Equilibria in Electric Power Exchane Auction Markets

John R. Birge
R.R. McCormick School of Engineering and Applied Science
Northwestern University

Joint work with Chonawee Supatgiat, Enron, and Rachel Zhang, Cornell

Outline

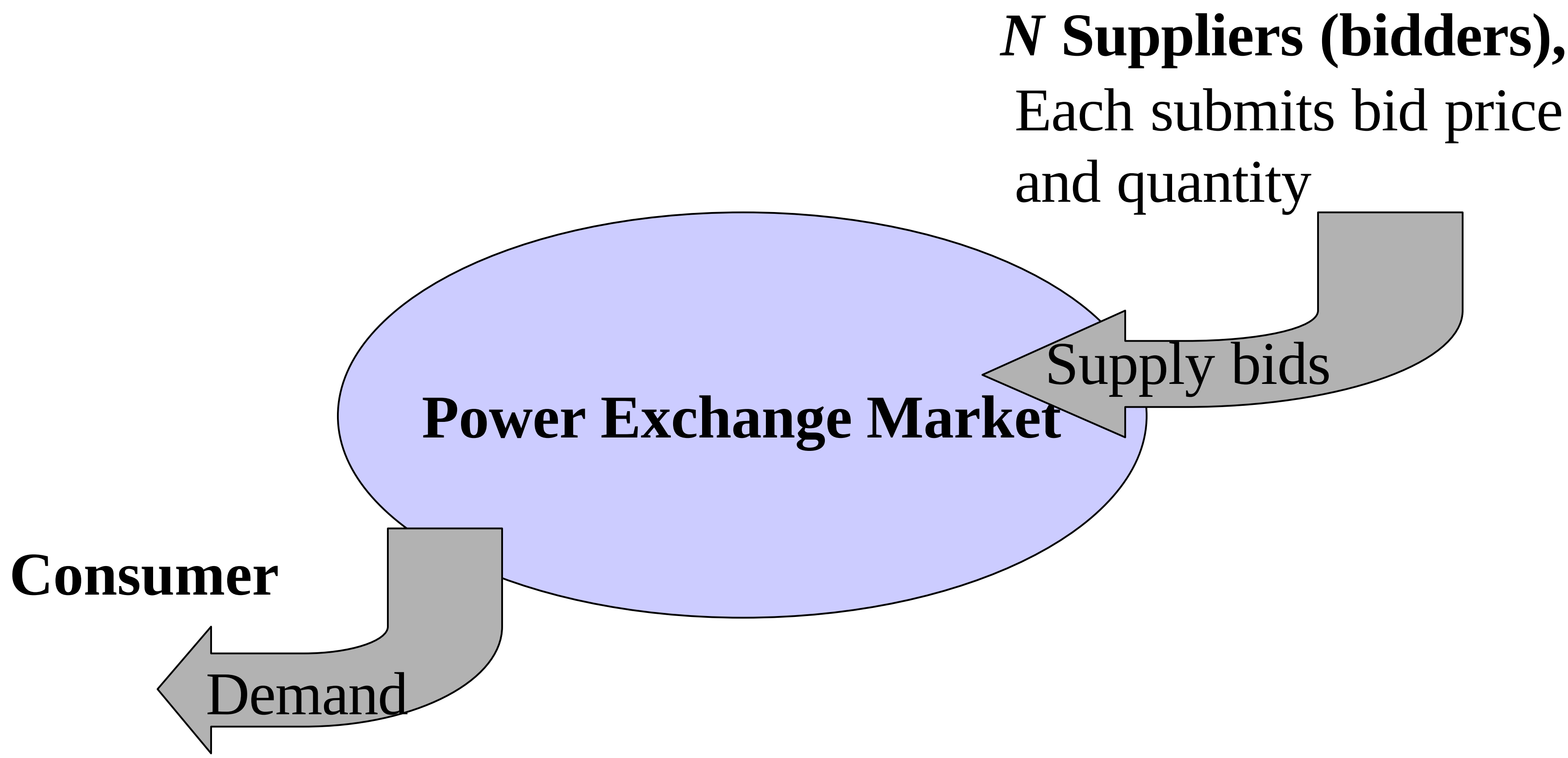
I. Problem Overview

II. Model

III. Results

V. Summary and Extensions

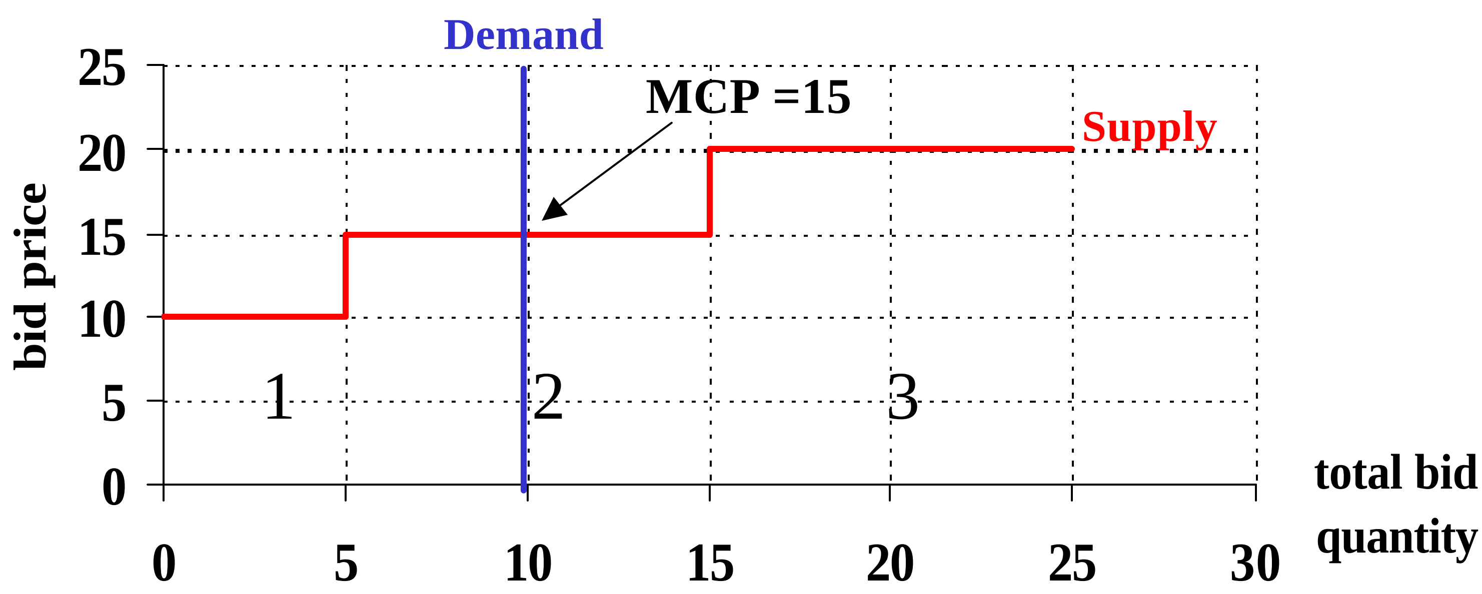
Competitive Markets



Market Clearing Process

Supplier 1 : 5MWh @ $10
Supplier 2 : 10MWh @ $15
Supplier 3 : 10MWh @ $20

Demand is 10



Problem: finding optimal bidding strategies and the resulting MCP

Market Overview

- Non-sealed bid, Multi-round
  - Bidders can see each other's bids and can adjust their prices as many times as they want
  - Market is closed when no bidder wants to adjust his/her bid price
- Selling at spot:
  - All dispatched units are traded at the same price

Model

- Demand D (or d) must be satisfied
- Bidder i has unit cost c_i
- Single bid per bidder : (x_i, p_i)
  - x_i = bid quantity of bidder i
  - p_i = bid price of bidder i : (assume discrete)

$$p_i \in \{le \mid l = 0,1,\ldots,O\}, \; i = 1,\ldots,N$$

  - $\mathbf{b} = [(x_1,p_1), \ldots, (x_N,p_N)]$

Model

- Market clearing price

$$MCP(\mathbf{b}, d) = \left\{ \min_j \; p_j : \sum_{i \in I(j)} x_i \geq d, I(j) = \{i : p_i \leq p_j\} \right\}$$

- Dispatch quantity of Bidder *i*

$$q_i(\mathbf{b}, d) = \begin{cases} 0 & \text{if } p_i > MCP(\mathbf{b}, d) \\ x_i & \text{if } p_i < MCP(\mathbf{b}, d) \\ \overline{q}_i(\mathbf{b}, d) & \text{if } p_i = MCP(\mathbf{b}, d) \end{cases}$$

* Dispatch of marginal bidders follows order determined from submission time of bids

Model

- Bidder *i* 's payoff

$$f_i(\mathbf{b}) = E_D\left[(MCP(\mathbf{b}, D) - c_i)q_i(\mathbf{b}, D)\right]$$

- Objective: Find Nash equilibrium $\{p_i^*, i = 1,\ldots, N\}$ such that

$$f_i\left(\left[(x_1, p_1^*),\ldots,(x_i, p_i),\ldots,(x_N, p_N^*)\right]\right) \leq f_i\left(\left[(x_1, p_1^*),\ldots,(x_i, p_i^*),\ldots,(x_N, p_N^*)\right]\right)$$

for all feasible p_i, for bidder i, and all $i = 1,\ldots, N$

Model

- Distinct bidders:

$$\left| c_i - c_j \right| > 2e \ , \quad \forall i \neq j$$

- Fixed bid quantity: bidders can adjust only bid price
- Given a number *x*,

  $\lfloor x \rfloor$ = max{*ie* | *ie* £ *x*, *i*=*0*,..,*O*} and

  $\lceil x \rceil$ = min{*ie* | *ie* ³ *x*, *i*=*0*,..,*O*}

Market Stability Condition

$$\bar{D} \le \sum_{\forall i \ne j} x_i \qquad \text{for } j = 1, \ldots, N$$

- $\bar{D}$ is the highest demand realization
- No one is guaranteed to be dispatched

Results

- Multiple equilibria
- Known demand:
  - At the highest MCP equilibrium point, every bidder *i* bids at $\lceil c_i \rceil$, except the marginal bidder *i* who bids at $\lfloor c_j \rfloor$
  - Unique marginal bidder if partially dispatched
- Stochastic demand:
  - Single marginal *i*
    - At any demand, bids at $\lceil p_j\text{-}e \rceil$
    - At the highest demand, bids at $\lfloor c_j \rfloor$
  - Two marginal bidders, *i*, *j*
    - They bid just above the cost of the bidder with a lower quantity

$$p_i = p_j = \lceil c_j \rceil \text{ where } x_j \leq x_i$$

Highest Equilibrium MCP

- Known demand: the highest possible equilibrium MCP must be in the set

$$\left\{\lfloor c_i \rfloor, i = 1, \ldots, N\right\}$$

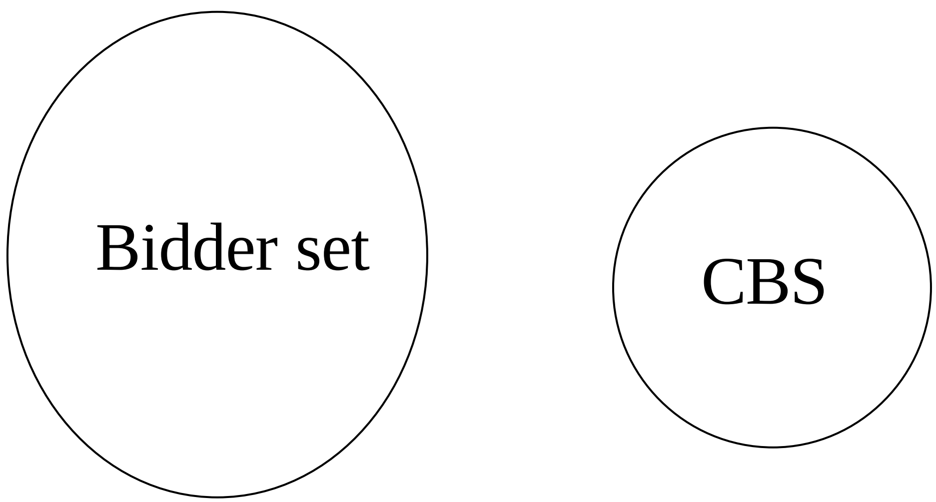
- Stochastic demand: the highest possible equilibrium MCP must be in

$$\left\{\lfloor c_i \rfloor, \lceil c_i \rceil, \lceil c_i \rceil + e,\ i = 1, \ldots, N\right\}$$

Competitive Bidder Set (CBS)

- CBS: bidders with the lowest costs and satisfy the market stability condition

$$\overline{D} \leq \sum_{\forall i \neq j} x_i \quad \text{for } j = 1, \ldots, N$$



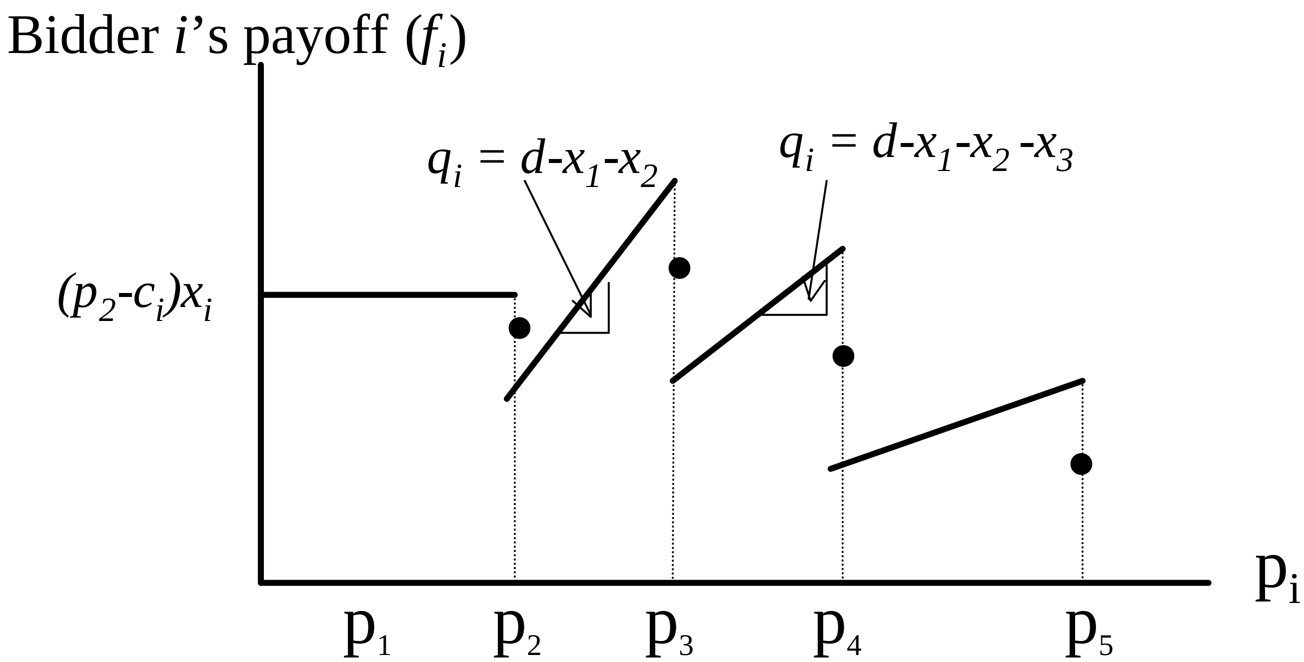
Competitive Bidder Set (CBS)

At an equilibrium point,

- All bidders outside the CBS are not dispatched
- When demand is known, at least one bidder in the CBS is not dispatched
- 2ε plus the highest cost among the bidders in the CBS is an upper bound on the MCP

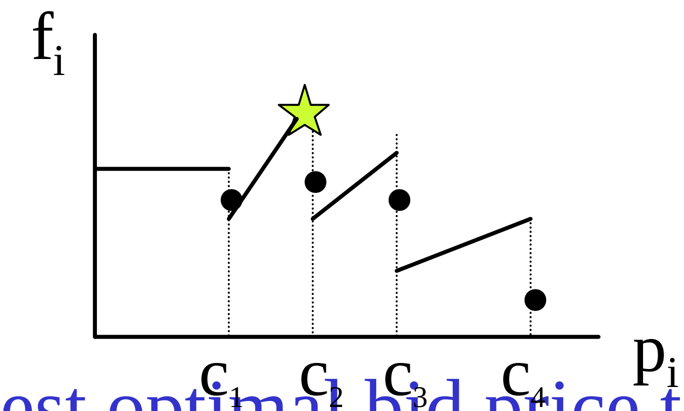
Payoff function

- Given other bidders' bid prices and demand



Algorithm for Finding the Highest MCP Equilibrium Point with Deterministic Demand

- Constructing CBS
- Condition on each bidder to be a marginal while others bid at cost
- Find the optimal bid price



- Pick the one with the highest optimal bid price to be the marginal bidder; others bid at costs

A Special Case: Identical Quantity

- Assume $x_i = x$ for all $i=1,\ldots,N$
- Equilibrium point with the highest MCP

$$p_i^* = \lfloor c_{i+1} \rfloor \quad \forall i = 1,\ldots, N-1$$
$$MCP = \lfloor c_k \rfloor$$

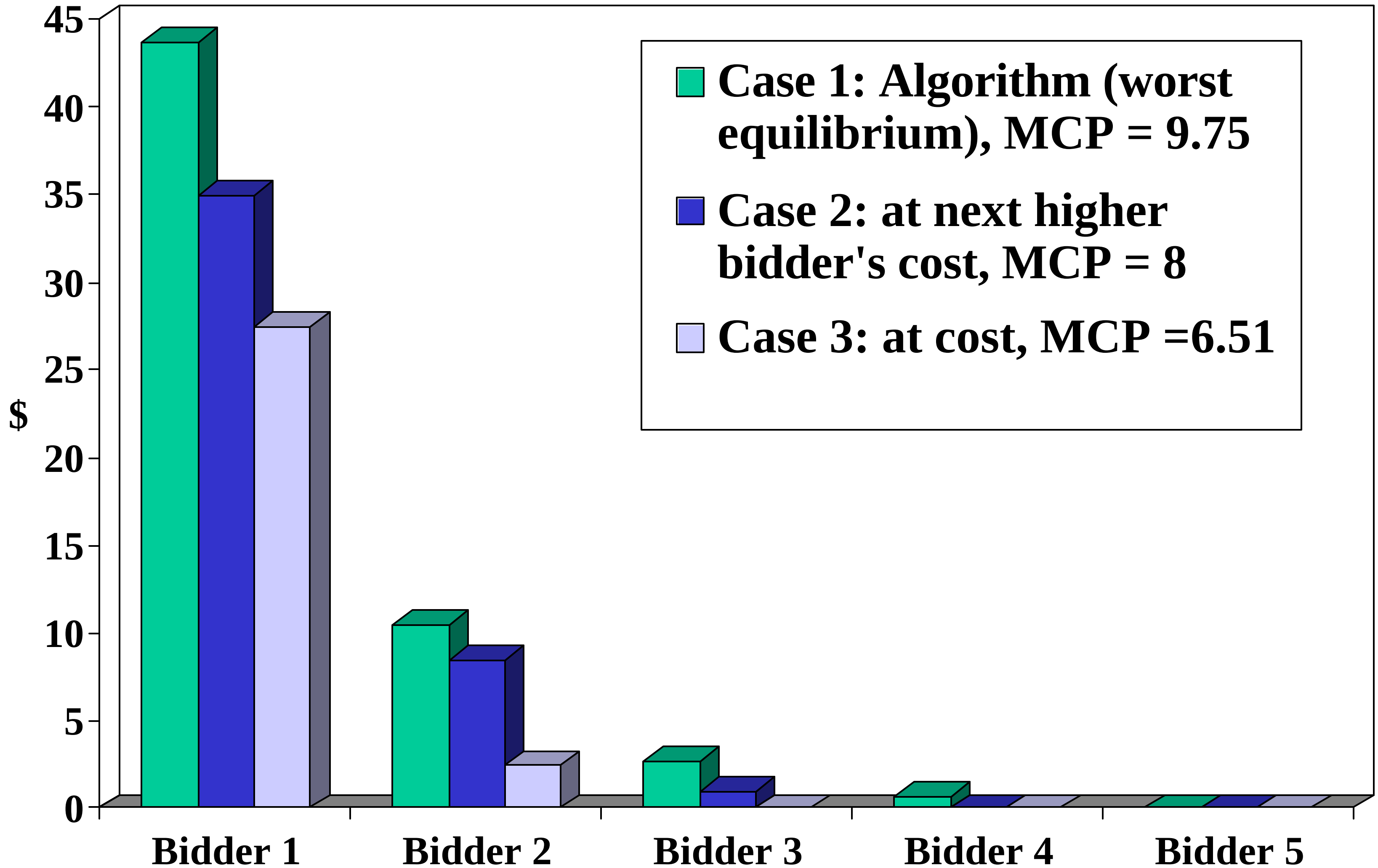
where *k* is the first undispatched bidder, given all bidders bid at their costs.

A Numerical Example

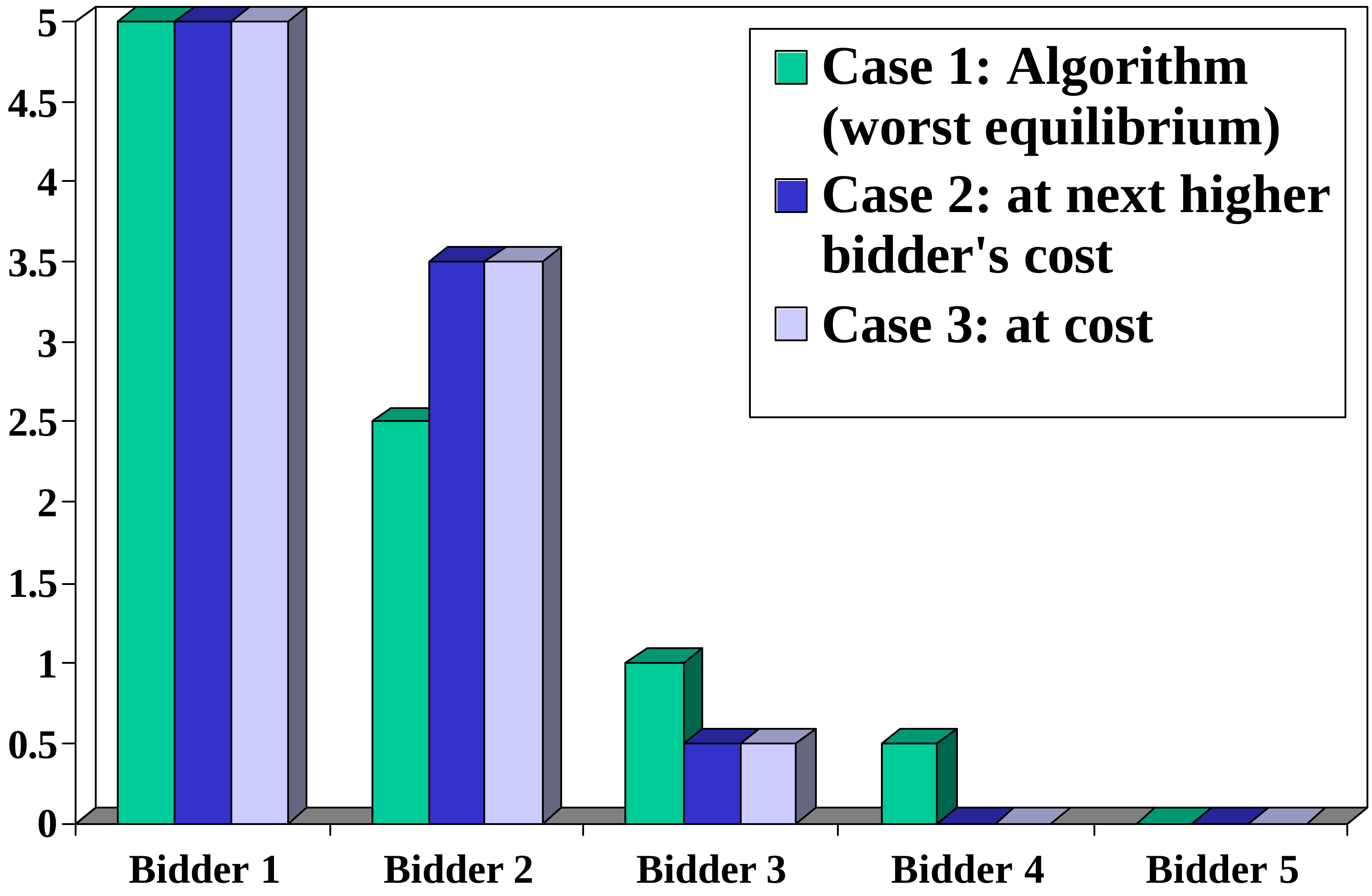
i	c_i	x_i
1	1.01	5
2	6.01	5
3	7.01	1
4	9.01	1
5	10.51	11

- $e = 0.01$, $D = 7$ or 11 w.p. 0.5

Comparison of Payoff



Comparison of Dispatch Quantity



Other Problems-Price Down as Demand Rises

Demand

- Period 1 - demand=50
- Period 2 - demand=100 or 200 equally likely

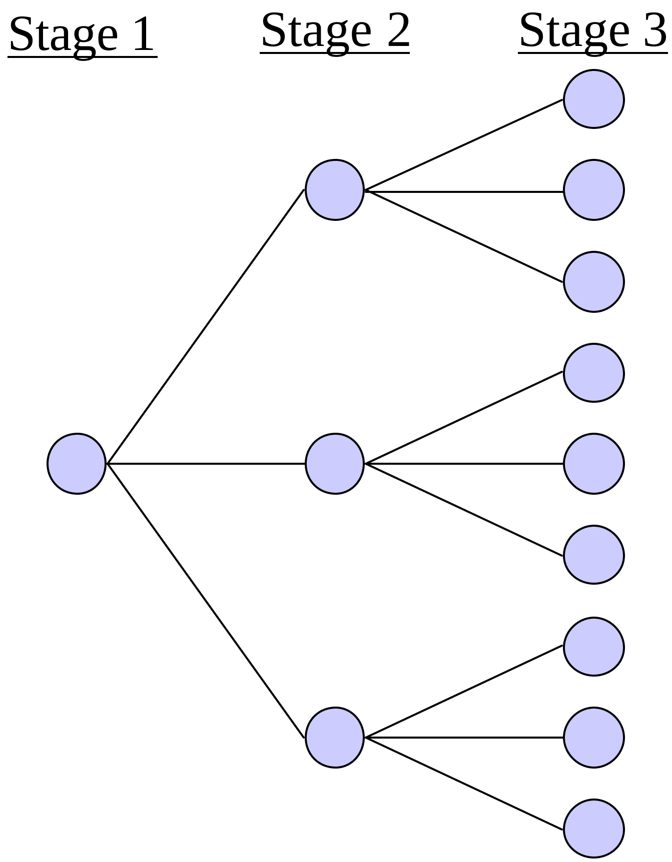
Capacities

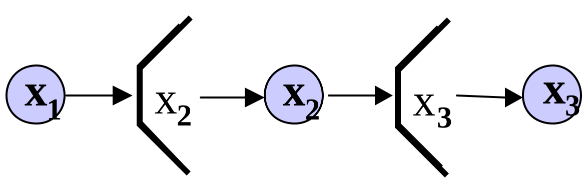
- Hydro - 100 total
- Thermal - 60 at once
- Backstop - ∞

Suppose Period 2 Demand = 100

- Hydro - Bid only in Period 2, 100 at 5 - ε
- Thermal - Bid 5, Backstop - Bid 50

General Equilibrium Solution: Multistage Stochastic Linear Program





$$
\begin{aligned}
\min \quad & c_1 x_1 + Q_2(x_1) \\
s.t. \quad & W_1 x_1 = h_1 \\
& x_1 \geq 0
\end{aligned}
$$

$$
Q_t\left(x_{t-1,a(k)}\right) = \sum_{x_{t,k} \in \Xi_t} prob\left(x_{t,k}\right) \mathrm{Q}_{t,k}\left(x_{t-1,a(k)}, x_{t,k}\right)
$$

$$
\mathrm{Q}_{t,k}\left(x_{t-1,a(k)}, x_{t,k}\right) = \begin{aligned}[t]
\min \quad & c_t\left(x_{t,k}\right) x_{t,k} + Q_{t+1}\left(x_{t,k}\right) \\
s.t. \quad & W_t x_{t,k} = h_t\left(x_{t,k}\right) - T_{t-1}\left(x_{t,k}\right) x_{t-1,a(k)} \\
& x_{t,k} \geq 0
\end{aligned}
$$

- $Q_{N+1}(x_N) = 0$, for all x_N,
- $Q_{t,k}(x_{t-1,a(k)})$ is a piecewise linear, convex function of $x_{t-1,a(k)}$

Nested Decomposition

- In each subproblem, replace expected recourse function $Q_{t,k}(x_{t-1,a(k)})$ with unrestricted variable $q_{t,k}$
  - Forward Pass:
    - Starting at the root node and proceeding forward through the scenario tree, solve each node subproblem

$$
\begin{aligned}
\hat{Q}_{t,k}\left(x_{t-1,a(k)}, x_{t,k}\right) = \quad \min \quad & c_t\left(x_{t,k}\right)x_{t,k} + q_{t,k} \\
s.t. \quad & W_t x_{t,k} = h_t\left(x_{t,k}\right) - T_{t-1}\left(x_{t,k}\right)x_{t-1,a(k)} \\
& E_{t,k}x_{t,k} + q_{t,k} \geq e_{t,k} \quad (optimality\ cuts) \\
& D_{t,k}x_{t,k} \geq d_{t,k} \quad (feasibility\ cuts) \\
& x_{t,k} \geq 0
\end{aligned}
$$

    - Add feasibility cuts as infeasibilities arise
  - Backward Pass
    - Starting in top node of Stage t = N-1, use optimal dual values in descendant Stage t+1 nodes to construct new optimality cut. Repeat for all nodes in Stage t, resolve all Stage t nodes, then t $\rightarrow$ t-1.
  - Convergence achieved when

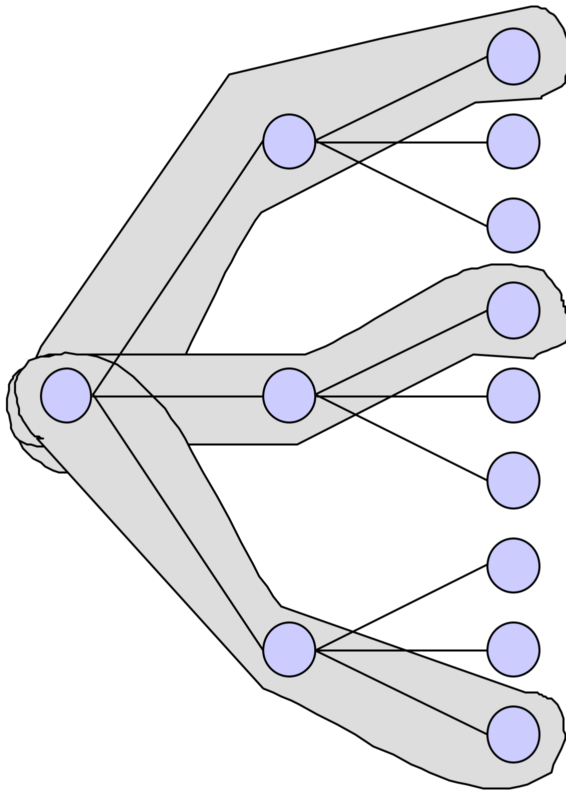
$$q_1 = Q_2\left(x_1\right)$$

Pereira-Pinto Method

1. Randomly select *H* *N*-Stage scenarios
2. Starting at the root, a forward pass is made through the sampled portion of the scenario tree (solving ND subproblems)
3. A statistical estimate of the first stage objective value $\bar{z}$ is calculated using the total objective value obtained in each sampled scenario

the algorithm terminates if current first stage objective value $c_1x_1 + q_1$ is within a specified confidence interval of $\bar{z}$

4. Starting in sampled node of Stage t = N-1, solve all Stage t+1 descendant nodes and construct new optimality cut. Repeat for all sampled nodes in Stage t, then repeat for t = t - 1



Abridged Nested Decomposition

- Also incorporates sampling into the general framework of Nested Decomposition
- Also assumes relatively complete recourse and serial independence
- Samples both the subproblems to solve and the solutions to continue from in the forward pass

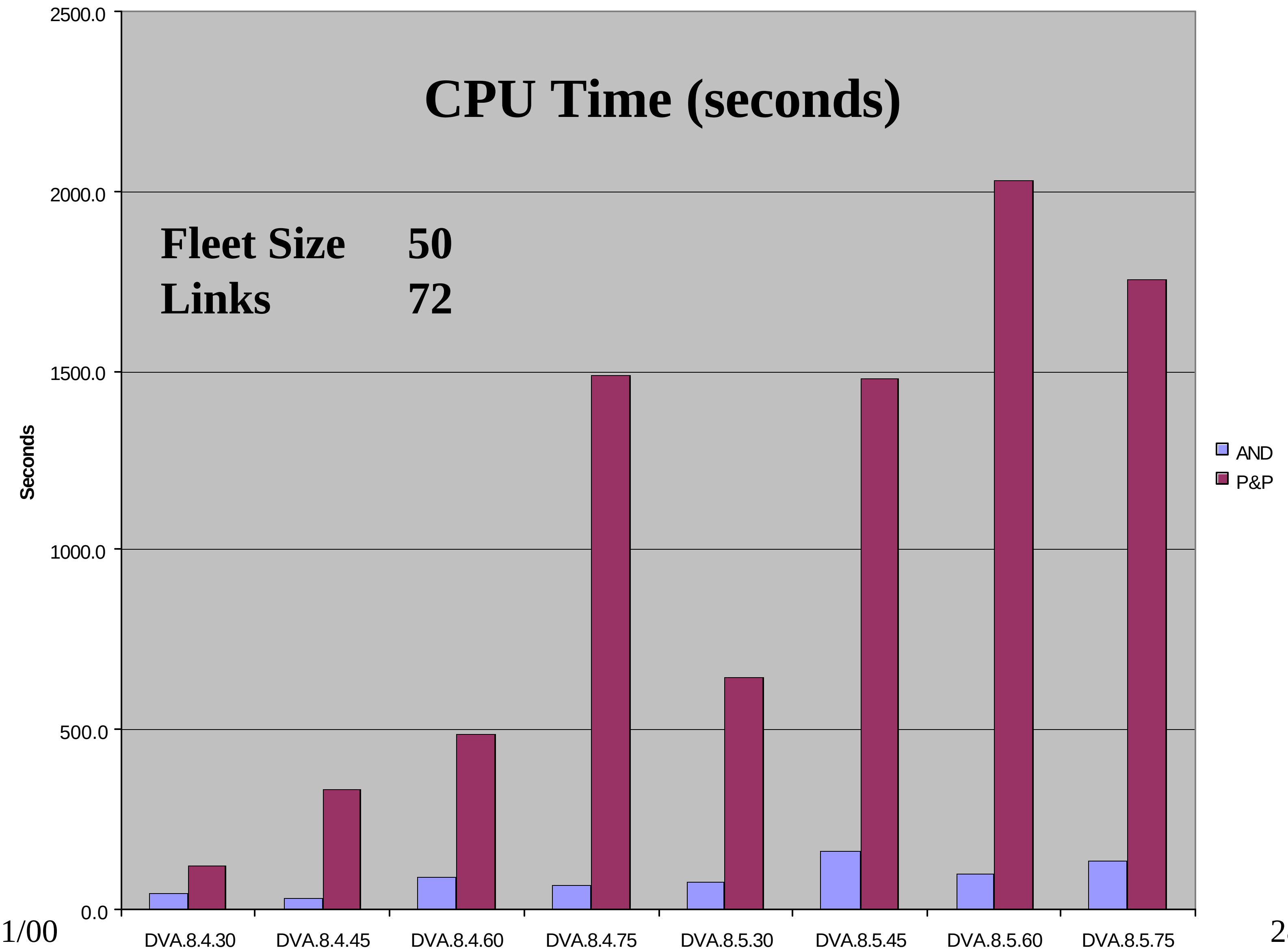
Computational Results

- Implementation of Pereira & Pinto Method and Abridged Nested Decomposition
  - written in C, run on Sun SPARC 20 workstation
  - uses CPLEX to solve subproblems
- Pereira & Pinto Method
  - uses a sample size of 30 for each problem
- Abridged Nested Decomposition
  - number of Stage t subproblems solved from each Stage t-1 branching value: 15
  - initial number of Stage t branching values: 2
    - number of Stage t branching values increases with each failed convergence test
- Both methods terminate when first stage objective value is within one standard deviation of statistical estimate

Computational Results

- Test Problems
  - Dynamic Vehicle Allocation (DVA) problems of various sizes
    - set of homogeneous vehicles move full loads between set of sites
    - vehicles can move empty or loaded, remain stationary
    - demand to move load between two sites is stochastic
  - DVA.*x*.*y*.*z*
    - *x* number of sites (8, 12, 16)
    - *y* number of stages (4, 5)
    - *z* number of distinct realizations per stage (30, 45, 60, 75)
  - largest problem has > 30 million scenarios

Computational Results (DVA.8)



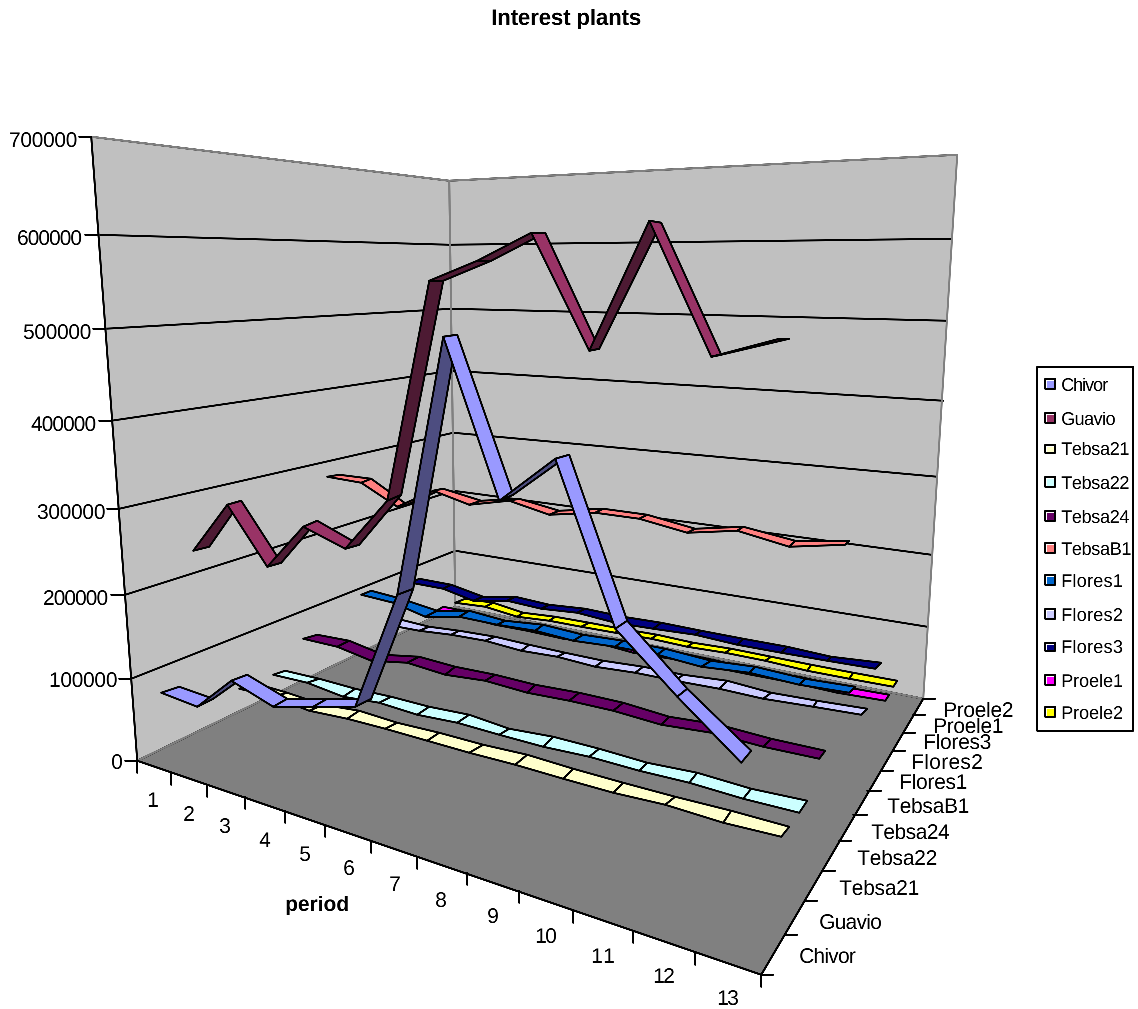
Model for Colombia Power

- Basic model in AMPL
- Solvable in CPLEX
- Assume serial correlation within a larger stage (may extend over several months)
- Single branch or multiple branches
- Abridged NDUM able to solve to optimality

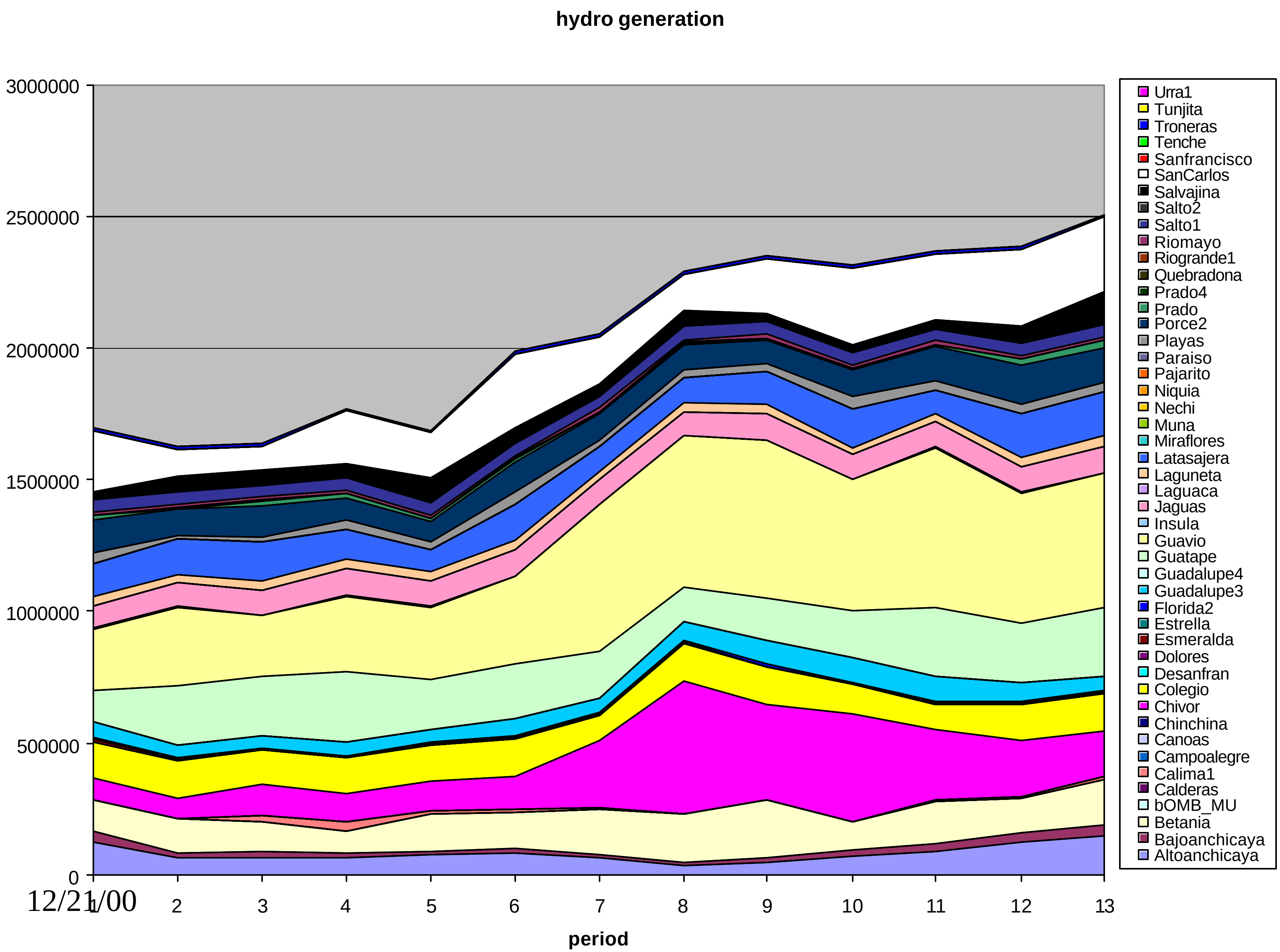
Basic Model

- Objective:
  - **minimize total generation costs**
- **subject to the operating constraints:**
  - **meet load constraints**
  - **thermal capacity constraints**
  - **hydro maximum/minimum flow constraints**
  - **export/import capacity constraints**
  - **minimum/maximum reservoir level**
  - **penalty on alert levels and minimum levels**

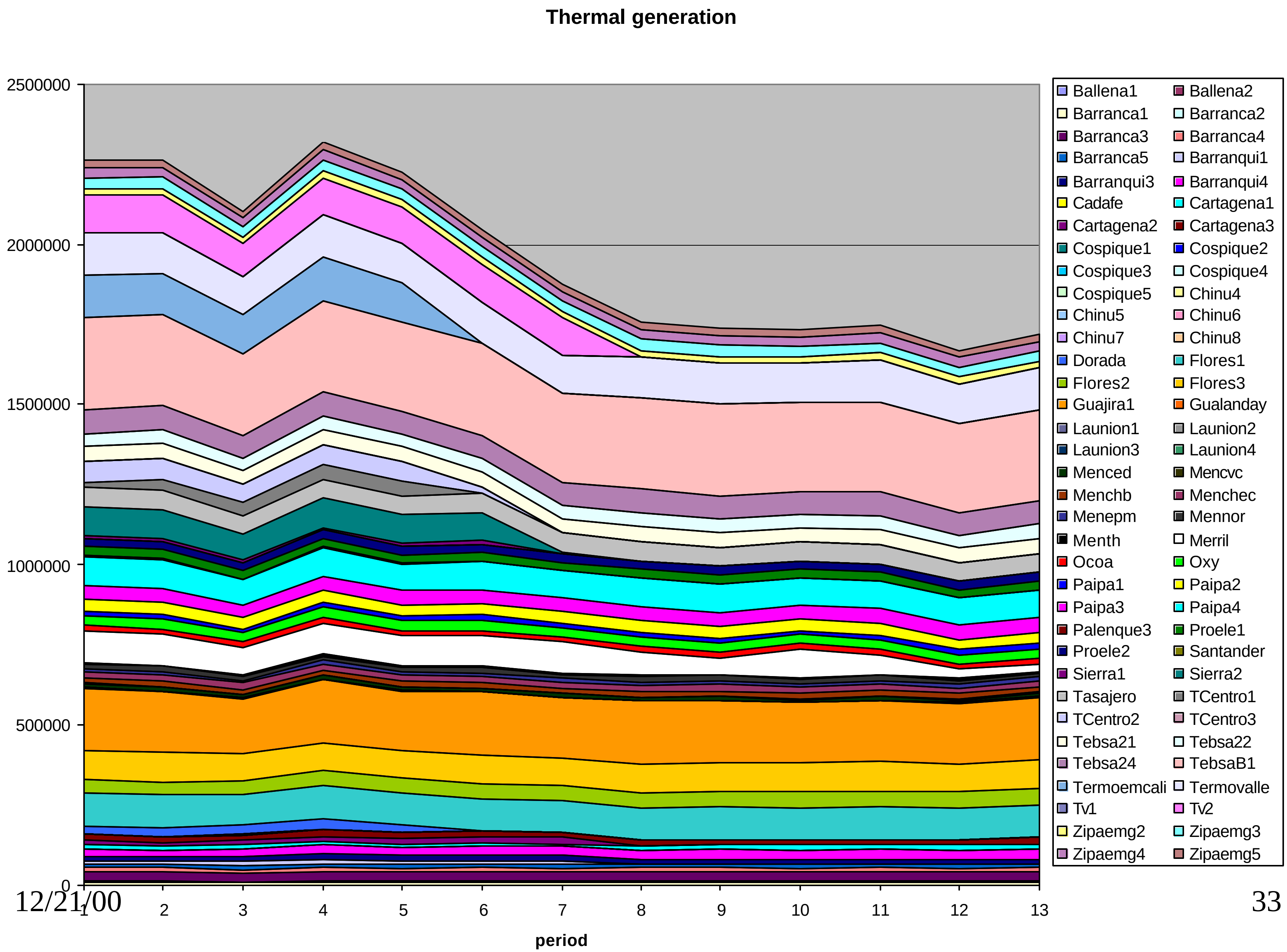
Example Results (selected plants)



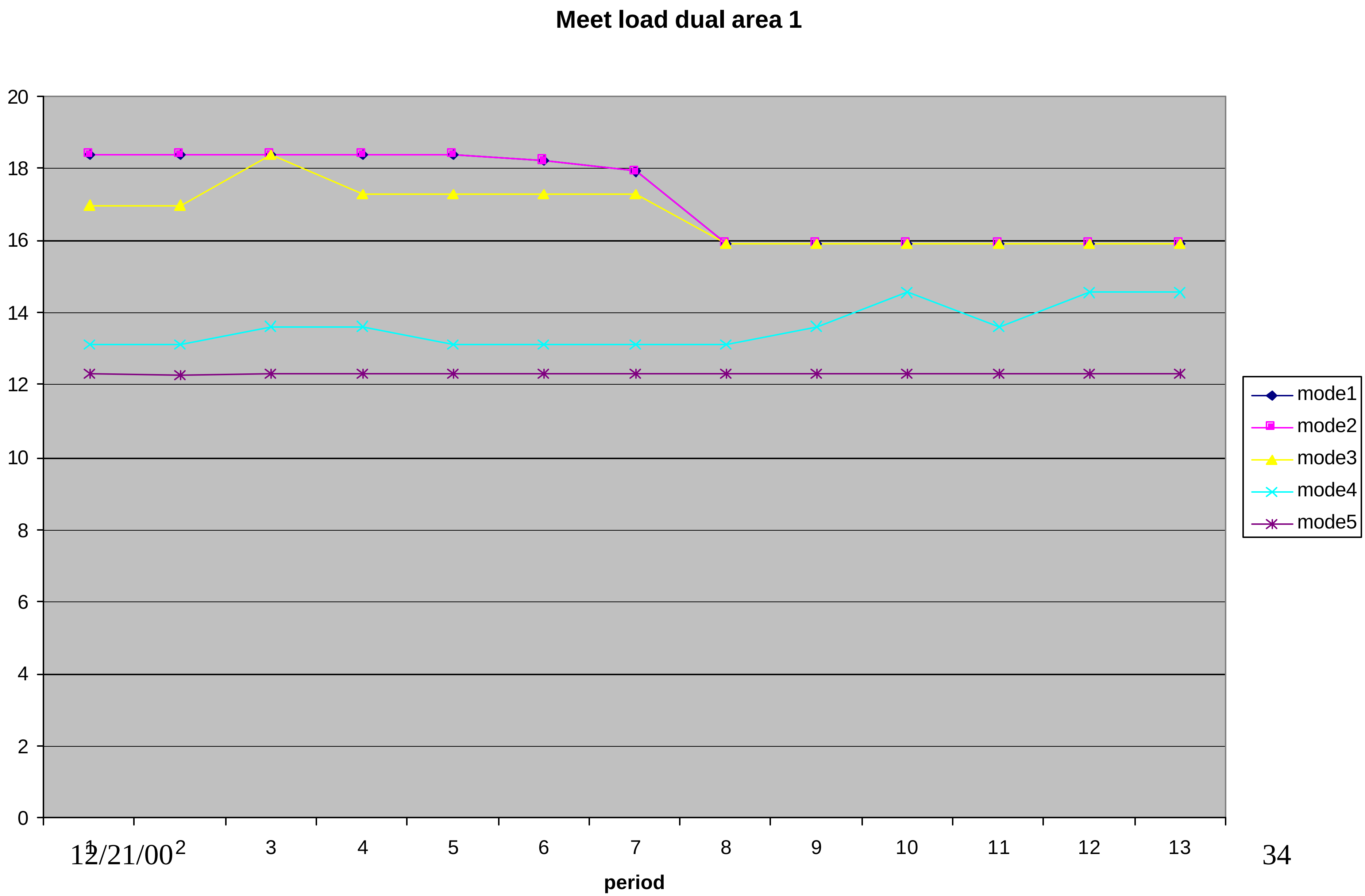
Example: Hydro Generation



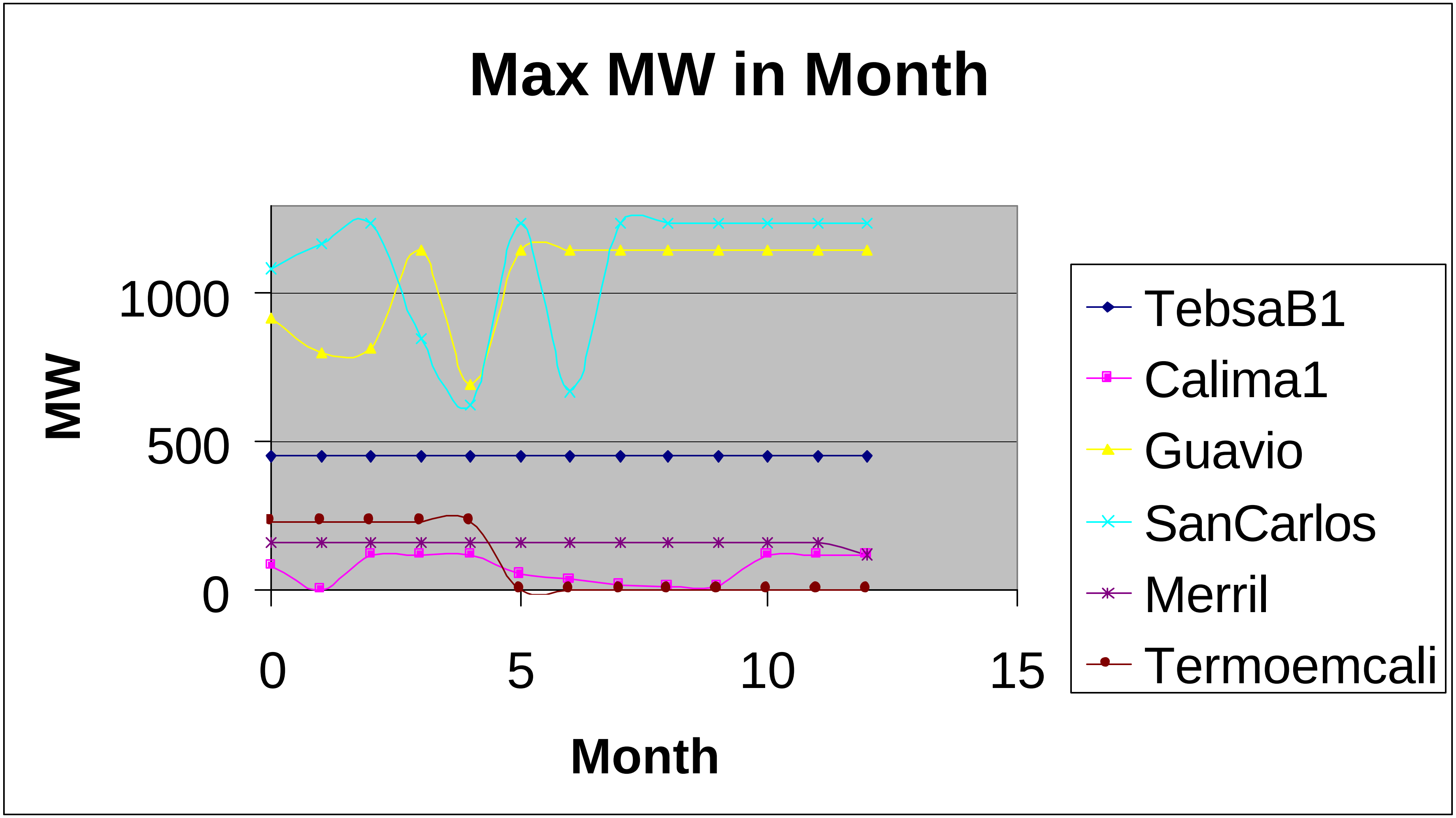
Example: Thermal Generation



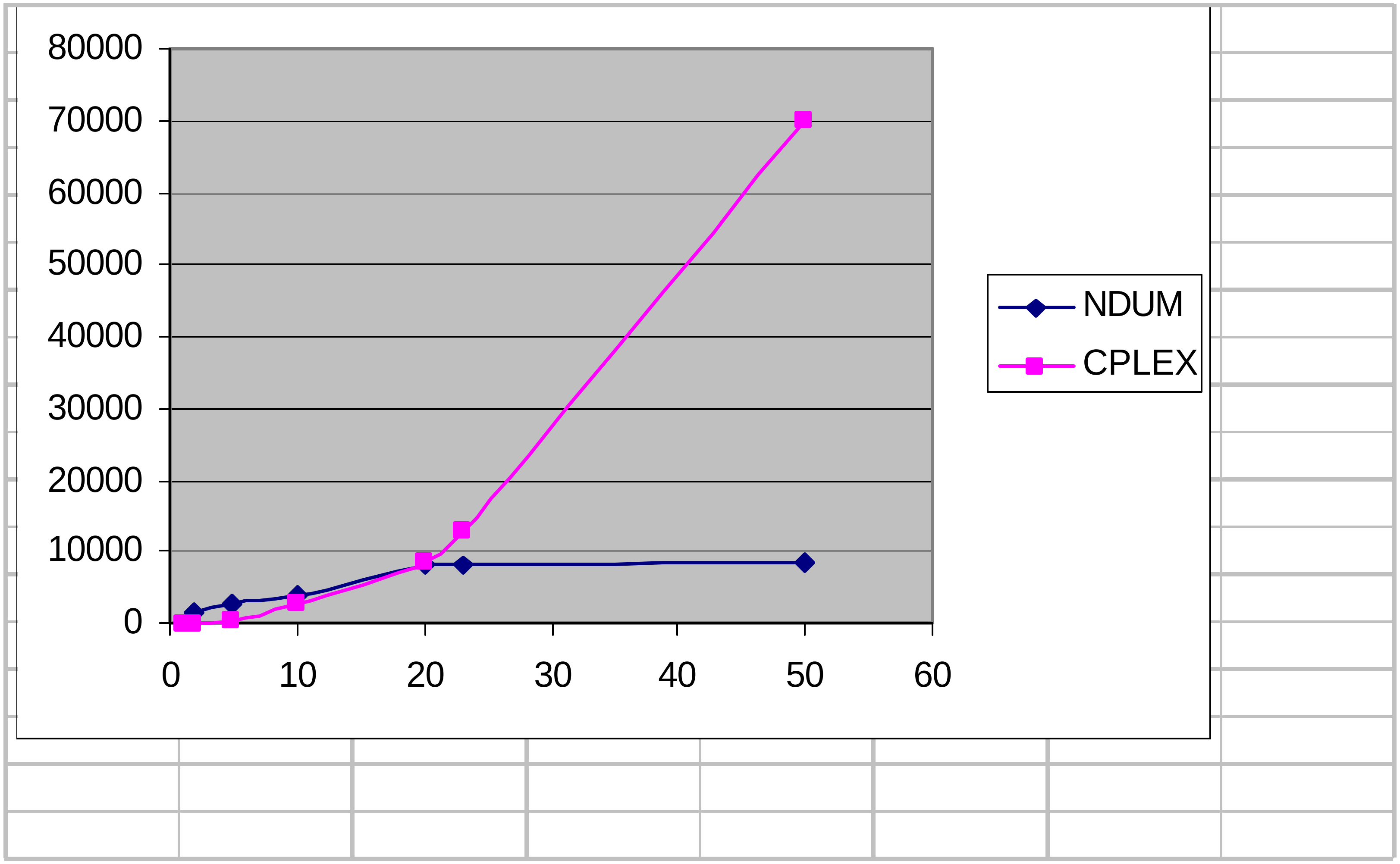
Example: Dual Prices



Example: Max MW



NDUM and CPLEX v. No. of Scenarios



Summary

- Multiple equilibria
- Demand is known
  - MCP <= $\lfloor c_N \rfloor$
  - marginal i at $\lfloor c_j \rfloor$, others i at $\lfloor c_i \rfloor + e$
- Demand is stochastic
  - MCP <= $\lceil c_N \rceil + e$
  - marginal i at p_j-e, $\lceil c_k \rceil$, or $\lceil c_k \rceil + e$, others i at $\lfloor c_i \rfloor + e$
- Algorithm to find highest MCP equilibrium point
- Abridge Nested Decomposition for Solving Individual Problems

Extensions

- Include start-up cost of generation
- Analyze multi-period problems: cost depends on the dispatch of the previous period
- Allow multiple bids per bidder

Worst Equilibrium point

- Worst equilibrium MCP = highest possible bid price

$$p_i^* = c_i \quad \text{for } i = 1, \ldots, N-1$$

$$\sum_{i=1}^{N-1} x_i^* = d - e$$

$$p_N^* = Oe$$

$$x_N^* \geq e$$