

SEMIDEFINITE PROBLEMS IN STRUCTURAL OPTIMIZATION AND THEIR SOLUTION BY PENNON

Michal Kočvara and Michael Stingl
Institute of Applied Mathematics, University of Erlangen-Nuremberg

Free Material Optimization

Find the best material within a given domain able to carry the given loads. The material properties can vary from point to point; the only constraint: physical availability.

MOPED (Material Optimization in Engineering Design):

software package for free material optimization of general two-dimensional structures, including graphical postprocessing.

Single- and multiple-load, 2D and 3D problems

large-scale convex nonlinear program (300 000 var., 100 000 quadr. constr.)
semidefinite program (60 000 var., 20 000 LMI constr.)

? software ?

Global Stability Constraints

Elastic instability is often of critical importance when designing a real-world structure, like a bridge or an aircraft. Such structures may fail not as a result of high stresses, but owing to insufficient elastic stability.

Global stability constraints formulated as a **nonconvex** matrix inequality

$$A(t) + G(t; u) \succeq 0$$

where t is the design and u the state variable, $A(t)$ the stiffness matrix and $G(t; u)$ the geometry stiffness matrix.

Optimal design problem: **nonconvex semidefinite program.**

? software ?

PENNON

general-purpose code for large scale nonlinear and semidefinite programming

- ✓ Convex NLP: generalized Augmented Lagrangian method based on PBM algorithm by Ben-Tal/Zibulevsky
- ✓ Convex SDP: via special penalty function for matrix inequalities; treatment of different data sparsity patterns
- ✓ Nonconvex NLP: based on trust region algorithm for inexact unconstrained minimization steps
- ✓ Nonconvex SDP, in particular BMI: combination of above
- ✓ Combination of NLP and SDP constraints: so far the only optimization code of this kind

Independent benchmarks by H. Mittelmann (<http://plato.la.asu.edu/bench.html>):

PENNON is one of the fastest codes for convex NLP and SDP, particularly for large sparse problems.

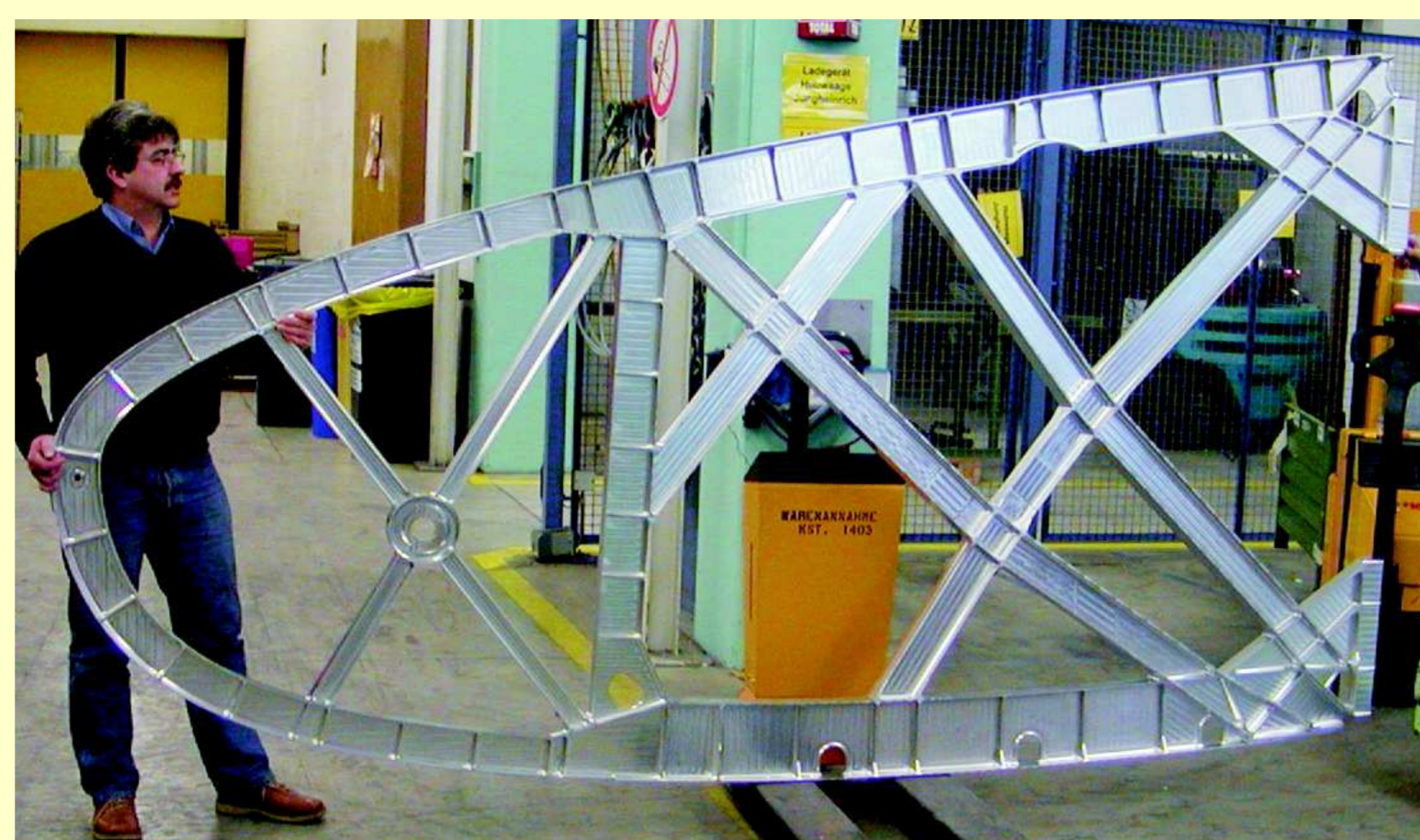
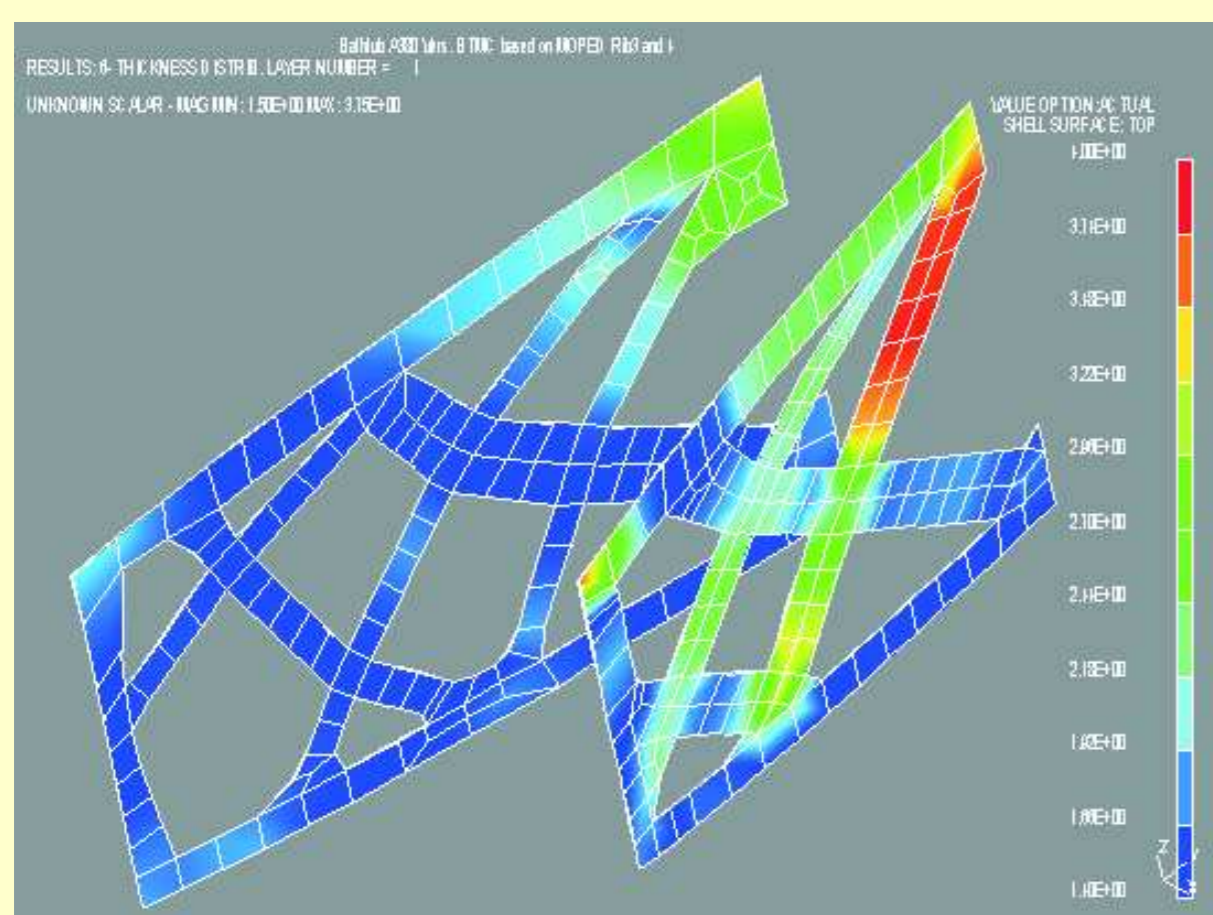
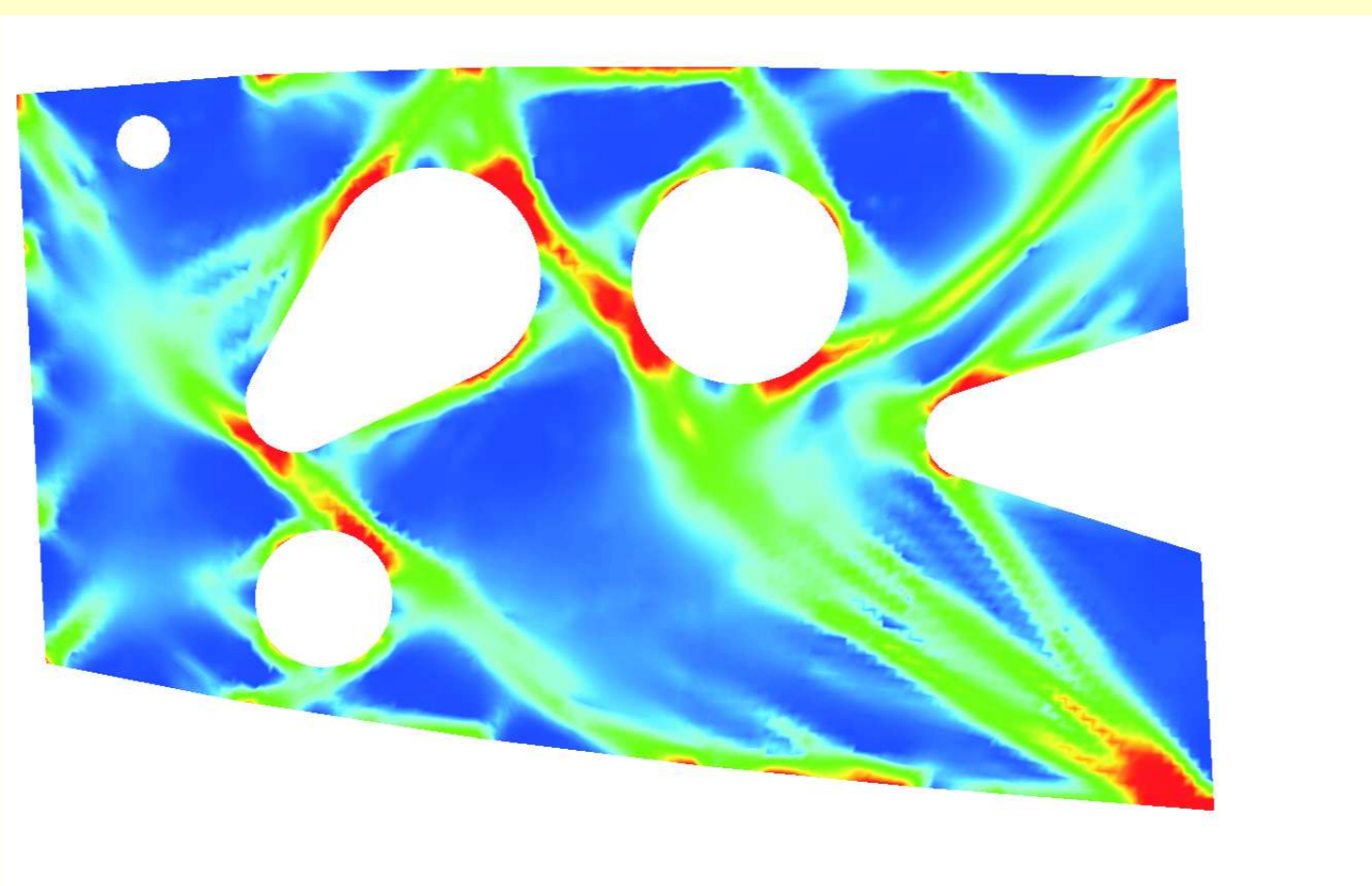
SDP version implemented on the NEOS server: <http://www-neos.mcs.anl.gov>

problem	no. of var.	size of matrix	CSDP	SDPT3	SeDuMi	PENNON
mater4	4807	12498	350	295	97	29
mater5	10143	26820	3434	m	202	78
mater6	20463	56311	201916	m	533	233

problem	no. of var.	size of matrix	CSDP	SDPT3	SeDuMi	PENNON
vibra3	544	1185	260	45	2871	34
vibra4	1200	2545	617	294	32280	191
vibra5	3280	6801	8489	3601	>100000	2724

Applications

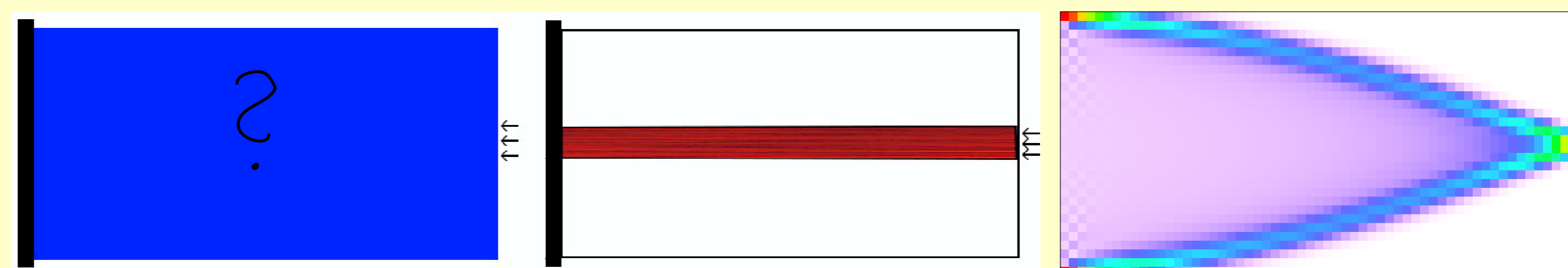
Airbus A380, design of "leading edge"



Weight saving of up to 17%, compared to traditional design.

2D model problems with stability constraints

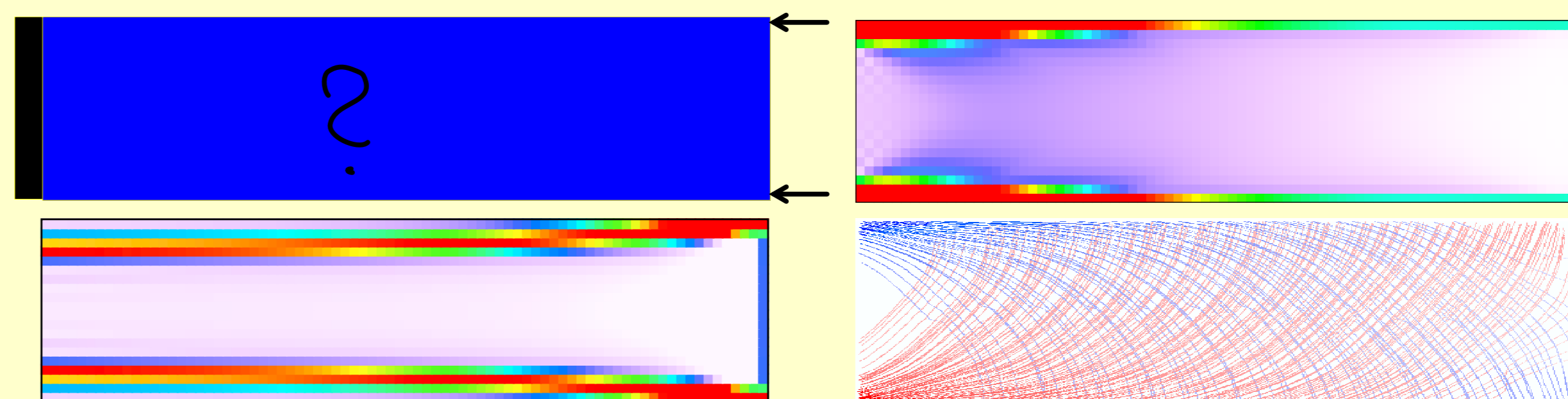
First solve problems with vibration constraints \rightarrow linear SDP with $A(t) + \lambda M(t) \succeq 0$ ($\lambda \dots$ min. allowed eigenfrequency, $M \dots$ mass matrix)



initial design

min. weight result

+ vibration constr.



initial design

min. weight result

3 sec CPU

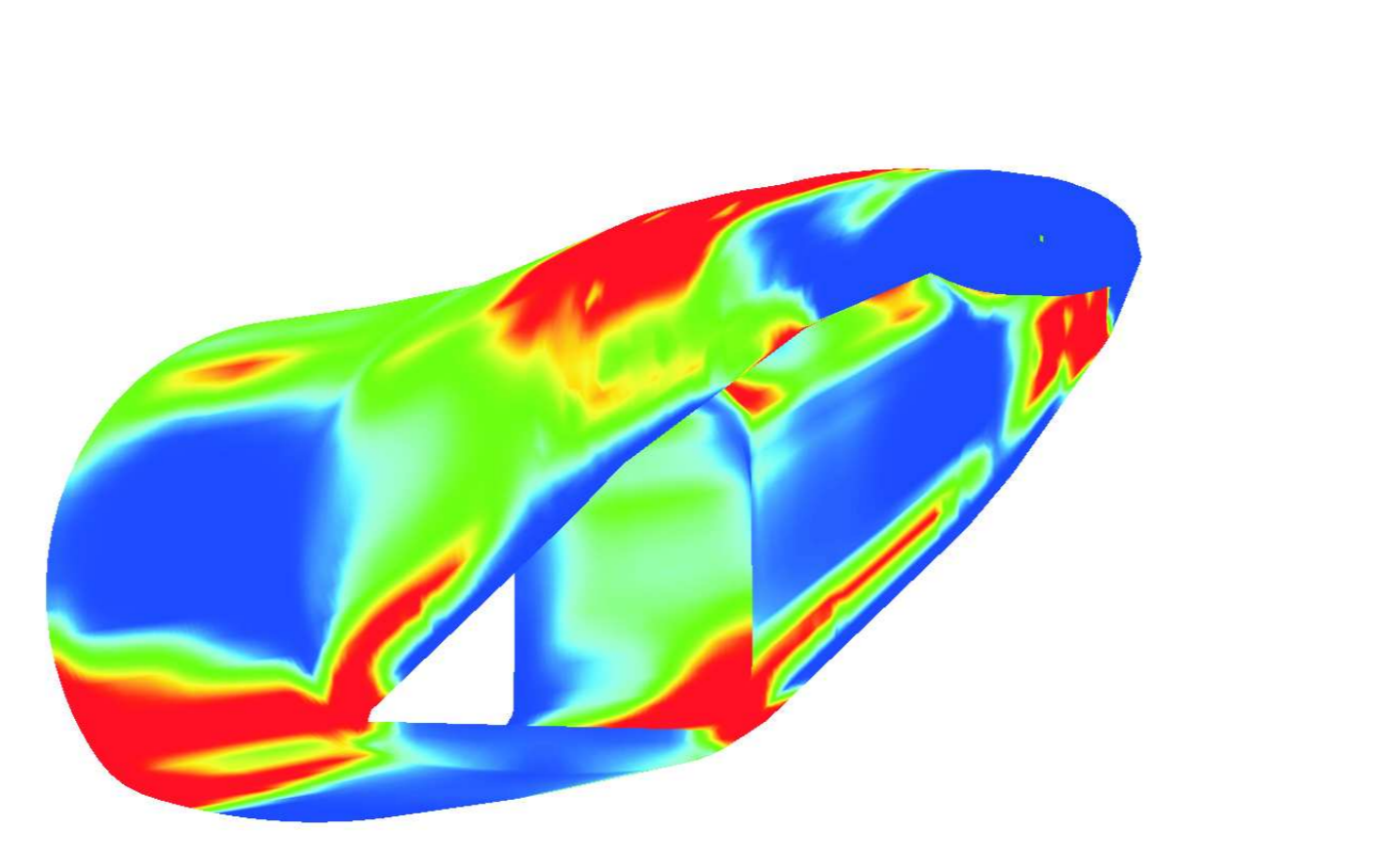
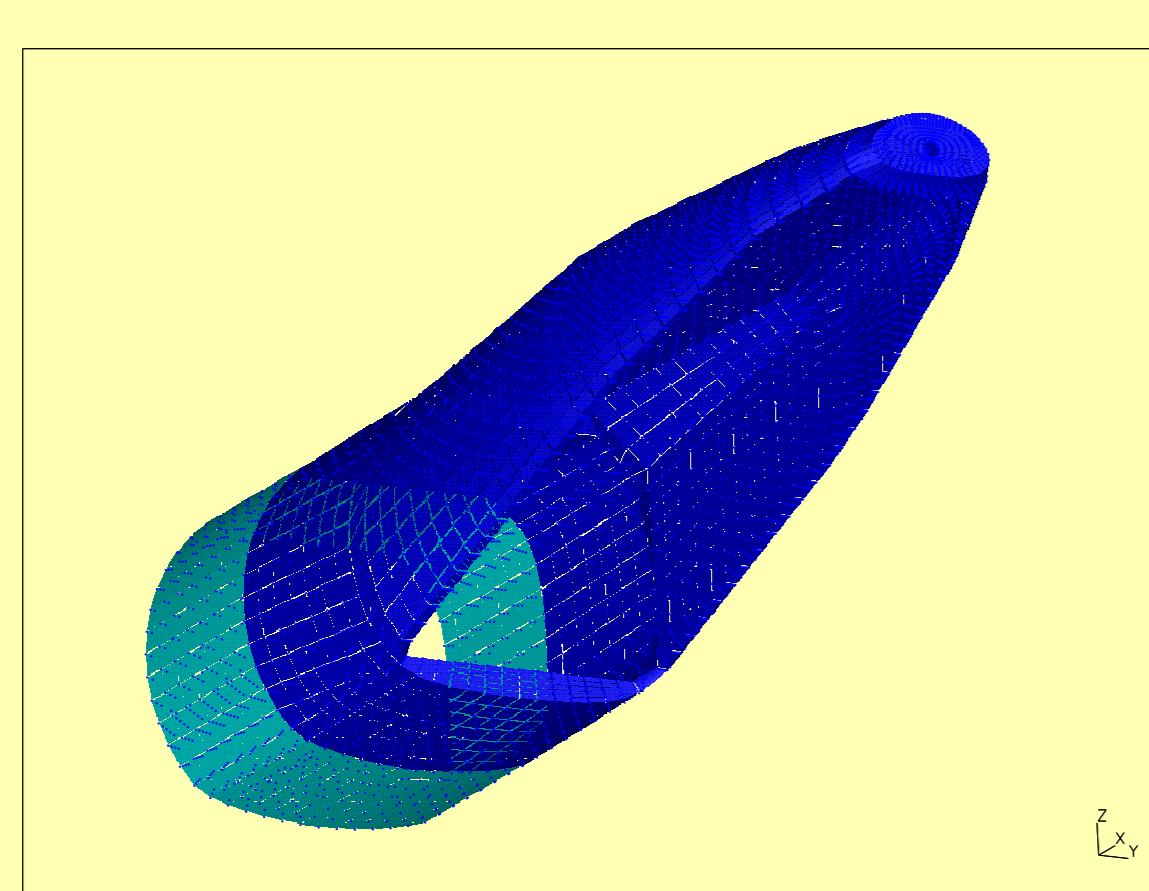
vibration constr., density

fibre layout

15 minutes CPU

Computationally very demanding

Design of rear fuselage of cargo aircraft



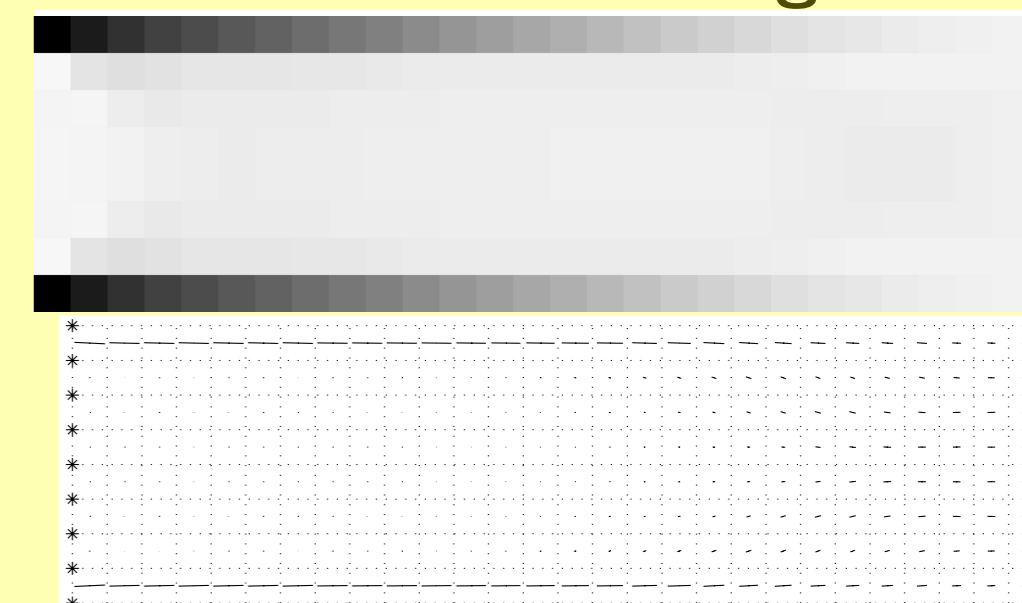
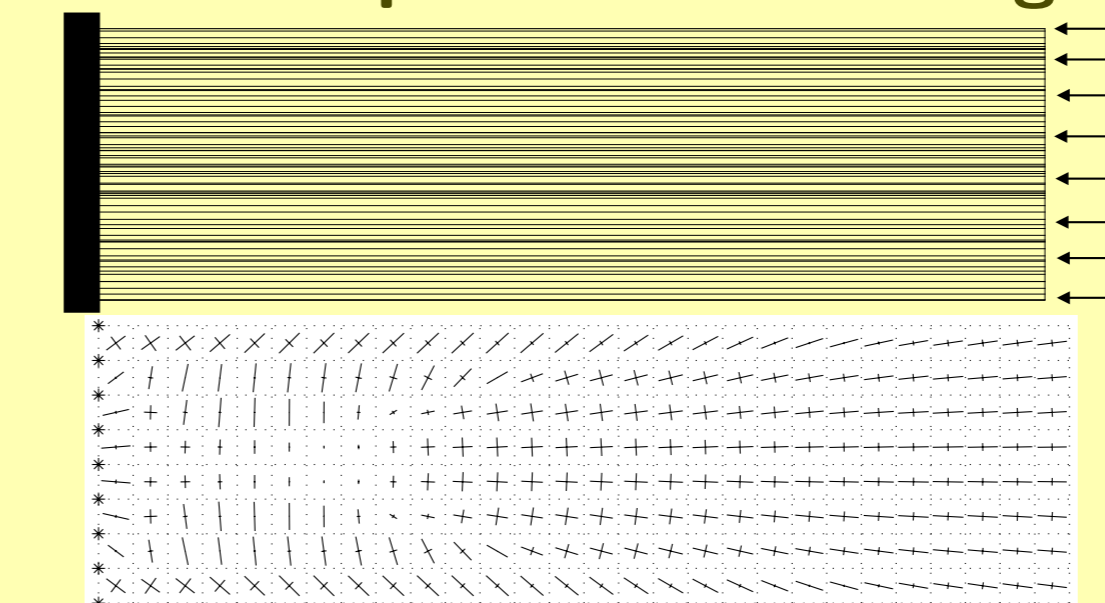
Problems with up to 100 000 finite elements.

Robust structural design (w.r.t. small random loads)

Ben-Tal/Nemirovski: LARGE scale SDP, numerically unsolvable. **Cascading:** solve a sequence of "small" multiple-load problems with most dangerous loads.

min. compliance FMO design

robust FMO design



robust strains

stresses