- $F_0$ not sensitive to grid structure
- Use of border region improves $F_0$ result
# Flow Rates for Channel System

<table>
<thead>
<tr>
<th>Model</th>
<th>$Q_x$</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine</td>
<td>52</td>
<td>-</td>
</tr>
<tr>
<td>Uniform ($r=0$)</td>
<td>29</td>
<td>45%</td>
</tr>
<tr>
<td>Uniform ($r=1$)</td>
<td>39</td>
<td>26%</td>
</tr>
<tr>
<td>FBG ($r=0$)</td>
<td>29</td>
<td>45%</td>
</tr>
<tr>
<td>FBG ($r=1$)</td>
<td>44</td>
<td>16%</td>
</tr>
</tbody>
</table>

- Use of border region improves $Q_x$ result
- $Q_x$ varies with grid structure for $r=1$
Detailed Fault Zone System

300 x 300 Fine Model

- host rock, $k = 200$ md
- gouge, $k = 0.1$ md
- slip surface (fracture), $k \sim 2 \times 10^5$ md

(model from Myers, Aydin, Jourde, Flodin)
Detailed Fault Zone - Streamlines and Flow-Based Grid

Permeability Field
Streamlines for Fine Model
60x60 Coarse FBG
**Detailed Fault Zone - Flow Results**

**Fault-Parallel Flow**

- $Q_y = 2200$, Fine Grid
- $= 1616$, FBG (27% error)
- $= 680$, Uniform (69% error)

**Fault-Perpendicular Flow**

- $Q_x = 10.8$, Fine Grid
- $= 9.4$, FBG (13% error)
- $= 7.2$, Uniform (33% error)
Flow-Based Grids in 3D Channel System

Permeability field - channels

Flow-based grid: two layers

(from Castellini, 2001)
Highly Coarsened Flow Models: Upscaling Two Phase Flow Parameters

- Fine scale saturation equation: \( \frac{\partial S}{\partial t} + \mathbf{v} \cdot \nabla f(S) = 0 \)

- Volume averaging and modeling of higher moment terms
  - Coupling to large scale flow appears directly
  - Generates moment equations, approximation required

- Use of upscaled or pseudo relative permeability
  - \( f^*(S) \) replaces \( f(S) \) in saturation equation
  - BC’s applied to local problem (solved to compute \( f^* \))
    attempt to mimic global flow
Volume-Averaged and Fluctuating Saturation Equations

\[ S(x, y) = \overline{S} + S'(x, y) \]
\[ \mathbf{v}(x, y) = \overline{\mathbf{v}} + \mathbf{v}'(x, y) \]

- \( \overline{S} \) and \( S' \) equations:

\[ \frac{\partial \overline{S}}{\partial t} + \overline{\mathbf{v}} \cdot \nabla f + \overline{\mathbf{v}}' \cdot \nabla f' = 0 \]
\[ \frac{\partial S'}{\partial t} + \overline{\mathbf{v}} \cdot \nabla f' + \mathbf{v}' \cdot \nabla \overline{f} + \mathbf{v}' \cdot \nabla f' = \overline{\mathbf{v}}' \cdot \nabla \overline{f}' \]
Coarse Scale Saturation Equation

• Consider unit mobility ratio $M = 1$ displacement
  ($k_{rw} = S$, $k_{ro} = 1 - S$, $\mu_o = \mu_w$, $f = S$)

  \[
  \frac{\partial S}{\partial t} + \bar{v} \cdot \nabla S + \nabla \cdot (\bar{v} S^\prime) = 0
  \]

  \[
  \frac{\partial S^\prime}{\partial t} + \bar{v} \cdot \nabla S^\prime + \bar{v}^\prime \cdot \nabla S + \bar{v}^\prime \cdot \nabla S^\prime = \bar{v}^\prime \cdot \nabla S^\prime
  \]

• Use $S^\prime$ equation to estimate $\bar{v} S^\prime$ term

• Form discrete (finite volume) equation by integrating over coarse block boundaries ($\partial D$)
Saturation Equation - Finite Volume Representation ($M = 1$)

• Final discrete equation given by:

$$\frac{1}{\Delta t} \Delta_t \bar{S} + \frac{1}{A} \int_{\partial D} v_j n_j \bar{S} dl = \frac{1}{A} \int_{\partial D} D_{ij}(\mathbf{x}, t) n_i \nabla_j \bar{S}(t, \mathbf{x}) dl$$

$$D_{ij}(\mathbf{x}, t) = \frac{1}{h_E} \int_{\partial E} \left[ \int_0^t v_i'(\mathbf{x}) v_j'(\mathbf{x}(\tau)) d\tau \right] dl$$

• Non-local dispersivity $D_{ij}$ expressed in terms of a two-point correlation function
Approximate $D_{ij}$ for $M=1$ Case

- Introduce following approximation for systems of moderate to large correlation length (pre-asymptotic):

\[
D_{ij}(x, t) = \frac{1}{h_E} \int \left[ \int_{0}^{t} v'_i(x)v'_j(x(\tau))d\tau \right] dl \\
\approx \alpha(\sigma, l_x, l_z) \left| v'_i(x) \right| L_j(x, t)
\]

$\alpha(\sigma, l_x, l_z)$ - semi-empirical function, $\alpha=(\sigma/2)^4$

$|v'_i|$ - magnitude of velocity fluctuation (fine, local)

$L_j$ - distance traveled along streamline (coarse, global)
Coarse Scale Equations - Two Phase Flow

• Non-unit mobility ratio \( (k_{rw} = S^2, \ k_{ro} = (1 - S)^2, \ \mu_o / \mu_w = M) \)

• Pressure equation solved at every time step \( (v_j \text{ changes}) \)

• Use \( f(S) = f(S + S') \approx f(S) + f_S(S)S' + \frac{1}{2} f_{SS}(S)S'^2 + \ldots \)

• Coarse scale FV saturation equation becomes:

\[
\frac{1}{\Delta t} \Delta_t \bar{S} + \frac{1}{A} \int_{\partial D} v_j f(S) n_j dl + \frac{1}{A} \int_{\partial D} f_S(S) v'_j S' n_j dl + \\
\frac{1}{A} \int_{\partial D} \frac{1}{2} v_j f_{SS}(S) S'^2 n_j dl = 0
\]
Numerical Approximations for Two Phase Flow Solution

- Neglect $S' S''$ subgrid term relative to $v' S'$ term based on numerical experiments

- Approximate $v' S'$ subgrid term (to leading order) as:

$$F_{vS}(x,t) \approx -\alpha \int_{D} |v'_i(x,t)| f_S^2(\bar{S}) \left[ L_j \nabla_j \bar{S} - \frac{1}{2} (L_j \nabla_j \bar{S})^2 f_{SS}^2 + ... \right] n_i dl$$

- Recover $M=1$ subgrid model in appropriate limit ($f = S$)

- Solve $\nabla \cdot [\lambda(\bar{S}) k^* \nabla p] = 0$ for $p$ and $v$ at each time step
Numerical Solution Procedure

• Generate fine model (100 × 100) of prescribed parameters

• Form coarse grid (~10 × 10) and compute $k^*$ and $v_j'$ for each coarse block via local solutions

• Solve saturation equation using second order TVD scheme or by integrating along streamlines (for $D_{ij} = 0$ and $M = 1$)

fine grid: $100 \times 100$, $l_x \gg l_z$, $L_x = 5L_z$
Two levels of scale up, \( l_x = 0.3, \ l_z = 0.01, \ \sigma = 1.5 \)

**Unit Mobility Ratio Displacement Results**

\[
\begin{align*}
F_o \times F_o & = 100 	imes 100, \\
16 \times 15 (D=0) & = 16 \times 15 \text{ (w/ } D) & \text{ (w/ } D) & \\
11 \times 9 (D=0) & = 11 \times 9 \text{ (w/ } D) & \text{ (w/ } D) & \\
\end{align*}
\]
Oil Rates for Two Phase Flow Simulations

\[ l_x = 0.2, \ l_z = 0.02, \ \sigma = 1.5, \ M = 10 \]

- **fine model**
- **coarse model, \( F_{vs} = 0 \)**
- **coarse model, with \( F_{vs} \)**
Saturation Contours, Two Phase Flow
\( l_x=0.2, \ l_z=0.02, \ \sigma=1.5; \ M=10; \ S = 0.05, 0.4 \)

time = 0.1 pvi
time = 0.15 pvi

---

- **fine model (on \( 10 \times 10 \))**
- **coarse model, \( F_{vs}=0 \)**
- **coarse model, with \( F_{vs} \)**
Oil Rates for $M = 10$ Simulations

$l_x = 0.5, \ l_z = 0.025, \ \sigma = 2$

$l_x = 0.6, \ l_z = 0.03, \ \sigma = 2$

fine model
coarse model, $F_{vs} = 0$
coarse model, with $F_{vs}$
Fractal Permeability Field

\[ \gamma(h) \approx Ch^\beta \]

\[ \beta = 1, \quad \sigma_{\log k} = 2 \]

(algorithm from Wonho Oh, 1998)
Oil Rates for $M=3$ Simulations
Fractal Permeability Field ($\beta=1$, $\sigma=2$)
Saturation Contours, Two Phase Flow
Fractal $k$, $\sigma = 2$; $M = 3$; $S = 0.05, 0.4$; $t = 0.1$ pvi

10 x 10 coarse models

20 x 20 coarse models

- fine model
- coarse model, $F_{vs} = 0$
- coarse model, with $F_{vs}$
Summary (1)

• Presented new methodology for gridding and upscaling of complex systems in 2D
  – Flow-based grid generation
  – Border region procedure for computing $k^*$
  – Full permeability tensors in global simulation

• Demonstrated significant improvement in accuracy over current procedures
  – Grid resolves dominant features
  – Upscaling captures connectivity effects
Summary (2)

• Presented new procedure for subgrid modeling of transport in layered systems
  – Method introduces coupling between local fine scale effects and global coarse grid flow
  – Provides significantly improved results relative to coarse grid simulations without subgrid treatment
Future Directions

• Flow-based grid generation and $k^*$ calculation
  – Explore more general flow-based grid generation schemes and test on other types of geological models
  – Extend to 3D systems and to problems with strong gravitational effects

• Subgrid modeling of transport
  – Investigate application for other flow scenarios
  – Consider modeling of other subgrid effects ($S^2$ term)
Channel System - k Field and Grid

Permeability Field (130 x 100)

\[ k_{\text{sand}} \sim 300 \text{ md}, \quad k_{\text{shale}} \sim 5 \text{ md} \]

Flow-Based Grid (29 x 20)

(Stanford V permeability field)
Flow Results for Channel System

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