



Non Linear Analysis of Spatial Time Series

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Introduction

- Agenda
- Motivation
 - Global climate change
 - Environmental science
 - Ecological science
- Contents



Contents

- Description of STARMA models
- Modeling procedure
- Order determination
- Application to real data
- Summary and Extensions



STARMA Model

- Pfeifer and Deutsch(1980)
- R.v. Z measured at N sites
- Incorporates spatial dependencies
 - Matrix of weights
 - System of neighborhoods



STARMA($p_{l_1, l_2, \dots, l_p}, q_{m_1, m_2, \dots, m_q}$)

$$z(t) = \sum_{k=1}^p \sum_{l=0}^{l_k} \mathbf{f}_{kl} W^{(l)} z(t-k) - \sum_{k=1}^q \sum_{l=0}^{l_k} \mathbf{q}_{kl} W^{(l)} \mathbf{e}(t-k) + \mathbf{e}(t)$$

- $Z(t)$: $N \times 1$, $t=1, \dots, T$
- $W^{(l)}$: $N \times N$

$$\mathbf{e}(t) \sim N(0, G)$$

$$G = \mathbf{s}^2 I$$

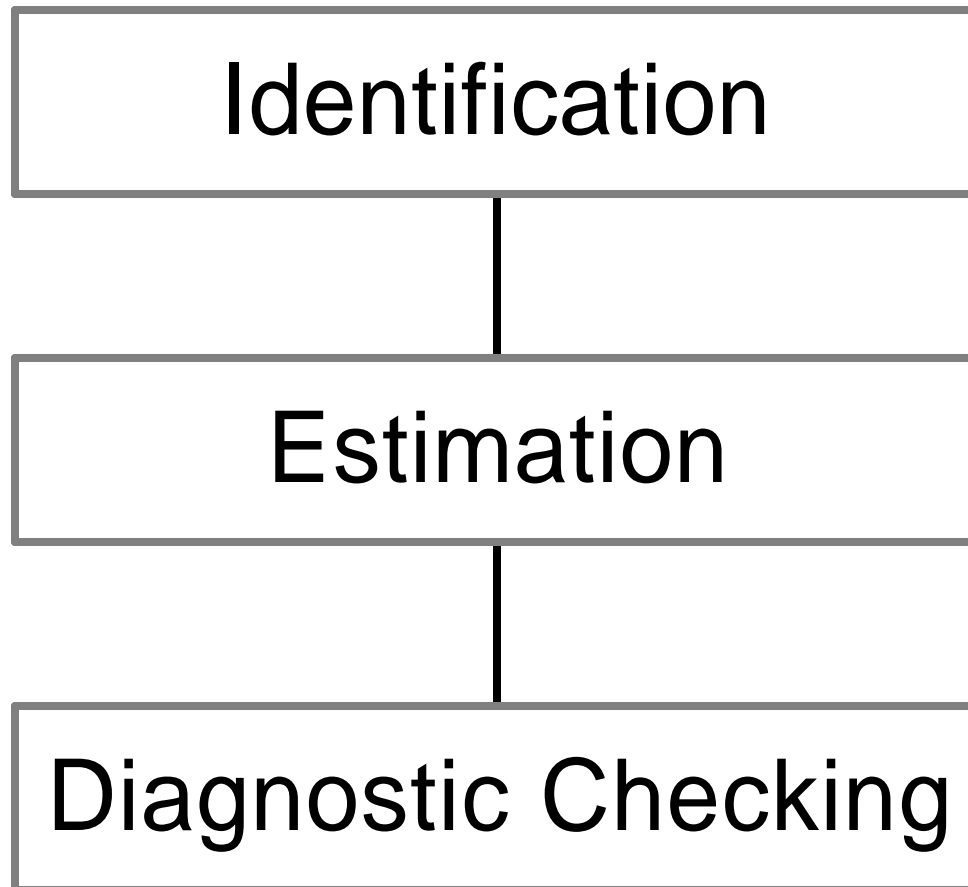


Weighting Matrices $W^{(1)}$

- Fixed beforehand
- Reflect physical features
- Regular spaced systems:
 - equal weights
- Irregularly spaced systems
 - inversely proportional to distance



Modelling procedure





Identification

- Space-time autocovariance function

$$\mathbf{g}_{lk}(s) = E \left\{ \frac{[W^{(l)} z(t)] [W^{(k)} z(t+s)]}{N} \right\}$$

- Space-time autocorrelation function

$$\mathbf{r}_l(s) = \frac{\mathbf{g}_{l0}(s)}{[\mathbf{g}_{ll}(0)\mathbf{g}_{00}(0)]^{\frac{1}{2}}}$$

- Sample estimates

$$\hat{\mathbf{g}}_{lk}(s) = \frac{1}{N} \sum_{t=1}^{T-s} \frac{[W^{(l)} z(t)] [W^{(k)} z(t+s)]}{T-s}$$



Identification

- Space-time analogue of Yule-Walker equations

$$\mathbf{g}_{h0}(s) = \sum_{j=1}^k \sum_{l=0}^l \mathbf{f}_{jl} \mathbf{g}_{hl}(s-j)$$

$$s = 1, \dots, k$$

$$h = 0, \dots, l$$

Space-time partial autocorrelation function: $\phi_{k\lambda}$



Estimation

- Assumption: orders are known
- Conditional maximum likelihood estimation
- $\Phi=(\phi)_{kl}$, $\Theta=(\theta)_{kl}$, σ^2 maximize:

$$l_*(\Phi, \Theta, \mathbf{s}^2) = -\frac{TN}{2} \ln(2p) - \frac{TN}{2} \ln(\mathbf{s}^2) - \frac{S_*(\Phi, \Theta)}{2\mathbf{s}^2}$$

Minimize $S_*(\Phi, \Theta) = \mathbf{e}' I \mathbf{e} = \sum_{i=1}^N \sum_{t=1}^T \mathbf{e}_i^2(t)$



Estimation Issues

- Nonlinear nature of the problem with STMA terms
- Initial estimates
- Order determination



Initial Estimates and Order Determination

- Hannan and Rissanen(1982) - ARMA
- Extension to STARMA

- High order STAR($k_1, \dots, 1$)

- Y-W

$$BIC = \ln\left(\left|\tilde{\Sigma}\right|\right) + 2m \frac{\ln(T)}{T}$$

- Determine residuals

- Write model in general linear form : $Y=X\beta+\varepsilon$

- $\Phi, \Theta:(p,q)$ minimize BIC

- Numerical estimation of Φ, Θ



Diagnostic Checking

- 95% confidence intervals for the residuals space-time autocorrelations

$$\text{var}(\hat{\mathbf{r}}_{10}(s)) \approx \frac{1}{N(T-s)}$$

- Test the hypotheses that $\mathbf{G} = \sigma^2 \mathbf{I}$

- Use $M = T^{-1} \sum \hat{\mathbf{e}}_t \hat{\mathbf{e}}_t'$



Test the Hypotheses $G=D$

- Sample covariance matrix of the residuals

$$M = T^{-1} \sum \hat{e}_t \hat{e}_t'$$

- Test statistics:

– $H_0: G = \sigma^2 I$

– $H_0: G = D$

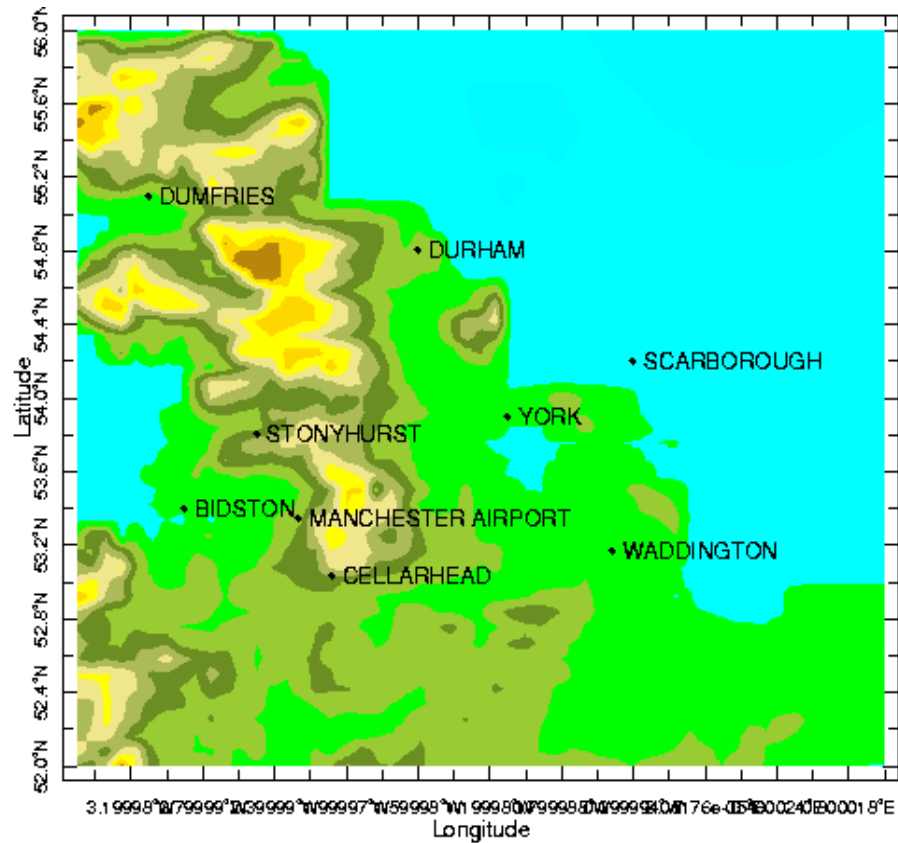
$$U = \frac{|M|}{\left[\frac{\text{tr}(M)}{N} \right]^N}$$

$$U = \frac{|M|}{\prod_{i=1}^N M_{ii}}$$



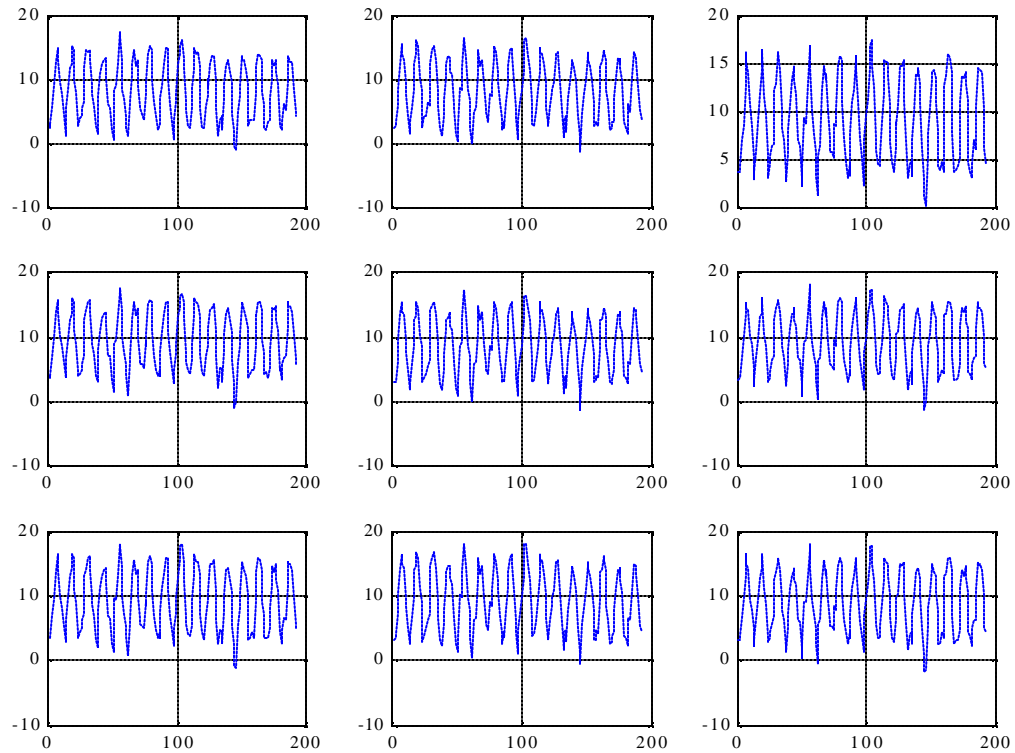
Data

- Monthly mean temperatures 1951-1966





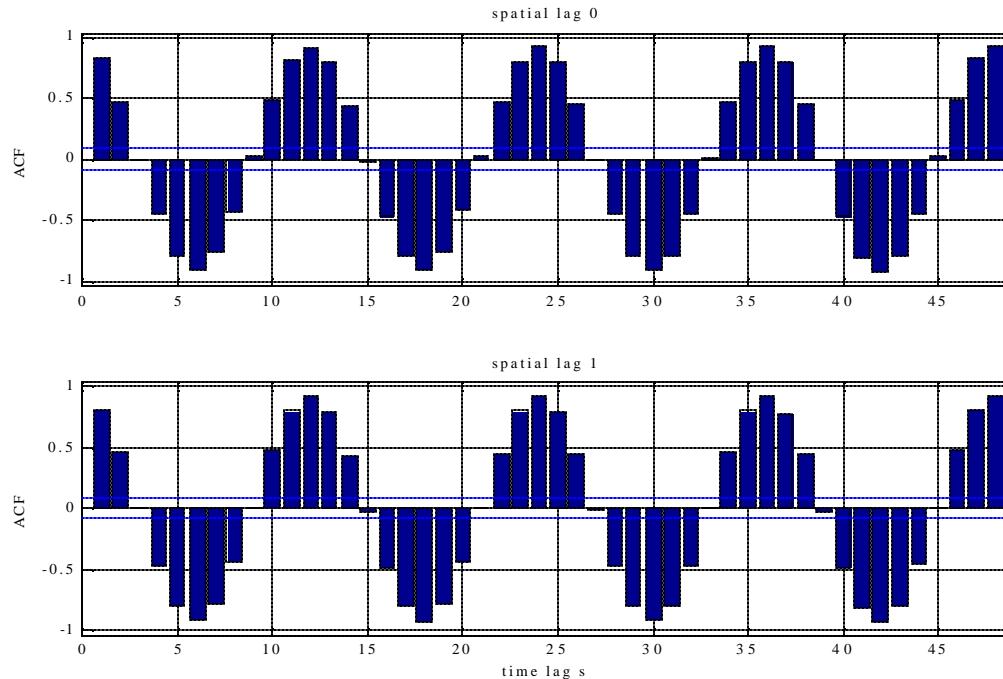
Data





Identification

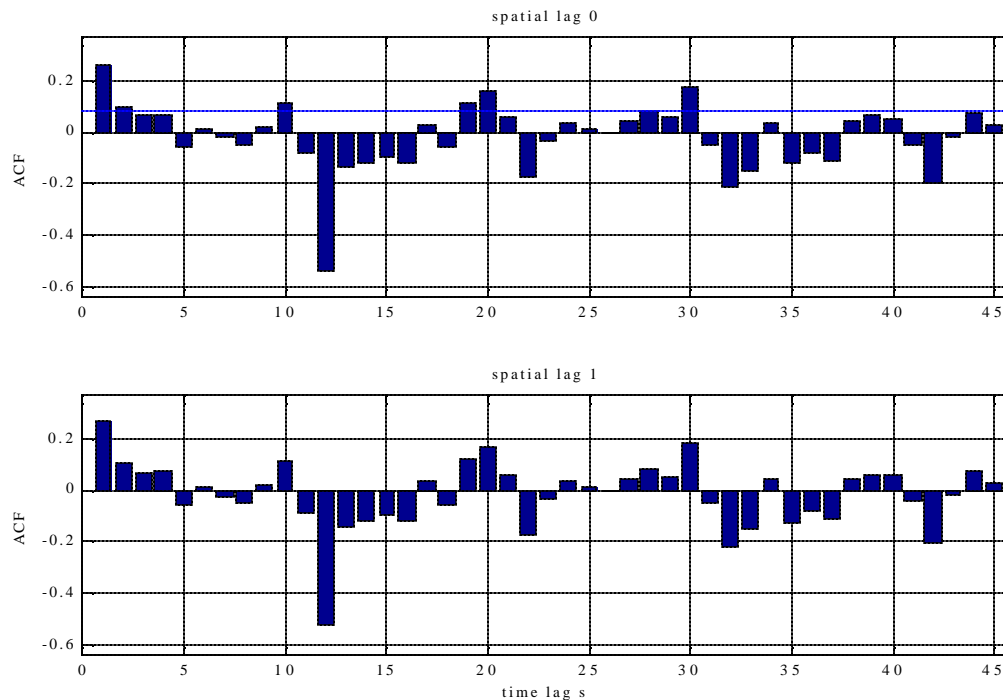
- Assumption : $G = \sigma^2 I$



Space-time autocorrelation function of original data



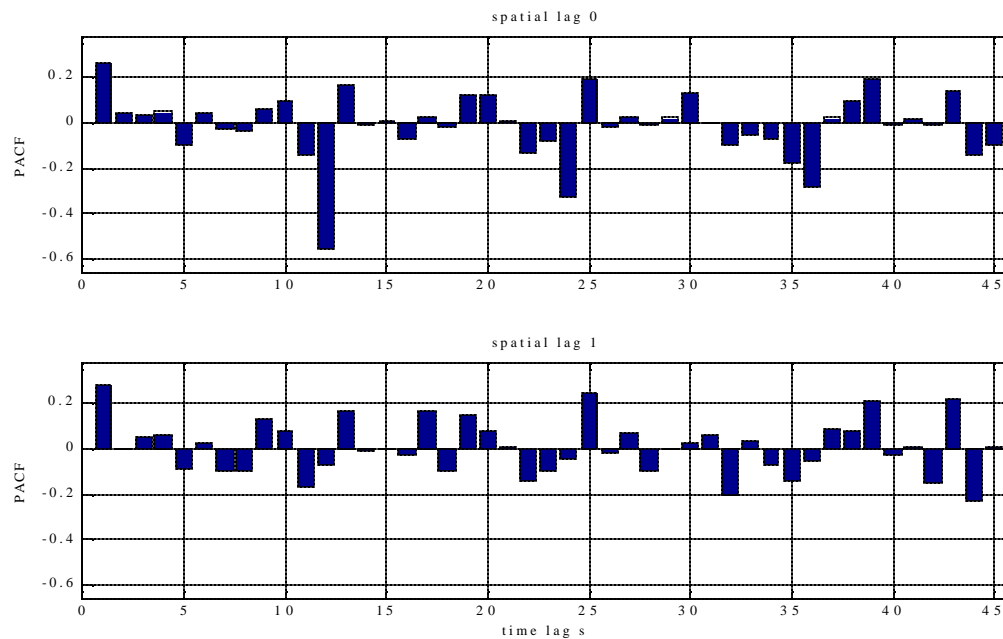
Identification



Space-time autocorrelation function of deseasonalised series



Identification



Space-time partial autocorrelation function of deseasonalised series

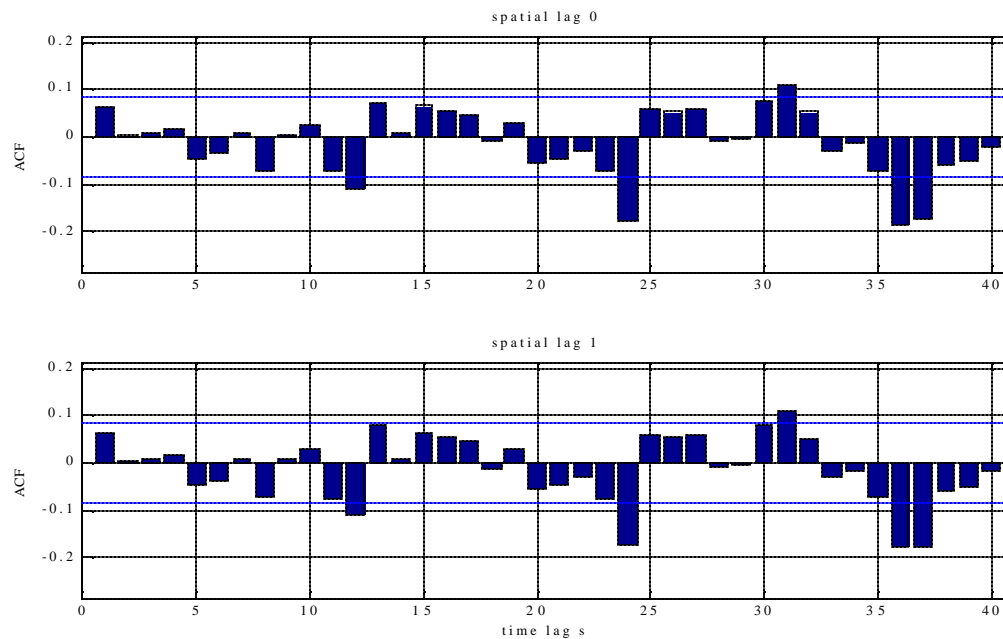


Estimation Results

- Assumption : $G = \sigma^2 I$
- STAR(24) spatial order 1



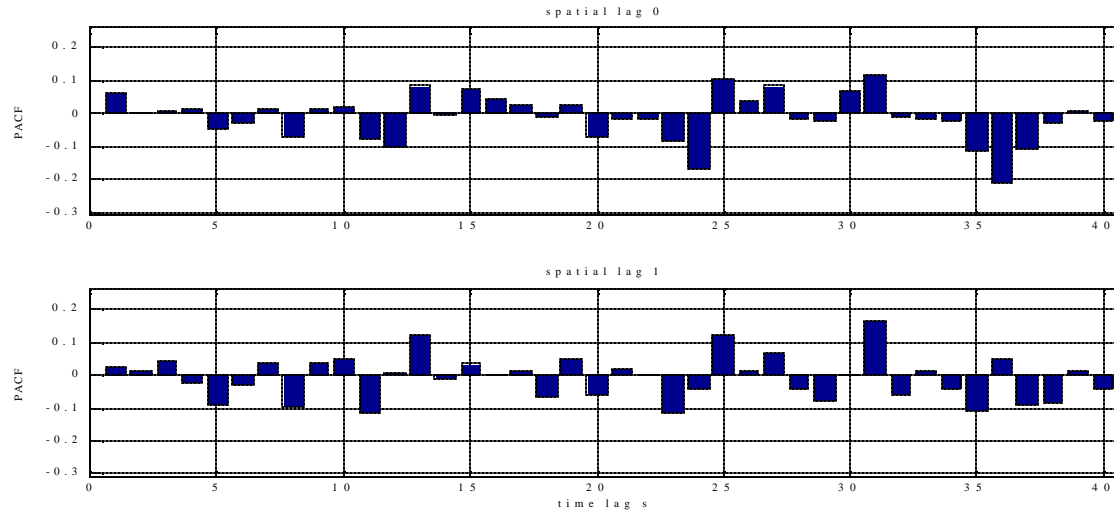
Diagnostic Checking



Space-time autocorrelation function of the residuals



Diagnostic Checking



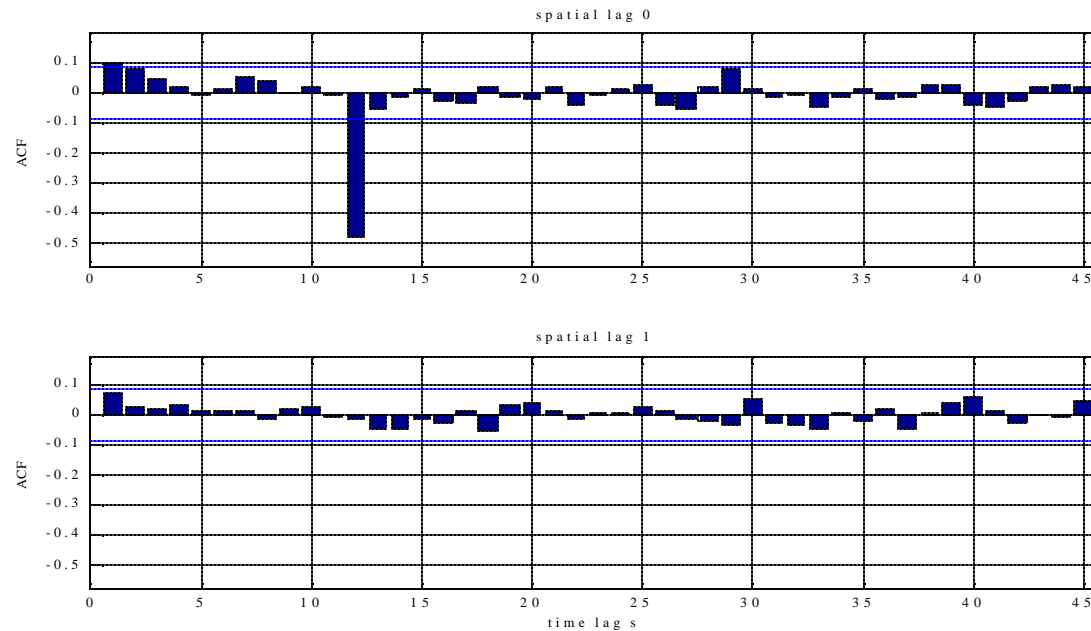
Space-time partial autocorrelation function of the residuals

Hypotheses $G = \sigma^2 I$ rejected



Identification

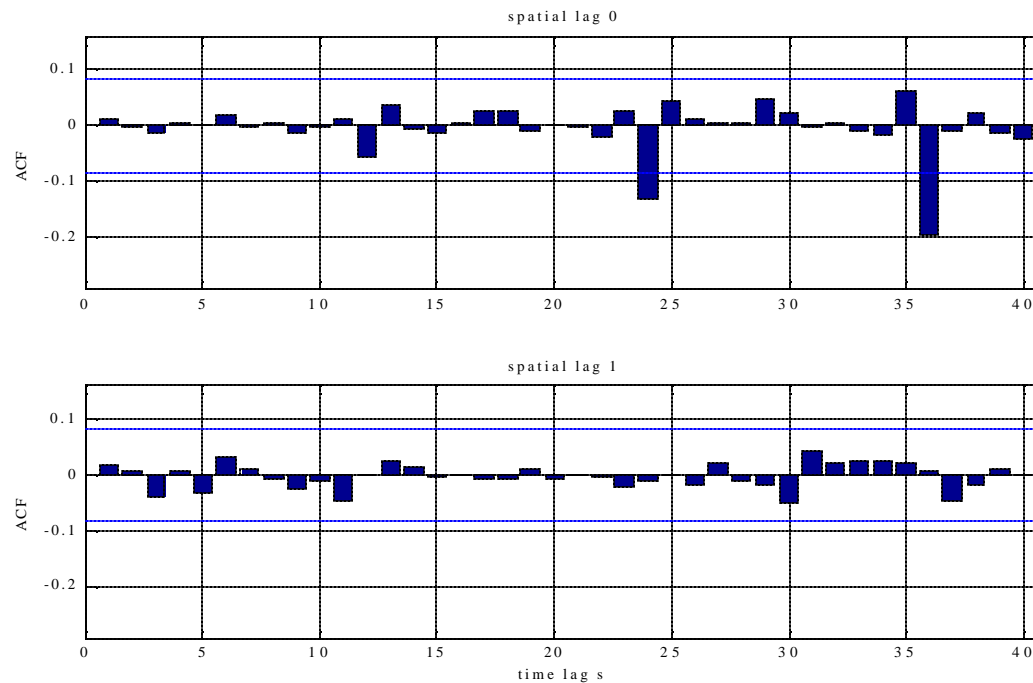
- Assumption : general G



Transformed Space-time autocorrelation function of the deseasonalised data



Diagnostic Checking



Space-time autocorrelation function of the residuals



Summary and Extensions

- Stationarity
- Gaussianity
- Linearity
- Periodically correlated spatial time series
- Non-gaussian – Pareto distribution
 - Subba Rao and Wong(1997)
- Non-linear – Bilinear
 - Dai and Billard(1998)