Airline Revenue Management and e-Markets

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Outline

Overview and basic economics

Revenue management models & methodology

Auctions and alternative price mechanisms

Research challenges

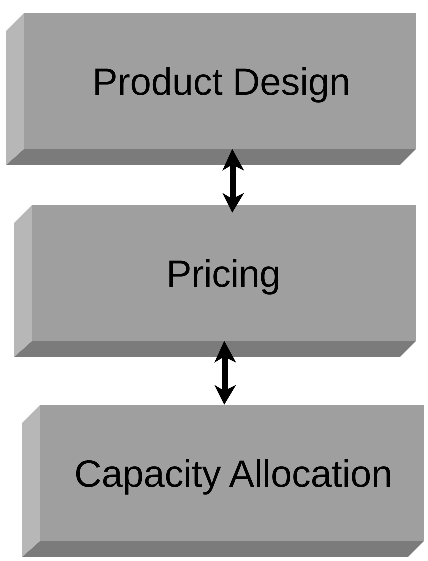
The airline economic environment

- Demand Side
  - Heterogeneity in customers
    - Product sensitivity:
      - schedule flexibility
      - cancellation options
      - service/brand
    - Price sensitivity
    - Purchase Behavior
  - Demand variability
    - Aggregate variability
    - Individual uncertainty

- Supply Side
  - Low marginal costs
  - Capacity constraints
  - “Lumpy” capacity
  - Network effects

- Competitive markets

Revenue management components



- Purchase restrictions
- Cancellation options
- Channel of distribution

- O-D market level
- Quasi-static (rigid)
- Set *ex ante*

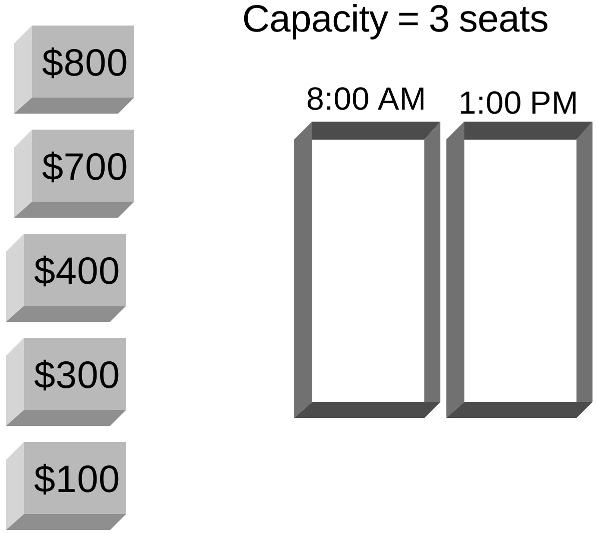
- Departure level
- Tactical, real-time
- *ex post* rationing

Goal: Maximize revenues from a fixed set of capacities

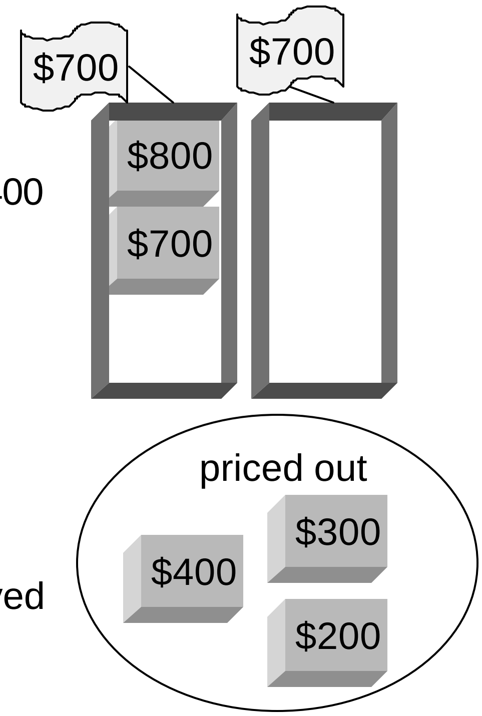
Example

5 customers with different valuations (unobservable)

2 Flights
Capacity = 3 seats



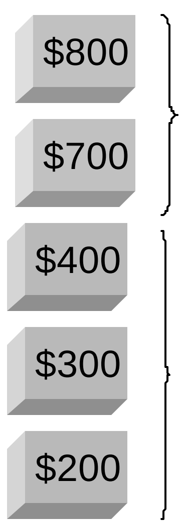
Best single price: $700
Revenue: 2 x $700 = $1400



Maximum obtainable revenue
$800 + $700 + $400 + $300 +$ 200 = $2400

58% of maximum achieved

Discrimination via a “sorting mechanism”



Customers returning by Saturday

Customers staying a Saturday

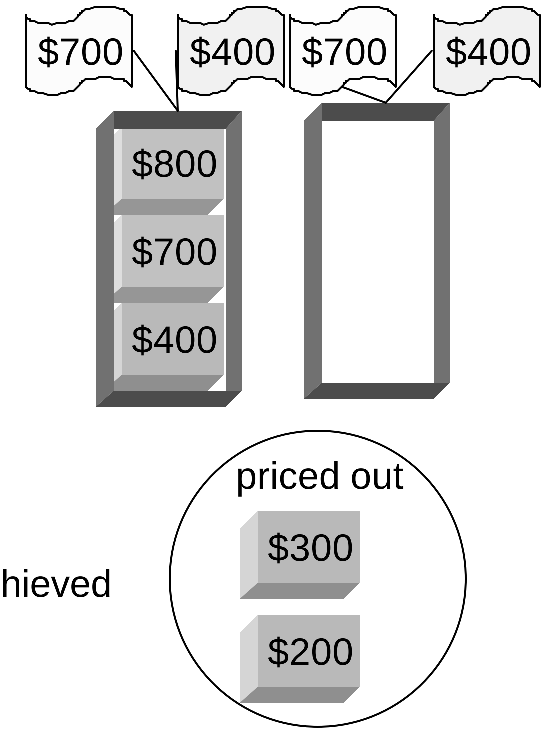
A trait that is correlated with willingness to pay allows for discrimination

- Saturday night stay
- Advance purchase req.
- Distribution channel (e.g. internet)

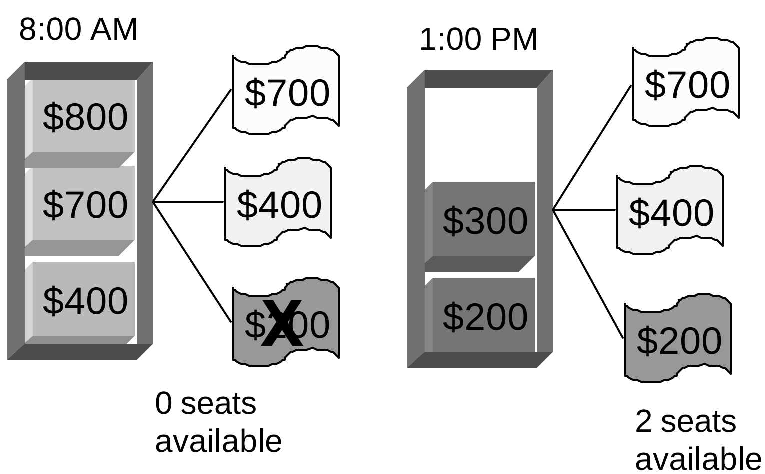
Price discrimination:
SA stay: $400
No SA stay: $700
Revenue:
2 x $700 + 1 x $400
= $1800

Maximum revenue
$2400

75% of maximum achieved



Introduce capacity-controlled discount



Reveune = 2 x $700 + 1 x $400 +
2 x $200 = $2200

92% of maximum

What is the economics at work here?

Monopoly price discrimination

Peak load pricing

Prescott equilibrium

Monopoly price discrimination

- This is the most common explanation given by practitioners
  - Discrimination based on willingness to pay
  - “Fences” to prevent “diversion” (sorting mechanism)
- Problems with this explanation
  - Many airline markets are highly competitive
    - Instantaneous price matching
    - Product matching
  - Entry & exit is not that difficult

Is sorting discrimination?

Yes (?) if it's not cost based ...

"It is not because of the few thousand francs which would have to be spent to put a roof over the third-class carriages or to upholster the third-class seats that some company or other has open carriages with wooden benches … What the company is trying to do is prevent the passengers who can pay the second-class fare from traveling third-class; it hits the poor, not because it wants to hurt them, but to frighten the rich."

Dupuit (1849) discussion of passenger railroad tariffs

Peak load pricing explanations

Gale & Homes (IJIO ’92, AER ’93)

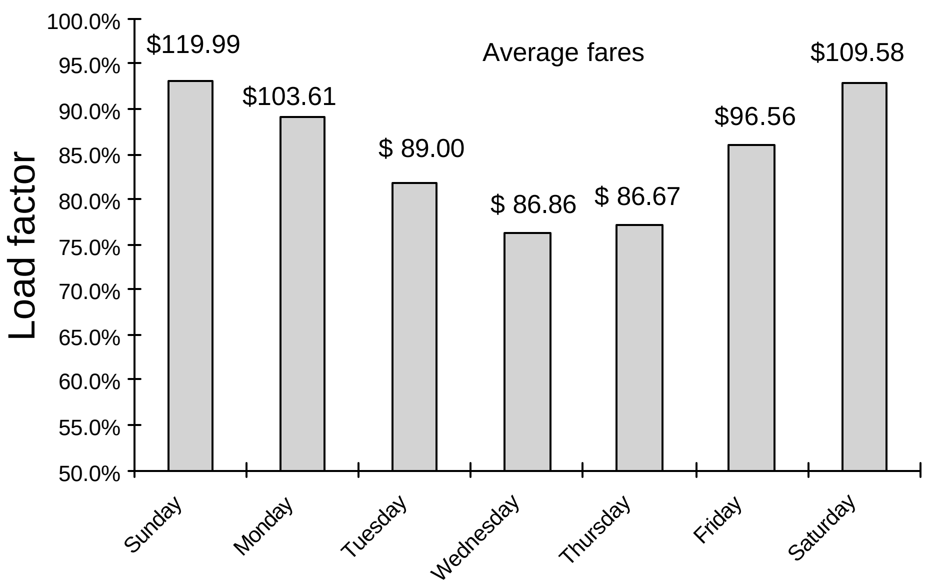
- Monopoly/oligopoly firm with two flights
- Peak flight uncertain (aggregate uncertainty)
- Advance purchase discounts induce consumers with weak preference over flight time to buy early

Dana (RAND ‘99)

- Competitive market model
- Peak flight uncertain
- Capacity limited fares induce self-selection

Carlton (AER ’78), Deneckere and Peck (RAND ’95)

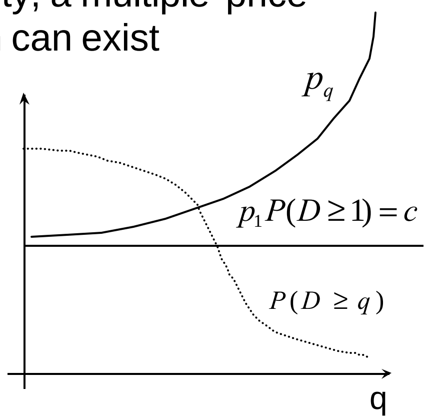
Peak Load Pricing Evidence (Det.- FL, Discount Airline)



Prescott *(JPE '75)* equilibrium

With demand uncertainty, a multiple-price competitive equilibrium can exist

$$p_1 P(D \geq 1) = c$$
$$p_2 P(D \geq 2) = c$$
$$\vdots$$
$$p_n P(D \geq n) = c$$

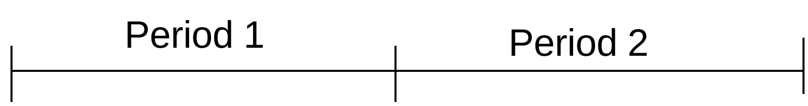


Assumes price rigidity (*ex ante* pricing) and no resale

Leads to cost-based explanation of advance-purchase sorting

Dana's competitive model (JPE '88)

Ex ante marginal cost of capacity = *c*



Type 1 customers
- low valuation
- certain need

Self-select period 1 discount

$$p_1 = c$$

Type 2 customers
- high valuation (given need)
- uncertain need

Self-select period 2 price

$$p_2 P(\text{Need}) = c$$

Competitive firms earn zero profits in each case

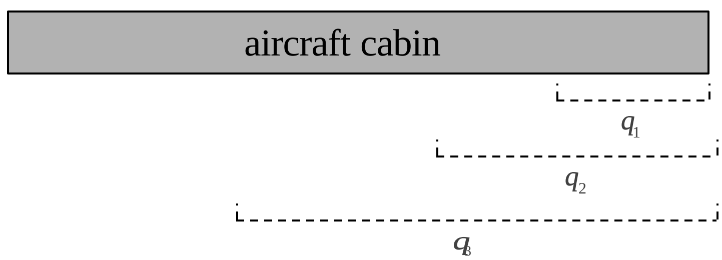
Some of the math models used to implement these ideas in practice

Revenue management literature

- Single-resource capacity allocation
  - Belobaba (1987, 1989)
  - Brumelle & McGill (1990,1993)
  - Robinson (1991)
  - Lee & Hersh (1993)
  - Stidham et al. (1994-97)
  - van Ryzin & McGill (1998)
- Overbooking
  - Rothstein (1971, 1974, 1985)
  - Shlifer and Vardi (1975)
  - Liberman & Yechiali (1978)
  - Bitran & Gilbert (1992)
  - Chatwin (1997)
  - Karesman & van Ryzin (1998)
- Network capacity allocation/bid price control
  - Glover et al. (1982)
  - Curry (1989)
  - Simpson (1989)
  - Williamson (1992)
  - Talluri & van Ryzin (1995,1997)
- Pricing
  - Kincaid & Darling (1963)
  - Stadje (1990)
  - Ladany & Arbel (1991)
  - Gallego & van Ryzin (1994,1997)

Single-leg, nested allocation model
(Brumelle & McGill, OR '93)

X_i	Demand for class $i = 1, \ldots, n+1$	(L before H)
f_i	Fare (net contribution) of class i	
	$f_1 > f_2 > \ldots > f_{n+1}$	
q_i	Nested protection level for class i and higher	



Optimal Protection Levels
(Brumelle & McGill, OR '93)

"Fill events"

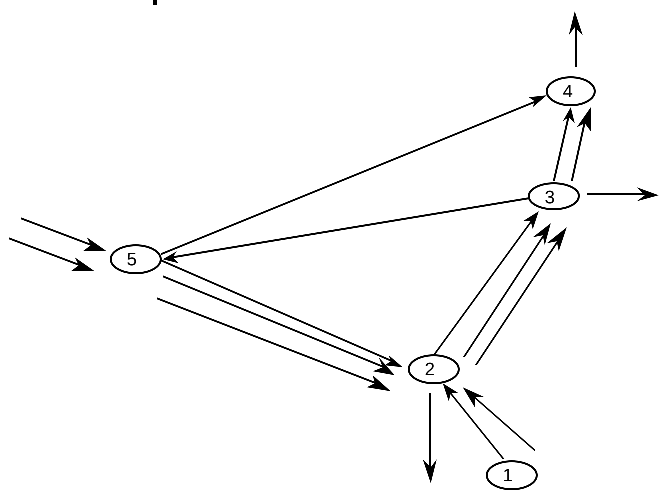
$$A_1(X,q) = \{X_1 > q_1\}$$
$$A_2(X,q) = \{X_1 > q_1, X_1 + X_2 > q_2\}$$
$$\vdots$$
$$A_i(X,q) = \{X_1 > q_1, X_1 + X_2 > q_2, \ldots, X_1 + \cdots + X_i > q_i\}$$

Optimality conditions:

$$\frac{f_{i+1}}{f_i} - P(A_i(X,q)) = 0 \qquad i = 1, \ldots, n$$

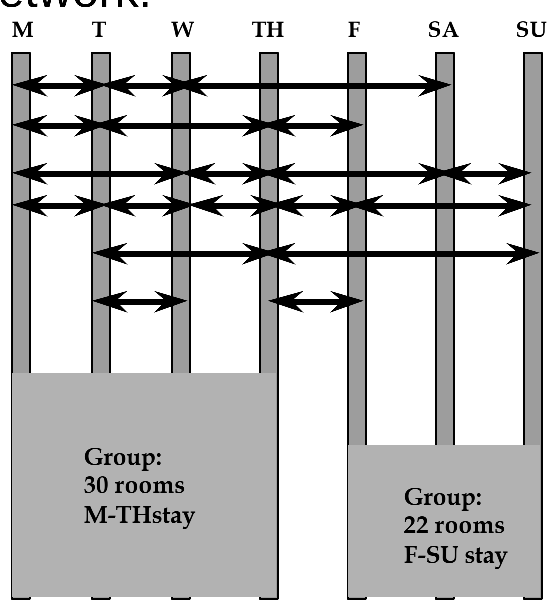
Solved via Monte-Carlo integration (Robinson)

Network problems: Airline



Objective:
Manage accept/deny at the network level.

Hotel network:



Network model – allocating paths on a network

$t=1,\ldots,k$

x_t m-vector of leg capacities

$A=[a_{ij}]$ incidence matrix (m x n)

$$a_{ij} = \begin{cases} 1 & \text{if itinerary } j \text{ uses leg } i \\ 0 & \text{otherwise} \end{cases}$$

ξ_t n-vector of randomly arriving revenues

u_t n-vector of 0-1 controls (accept/deny decisions)

Dynamic Program

$$V_t(x_t) = \max_u E\left[x^T u_t(x_t,x) + V_{t-1}(x_t - Au_t(x_t,x))\right]$$

Structure of an optimal allocation policy

An optimal policy is to accept revenue R_j for itinerary j if and only if

$$R_j \geq V_{t-1}(x) - V_{t-1}(x - A_j)$$

displacement cost
(opportunity cost)

Issues:

- Approximating control structure

- Approximating displacement cost

Approximate control structures

Bid prices

Given values $\mu_i(x,t), i = 1, \ldots, m$ for each leg,
accept a request for itinerary $j = (i_1, i_2, \ldots, i_k)$ if

$$R_j \geq \mu_{i_1}(x,t) + \mu_{i_2}(x,t) + \cdots + \mu_{i_k}(x,t)$$

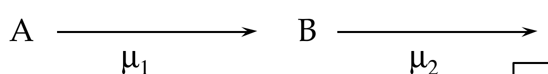
Displacement adjusted virtual nesting (DAVN)

$$R_j - \mu_{i_1}(x,t) + \mu_{i_2}(x,t) + \cdots + \mu_{i_k}(x,t)$$

Compute displacement-adjusted revenue for each itinerary and apply the resulting revenues and demand in a single-leg model on each leg.

Is a bid price policy always optimal? No

2 periods remaining; 1 seat available on each leg



t=1	Itinerary	Fare	Prob.		
	A,B,C	$ 500	0.40		
	A,B	$ 250	0.30		
	B,C	$ 250	0.30		
t=0	Itinerary	Fare	Prob.		
	A,B,C	$ 500	0.80		
	No Arrival		0.20		
	Opt. Policy	$ 440	10%	improvement	
	Opt. Bid Price	$ 400			

At $t=1$, we need

$$\mu_1 > 250$$
$$\mu_2 > 250$$
$$\mu_1 + \mu_2 \leq 500$$

Bid price control is unable to block <u>both</u> local customers <u>and</u> also allow through customer to book!

But bid prices are asymptotically optimal (Talluri & van Ryzin MS ’98)

Consider a sequence of problems indexed by q

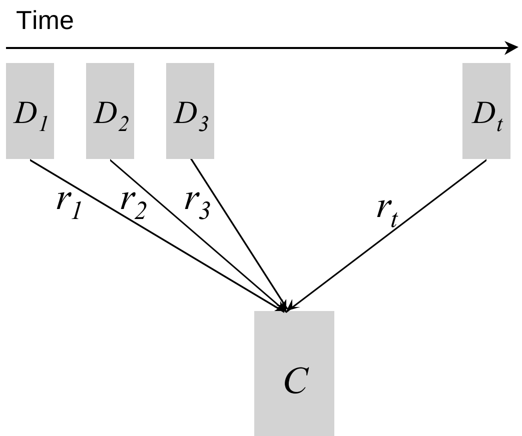
$R_t(q) =_D R_{\lceil t/q \rceil}$	random revenue/arrival
$x(q) = qx$	remaining leg inventory
$k(q) = qk$	remaining time

Then there exists a <u>static</u> bid price heuristic H :

$$\frac{V^H_{qk}(qx)}{V_{qk}(qx)} \geq 1 - O(q^{-1/2}) \rightarrow 1 \text{ as } q \rightarrow \infty$$

(cf. W. Cooper, U. Minn., ’00)

Bid prices are essentially a form of (internal) equilibrium prices



$$\max \sum_t r_t q_t$$

$$q_t \leq D_t$$

$$\sum_t q_t \leq C \qquad m$$

Dual price

Network approximations

Example: Deterministic LP

$$V_t^{LP}(x) = \max \quad \sum_j r_j y_j$$
$$Ay \leq x$$
$$0 \leq y \leq EY$$

Then, $\nabla V_t^{LP}(x) = l$ (provided gradient exists) and we accept R_j if ...

$$R_j \geq V_t^{LP}(x) - V_t^{LP}(x - A_j)$$
$$\approx \nabla^T V_t^{LP}(x) A_j$$
$$= \sum_{i \in A_j} l_i$$

Example: Probabilistic NLP

$$V_t^{NLP}(x) = \max \quad \sum_j r_j \, E \min\{Y_j, y_j\}$$
$$Ay \leq x$$
$$y \geq 0$$

Again, $\nabla V_t^{NLP}(x) = l$ (provided gradient exists) and we accept R_j if ...

$$\begin{aligned} R_j &\geq V_t^{NLP}(x) - V_t^{NLP}(x - A_j) \\ &\approx \nabla^T V_t^{NLP}(x) A_j \\ &= \sum_{i \in A_j} l_i \end{aligned}$$

Example: Randomized LP

(Talluri and van Ryzin '97)

perfect information (PI) approximation

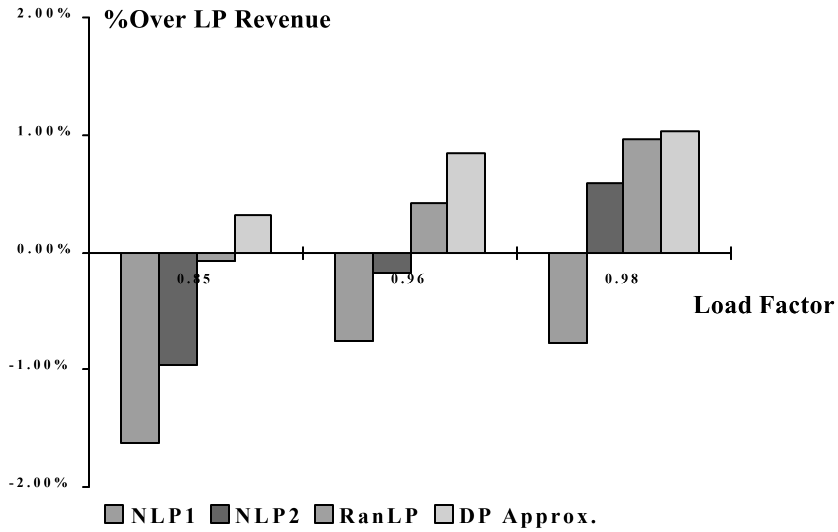
$$V_t^{PI}(x) = E \max \quad \sum_j r_j \, y_j$$
$$Ay \leq x \; : \; m(Y) \quad \text{dual price (r.v.)}$$
$$0 \leq y \leq Y$$

Then, estimate $\nabla V_t^{PI}(x)$ by interchanging differentiation and expectation and using simulation

$$\nabla V_t^{PI}(x) \approx \frac{1}{N} \sum_{i=1}^{N} m(Y_i)$$

Hotel network (real-world data)

42 days, 497 itins, 5 rate classes, 2 opts/day, 12 days, 450 rooms



How could/should auctions be used to accomplish the same thing?

Airline auctions and e-markets

- Practice
  - Priceline.com
  - Ticket auctions
  - Current trends
- Research
  - Basic economic theory
  - Auction design
  - Game theory based models
  - Integration of different mechanisms

New mechanisms are spreading in practice

- Third party auctioneers
  - Priceline.com
  - Expedia
  - TripBid.com
- Airline-operated auctions
  - South African Airways, Air Portugal, Cathay Pacific, Virgin
- Airline-owned consortia: Hotwire.com
  - America West, American Airlines, Continental, Northwest, United and US Airways
  - Scient (technology partner)
- Plans for futures/exchange market (A.D. Little)

Current unit sales volume is roughly 3-5%

Ex: Priceline.com

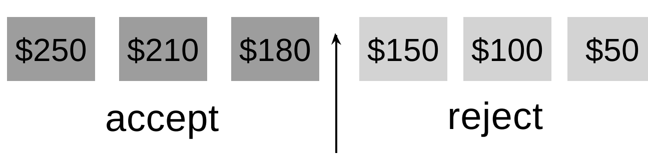
Approximately how it works

- Airlines provide priceline.com with
  - Itinerary/leg prices
  - Capacity allocations (max. number of seats)
  - Basis: traditional RM calculations
- Customers submit bids for O-D
  - Cannot specify flight time, connections, airline
- Priceline performs matching to maximize its spread

Airlines generally view priceline.com customers as independent segment

How airlines evaluate bids:

Customer bids



$$\nabla V_t(x) = 165$$

displacement cost

RM-based displacement cost becomes the reserve price for the auctioneer.

Some comments on the use of auctions in airlines

- Airline markets are already very “transparent”
  - CRS, travel agents, on-line price searches
  - Consumer and airline price transparency
- Capacity-controlled discounts currently provide significant price flexibility
- Demand is spread over time, so organizing single auction event seems impractical
- Industry scepticism and resistance

"We have mixed views on seats as a commodity."

"There's the potential to lose control of inventory, and we're not anxious to push that along."

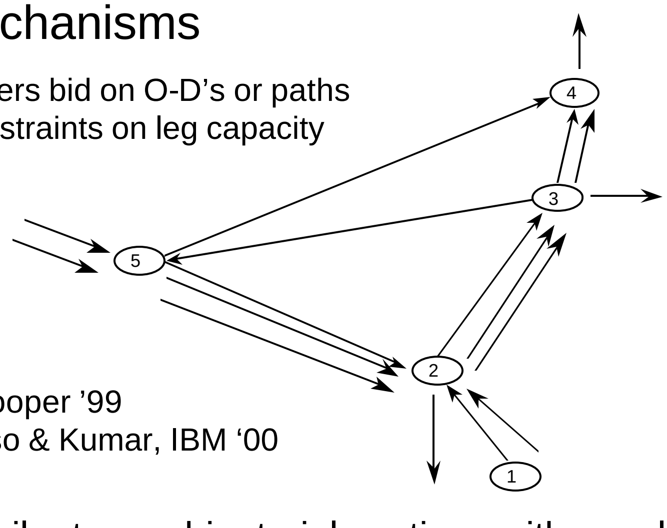
Steve Cossette
VP Distribution Planning
Continental Airlines
IATA Press Briefing

Research: Basic theory questions

- Competitive efficiency under price rigidity
  - Advance purchase equilibrium can reduce welfare (Dana JPE '98)
    - Capacity provided for low-value, advance purchase customers and is reserved for high-value, uncertain customers => Excess capacity allocated
    - Ex post price equilibrium is more efficient
- Relaxing rigidity (auctions/markets)
  - Risk aversion: Auctions create rationing risk & price risk
  - Transaction costs of auctions
  - Purchases/auctions over time
  - Investment incentives (lumpy capacity)

Research: Network auction mechanisms

Buyers bid on O-D's or paths
Constraints on leg capacity



Cooper '99
Eso & Kumar, IBM '00

Similar to combinatorial auctions with supply flexibility.

Research: Buyer's strategic behavior

Most RM models in practice do not consider strategic behavior – auction theory does

A first step: Some RM research now based on discrete choice models of consumer behavior

S.E. Anderssen of SAS (1998), *EJOR*
Belobaba & Hopperstad PODS Studies
Talluri & van Ryzin, Choice-based single-leg model

Some modest progress on choice-based inventory control

Cf. Mahajan & van Ryzin, *OR* to appear

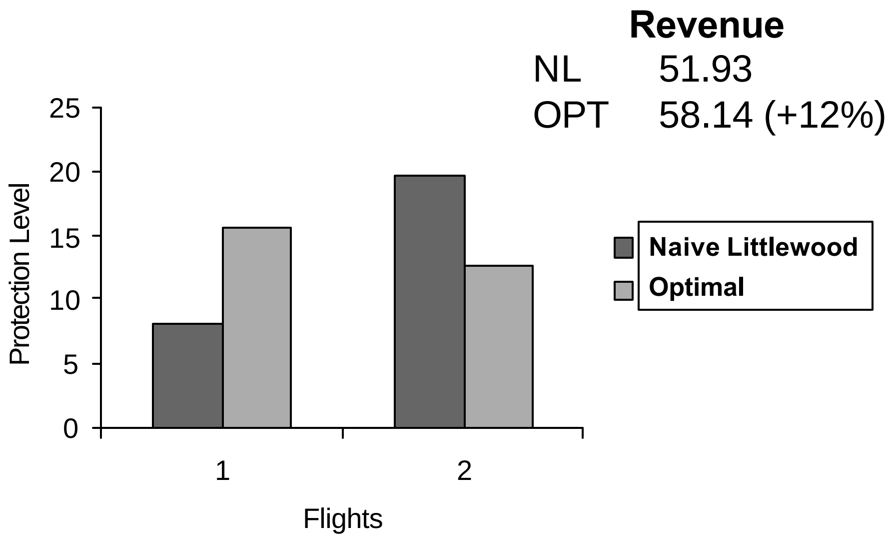
Ex:

	Utility (avg.)	Price
Flight 1	Low	High
Flight 2	High	Low

What happens if we ignore choice behavior?

Sample path gradient algorithm for standard inventory models (e.g newsboy/Littlewood).

Seat availability & revenue



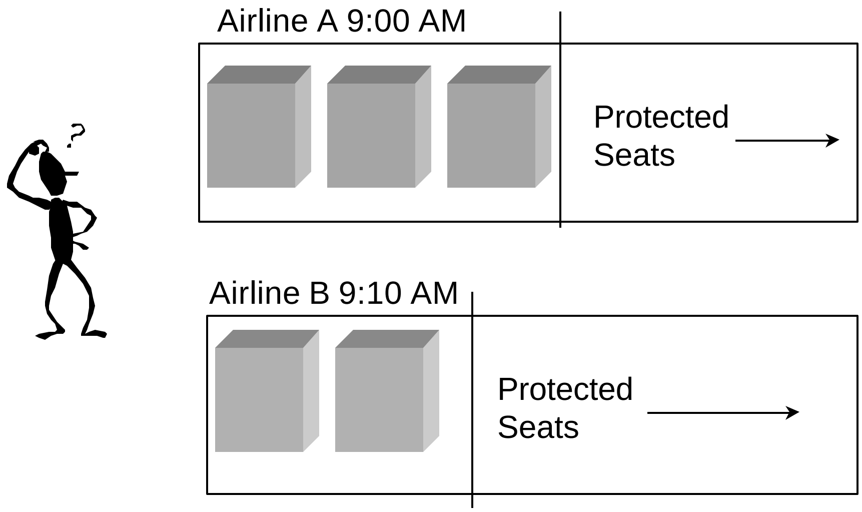
Research: Strategic interactions with other firms

Most RM models in practice do not consider strategic interactions with other sellers

Some early work in this direction

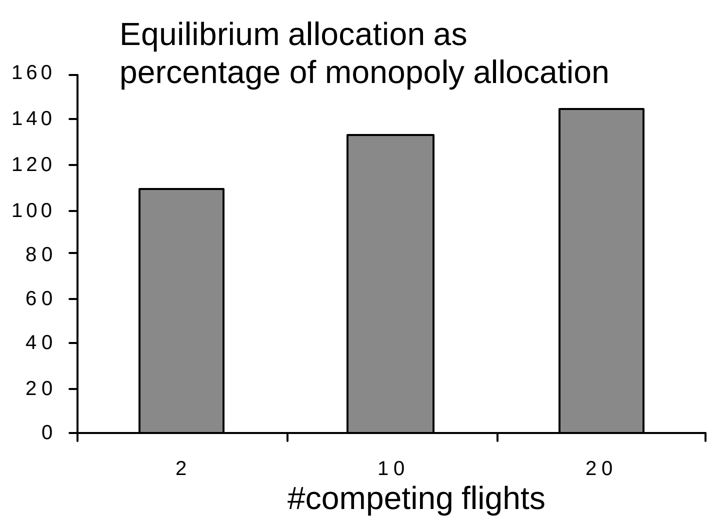
Lippman & McCardle, OR '97
Mahjan & van Ryzin, OR to appear
Nessim & Schumsky, U. Rochester, Working Paper

Ex: What effect does competition have on inventory allocations?

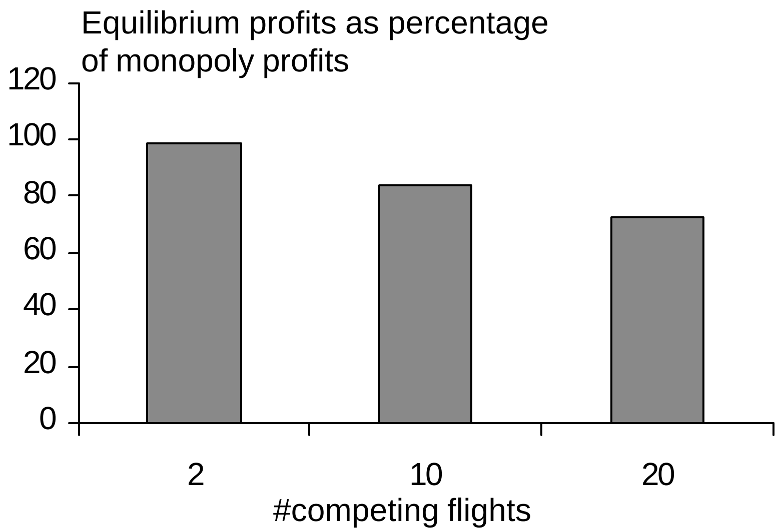


Competition causes “excess allocation” effect ….

Cf. Mahajan & van Ryzin, *OR* to appear



… leading to lower profits



Summary & conclusions

- RM based on a largely price-rigid world
- New markets relax price rigidity
- Consequences
  – Practice
    - Lots of entry by intermediaries
    - But airlines themselves are being quite cautious
  – Theory questions
    - How does it help? Whom does it help?
    - Incentives to join? Incentives to invest?
  – Math modeling questions
    - Strategic behavior (buyers & sellers)
    - Auction design (networks, inter-temporal)