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Nonparametric Deconvolution of Seismic Depth Phases

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Abstract: Accurate determination of the source depth of a seismic event is a potentially important goal for better discrimination between deeper earthquakes and more shallow nuclear tests. Earthquakes and explosions generate depth phases such as pP and sP as reflections of the underlying P signal generated by the event. The delay time between the original signal and the pP phase can be used to estimate the depth of the seismic event. Cepstral methods, first used by Tukey and later by others, offer natural nonparametric means for estimating general echo patterns in a single series. Here, we extend the single series methodology to arrays by regarding the ensemble of log spectra as sums of nonstationary smooth functions and a common additive signal whose periods are directly related to the time delays of the seismic phases. Detrending the log spectra reduces the problem to one of detecting a common signal with multiple periodicities in noise. Plotting an approximate cepstral F-statistic over pseudo-time yields a function that can be considered as a deconvolution of the seismic phases. We apply the array methodology to determining focal depths using three component recordings of earthquakes.

Key words: Cepstral F, array processing, signal detection, nuclear monitoring, earthquakes, depth estimation.

1. Introduction

One definitive way of ruling out seismic events as possible nuclear tests is to accurately determine the depth of the event, using the fact that nuclear explosions by their nature must be shallow, whereas earthquakes will be deep. The depths of events are usually estimated by measuring the time delay between the direct (P) and depth (pP and sP) phases. This time delay induces a periodicity in the sample spectrum that is directly related to the focal depth of the event. Hence, accurate determination of the delay time and subsequent depth can serve as a preliminary means for discriminating between earthquakes and explosions.

As an example, consider Figure 1, which shows an event in Northern Chile, as observed on a three component vertical array at Lajitas, Texas. Sampling is at 40 points per second and the data are filtered in a signal pass-band ranging from .6 to 4.5 cycles per second (Hz). The magnitude 5.3 event occurred in May,

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2000 and the depth, reported by the U.S. Geological Survey, was 57 km. For a depth of 57 km, the method of Kennett and Engdahl (1991) predicts an arrival of pP at 15.42 seconds or about 620 points at the sampling rate given above. The P and pP arrivals are marked approximately on the three components and there is obvious ambiguity in assigning the delay to be assigned to the second pP phase. The important feature of the second arrival that distinguishes from other apparent arrivals is that it must appear consistently on all three components. If the delay time is denoted by τ and there is a modification of the amplitude of the pP reflection by a multiplier θ, it is natural to express the received signal at each channel as $s(t) + \theta s(t-\tau)$, when $s(t)$ is the underlying signal.

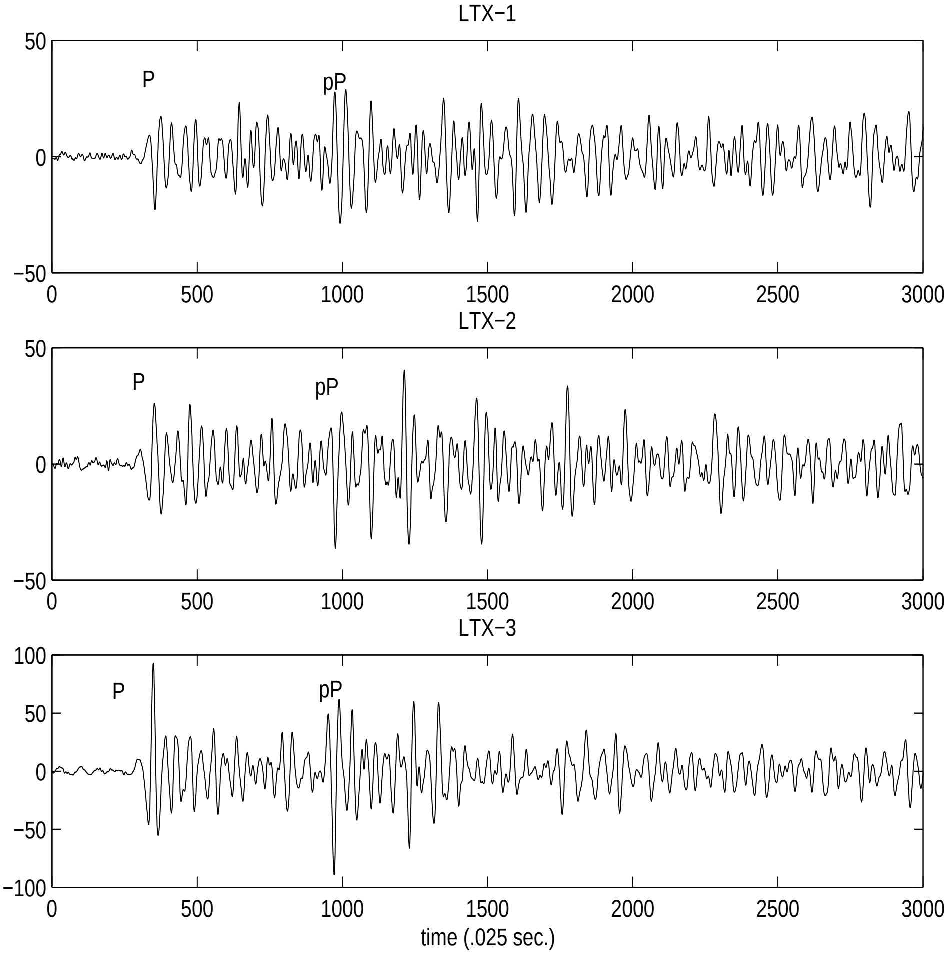

Figure 1: A Vertical and two horizontal components for northern Chile earthquake observed at Lajitas Texas. P and pP arrivals are visible after filtering (.6-4.5 Hz)

Since, the signal is received in echo form, it is natural to employ signal processing techniques that exploit this feature. The cepstrum was introduced as a technique for echo estimation by Bogert et al (1962) and has appeared in applications to speech and image processing as well as in seismology where a number of authors (see, for example, Kemerait, 1982, Baumgardt and Ziegler, 1988, Alexander, 1996, Shumway et al, 1998) have utilized it as a technique for modeling multiple arrivals. The idea behind the cepstrum as a tool for analysis is that there will be periodicities induced in the spectrum that are proportional to the delay times of the arrivals. These periodicities are often quite strong over a broad frequency range and are enhanced by looking at the log spectrum. In this paper, we exploit the above properties by thinking of the detrended log spectra at the different channels as the sum of a signal and noise, where the signal is roughly periodic and the same on each channel. This allows application of conventional methods for detecting a signal in a collection of stationarily correlated noise series as in Shumway (1971) and Shumway et al (1998).

In the next section, we develop a multiplicative signal and noise model that exhibits the log spectrum of the data in terms of an additive model as a function of frequency. In Section 3, the discrete Fourier transform (DFT) of the sample log spectra gives a signal plus noise model in the quefrency or pseudo-time domain that can be handled by the usual analysis of power techniques (see, for example, Shumway and Stoffer, 2000). The F-statistic obtained exhibits the echos at the proper delay times, giving the primary estimated output delay needed for determining depth. In section 4, the test procedure is applied to the Northern Chile earthquake shown in Figure 1.

2. Multiplicative Signal Models

We suppose here that N observed series $y_j(t), j = 1, 2, \ldots, N$ can be expressed as the convolution of a fixed unknown function $a_j(t)$ with a delayed stochastic unknown signal $s_j(t)$ and a noise process $n_j(t)$, assumed to be a linear process with square summable coefficients. The model for the observed data becomes

$$y_j(t) = a_j(t) \otimes [s_j(t) + \theta s_j(t-\tau)] \otimes n_j(t), \tag{1}$$

where we assume that the P phase reflection pP is delayed by τ points and scaled by a reflection parameter $|\theta| < 1$. The notation $a(t) \otimes b(t) = \sum_s a(s)b(t-s)$ denotes the convolution of the series $a(t)$ and $b(t)$. It is natural to handle (1) in the frequency domain because the theoretical spectrum of such a process will be of the form

$$\begin{aligned} f_{y_j}(\nu) &= |A_j(\nu)|^2 |1 + \theta \mathrm{e}^{-2\pi i \nu \tau}|^2 f_{s_j}(\nu) f_{n_j}(\nu) \\ &= |A_j(\nu)|^2 \big(1 + \theta^2 + 2\theta \cos 2\pi\nu\tau\big) f_{s_j}(\nu) f_{n_j}(\nu). \end{aligned} \tag{2}$$

where $A_j(\nu)$ is the Fourier transform of $a_j(t)$ and $f_{s_j}(\nu)$ and $f_{n_j}(\nu)$ are the spectra of the signal and noise respectively, with frequency ν measured in cycles per point over the range $-1/2 \leq \nu \leq 1/2$. The above form for the spectrum exhibits it as the product of multiplicative noise, a fixed signal function and a periodic component, with periodicities determined as a known function of the time delays. The dynamic range spanned by typical spectra tends to show the periodicities as being proportional to the magnitude of the spectral function. Taking logarithms helps stabilize the dynamic range and also leads to an additive model of the form

$$\begin{aligned} \log f_{y_j}(\nu) &= \log |A_j(\nu)|^2 + \log f_{s_j}(\nu) + \log\big(1 + \theta^2 + 2\theta \cos 2\pi\nu\tau\big) + \log f_{n_j}(\nu) \\ &= T_j(\nu) + \log\big(1 + \theta^2 + 2\theta \cos 2\pi\nu\tau\big) + \log f_{n_j}(\nu), \end{aligned} \tag{3}$$

where

$$T_j(\nu) = \log |A_j(\nu)|^2 + \log f_{s_j}(\nu) \tag{4}$$

trend function $T_j(\nu)$ is assumed to be smooth for each channel. In later arguments, we will identify the fixed additive function defined by the sum of the first two terms in (3) with a trend component that is different on each series. The common component in each series has the additive function whose period is proportional to the time delay τ. Hence it seems sensible to consider the Fourier transform of the log spectra as underlying data. For sampled data, consider modeling the $\log |Y_j(\nu)|^2 - T_j(\nu)$, where

$$Y_j(\nu_\ell) = n^{-1/2} \sum_{t=0}^{n-1} y_j(t) \mathrm{e}^{-2\pi i \nu_\ell t} \tag{5}$$

is the DFT of the original process and its squared value is the usual periodogram. We may use (3) at frequencies of the form $\nu_\ell = \ell/n, \ell = 0, 1, \ldots, n-1$ cycles per frequency point and think of the detrended version of (3) as a series in psueudo-time ν. Then, compute the sample periodogram again at delays of the form $d_k = k/n, k = 0, 1, \ldots, n-1$, i.e.,

$$C_j(d_k) = n^{-1/2} \sum_{\ell=0}^{n-1} \left(\log |Y_j(\nu_\ell)|^2 - T_j(\nu_\ell) \right) \mathrm{e}^{-2\pi i d_k \nu_\ell}. \tag{6}$$

The resulting sample cepstra should show peaks at delays corresponding to the periodicities in the spectra.

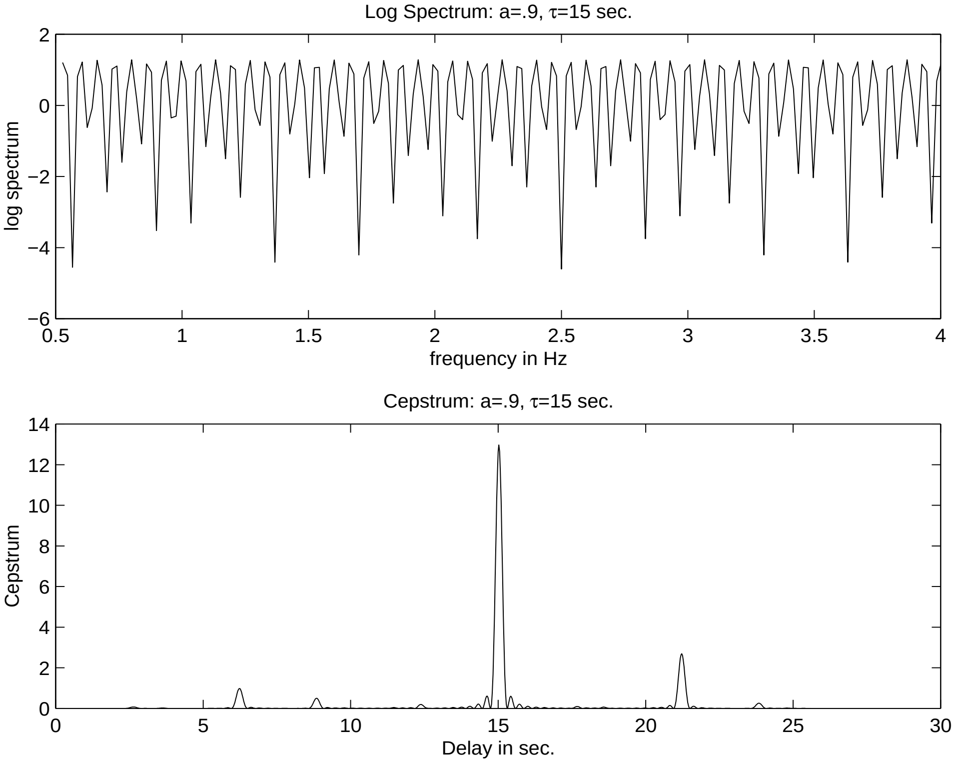

Figure 2: Plot of the function $g(\nu; \tau, \theta)$ in (7) for $\theta = .9, \tau = 600$ i.e, (top panel) and its predicted spectrum or cepstrum (bottom panel) showing pP arrival at about 15 seconds (600 points).

In order to check how this procedure might work in practice, consider the upper panel in Figure 2 which shows the function

$$g(\nu; \tau, \theta) = \log\left(1 + \theta^2 + 2\theta \cos 2\pi\nu\tau\right) \tag{7}$$

for $\tau = 600$ points and $\theta = .9$. The Fourier transform of $|g(\nu; \tau, \theta)|^2$ is shown in the lower panel and we note the peak at the correct delay of 15 seconds shows up in the component of (3) that contains the parameters of the reflection. There are, of course, some small peaks due to finite computations.

Before proceeding further to check the log spectra of the northern Chile earthquake as shown in Figure 3 for the bandwidth 1-5 Hz. It is clear that the periodicities noted for the underlying model all have strong trends which was the motivation for the term $\log |A_j(\nu)|^2$ in the model (3). However, there will always be a fairly smooth underlying trend function observed over the frequency band of interest. Hence, a cubic spline with a single knot is usual sufficient for detrending. In this context, we applied a regression spline model of the form

$$T_j(\nu) = a_{j0} + a_{j1}\nu + a_{j2}\nu^2 + a_{j3}\nu^3 + a_{j4}(\nu - \nu_0)_+^3, \tag{8}$$

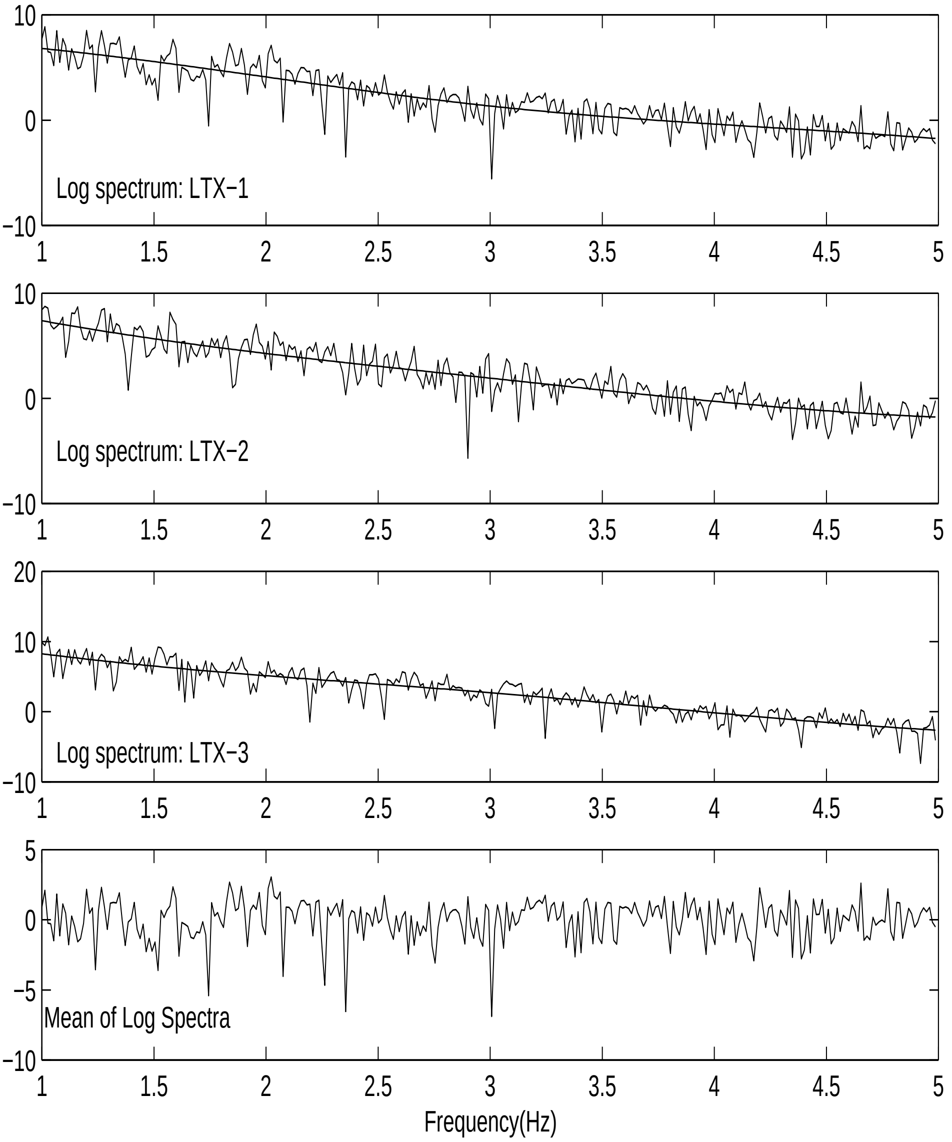

Figure 3: Log spectra for the three component data and the mean of the detrended log spectra. The smoother line is the cubic spline with one knot that was used to detrend the log spectrum. The mean series should show common periodicities corresponding to the pP time delay.

where ν_0 is the knot location and $(\nu-\nu_0)^3_+$ is zero for $\nu < \nu_0$. Placing the knot at the middle of the frequency leads to the smooth fitted lines shown in Figure 3. Adjusting each log spectrum for its fitted cubic spline at frequencies of the form $\nu_\ell = \ell/n$, say $\widehat{T_j(\nu_\ell)}$, and adjusting leads to a detrended series of the form

$$\log |Y_j(\nu_\ell)|^2 - \widehat{T_j(\nu_\ell)} = \log\left(1+\theta^2+2\theta\cos 2\pi\nu_\ell\tau\right) + \log f_{n_j}(\nu_\ell) \tag{9}$$

Before proceeding further, it is useful to consider the distribution theory for the residual spectra $f_{n_j}(\nu_\ell)$ in (9). We looked at the residuals from the mean detrended residuals shown in the bottom panel of Figure 3. The distributions were approximately normal for the first and third series and slightly skewed for the second. Some observations were quite far out in the left tails. The autocorrelation functions of all three residuals were essentially zero at all lags. The form of (9) also suggests a nonlinear regression approach with

θ and τ as the parameters but Figure 2 implies that the derivatives may be unstable so we opt for simply isolating periodicities via the nonparametric cepstral approach in the next section.

3. Cepstral Analysis of Power and the F Statistic

The residuals in (9) are nearly white and the DFTs (5) will be nearly Gaussian. Hence, the Fourier transform (6), applied to (9) will give a model of the form

$$C_j(d_k) = S(d_k) + N_j(d_k), \tag{10}$$

where the signal transform $S(d_k)$ have a peak at the time delay d_k corresponding to the periodicity in the function $g(\nu)$ in (7) (see Figure 2) . Noises will be uncorrelated, with variance equal to the cepstral variance $\sigma^2(d_k)$. Since we would like to determine the frequency where the primary signal lives, it is natural to apply the classical approach to detecting a fixed signal in noise, as proposed in Shumway (1971, 1998). Suppose that we ask for the test statistic for testing $S(d_0) = 0$ at a particular delay d_0, and suppose that we assume the variance of the noise to be constant, i.e. $\sigma^2(d)$ over some interval in the neighborhood of d_0. If L(odd) values of $\left\{C_j(d_0 + d_k), k = -\frac{L-1}{2}, \ldots, 0, \ldots, \frac{L-1}{2}\right\}$ are observed in some interval, define the total cepstral power as

$$\mathrm{TCP}(d_0) = \sum_{|k|<L/2} \sum_{j=1}^{N} |C_j(d_0 + d_k)|^2. \tag{11}$$

The total cepstral power, called the stacked cepstrum by Alexander (1996), has been used in the past for detecting time delays. We also write the mean or beam cepstral power as

$$\mathrm{BCP}(d_0) = N \sum_{|k|<L/2} |\bar{C}(d_0 + d_k)|^2, \tag{12}$$

where the mean is over $j = 1, 2, \ldots N$ channels. The cepstral noise power is

$$\begin{aligned} \mathrm{NCP}(d_0) &= \sum_{|k|<L/2} \sum_{j=1}^{N} |C_j(d_0 + d_k) - \bar{C}(d_0 + d_k)|^2 \\ &= \mathrm{TCP}(d_0) - \mathrm{BCP}(d_0). \end{aligned} \tag{13}$$

The statistic resulting from the test $C(d_0) = 0$ is given by

$$F_{2L,2L(N-1)}(d_0) = (N-1)\frac{\mathrm{BCP}(d_0)}{\mathrm{NCP}(d_0}, \tag{14}$$

which is distributed as central F with $2L$ and $2L(N-1)$ degrees of fredom when $\left\{S(d_0 + k) = 0, k = -\frac{L-1}{2}, \ldots, 0, \ldots, \frac{L-1}{2}\right\}$ and as non-central F otherwise. The non-centrality parameter is proportional to $2NL$ times the integrated signal power over the bandwidth and inversely proportional to the noise spectrum.

We may apply the procedure to the problem of determining the depth of the Northern Chile event shown in Figure 1. As mentioned, earlier the US Geological Survey estimated the depth at 57.2 km, which corresponded to a pP arrival 15.4 seconds after P. For this data the sampling rate is 40 points per second, leading to a folding frequency of 20 Hz. The data in Figure 1 were bandpass filtered to restrict the series to the interval from .6 to 4.5 Hz. The log spectra have already been exhibited in Figure 3 and we note that some periodicity is evident in the mean of the detrended log spectra. Note that the first three channels are $\log(|Y_j(\nu_\ell)|^2, j = 1, 2, 3$, whereas the last channel is the average of the three residuals on the left side of (9).

Two components of the cepstral power are shown in Figure 4, with the total power $\mathrm{TCP}(\cdot)$ as the dashed line and the beam power $\mathrm{BCP}(\cdot)$ as the solid line. For this analysis, we had 3000 data points and 1500

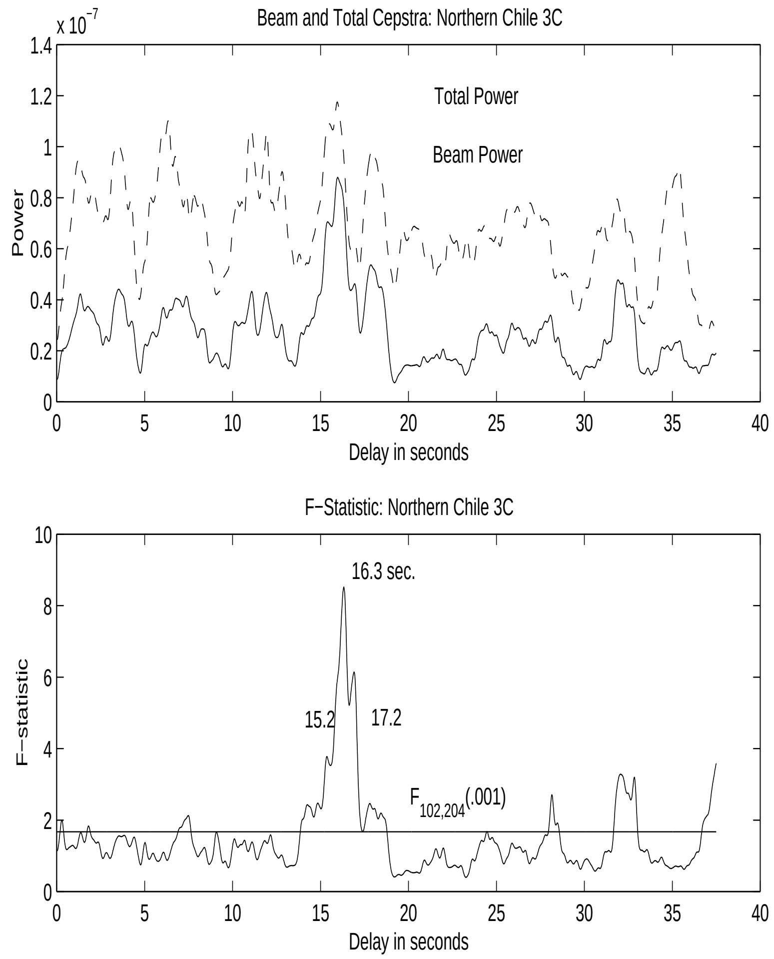

Figure 4: Analysis of cepstral power components (top panel) and F-statistics for northern Chile earthquake (bottom panel) showing estimated pP arrival at about 16 seconds. Estimated USGS depth for this event was 57.2 km, which would predict (Kennett and Engdahl, 1991) a travel time of about 15.5 seconds.

frequencies from zero to the folding frequency. Taking $L = 51$ would imply a bandwidth of about 1.25 sec in the pseudo-time domain. Since $N = 3$ in this case, the F-statistic shown in the bottom panel has $2(51) = 102$ and $2(51)(3 - 1) = 204$ degrees of freedom. We note the strong peak at 16.3 seconds, the .001 critical value $F_{102,204}(.001) = 1.66$ by a substantial amount. This peak corresponds to a depth of 61.1 km as compared to the quoted U.S. Geological Survey estimate of 57.2 km. If we suppose that upper and lower limits are defined as the narrow part of the peak, we might take 15.2 and 17.2 seconds as rough lower and upper limits; the depths corresponding to these two delays are 56.0 and 65.3 km. The broad limits implied suggest that determination of depth from delays may not be as accurate as assumed when time delays are read off the single series by an analyst.

4. Discussion

We have developed a nonparametric approach to estimating a single delay observed on multiple time series. The method is based on a decomposition of the detrended log periodogram into a smooth function specific to each component and a periodic function common to all components. The periodic function contains the delay information in its period. Transforming again to the pseudo time domain, we obtained a model implying that the transformed log periodogram could be represented as the sum of a deterministic signal and noise. Using known results for detecting a deterministic signal in stationary noise gave an F statistic that depended on the sum of the spectra and the spectrum of the mean.

Using the approach to estimate depth was illustrated for one single array recording a northern Chile earthquake. Large arrays and even combinations of arrays should lead to more accurate depth estimates. A large number of depth determinations using data from the Prototype International Data Center, Reviewed Event Bulletin and from the USGS bulletin were made in Bonner et al (2000). The results show that the method is highly successful for teleseismic distance, slightly less reliable at far regional distances and problematic at distances less than 10 degrees from the source. Other events, data and the software can be found on the website ***http://www.weston-geo.com/cepstral_data.html***.

5. Acknowledgements

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6. References

Alexander, S.S. (1996). A new method for determining source depth from a single regional station. *Seismic Research Letters*, 67, 63.

Baumgardt, D.R. and K. A. Ziegler (1988). Spectral evidence for source multiplicity in explosions: Application to regional discrimination of earthquakes and explosions. *Bull. Seismolog. Soc. Amer.*, 78, 1773-1795.

Bogert, G.M., M.J. Healy and J.W. Tukey (1962). The quefrency analysis of times series for echoes. In *Proceedings of a Symposium on Time Series Analysis*, ed. M. Rosenblatt. New York: John Wiley.

Bonner, J. L. D.T. Reiter and R.H. Shumway (2000). Application of a cepstral F-statistic for improved depth estimation. Proceedings of 22nd Annual DOD/DOE Seismic Research Symposium: Planning for Verification of and Compliance with the Comprehensive Nuclear Test Ban Treaty (CTBT). New Orleans, September 12-15, 2000.

Kemerait, R.C. and A.F. Sutton (1982). A multidimensional approach to seismic depth estimation. *Geoexploration*, 20, 113-130.

Kennett, B.L.N. and E.R. Engdahl (1991). Travel times for global earthquake location and phase identification. *Geophys. J. Int.*, 105, 429-465.

Shumway, R.H. (1971). On detecting a signal in N stationarily correlated noise series. *Technometrics,*, 10, 523-534.

Shumway, R.H., D.R. Baumgardt, and Z.A. Der (1998). A cepstral F statistic for detecting delay-fired seismic signals. *Technometrics*, 40, 100-110.

Shumway, R.H. and D.S. Stoffer (2000). *Time Series Analysis and Its Applications.* New York: Springer-Verlag.