Random Domain Decompositions for Stochastic PDEs with Multiplicative Noise

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Stochastic PDEs & Physical Systems

- Most physical systems are fundamentally stochastic (Frish, 1996; Papanicolaou, 1973; Van Kampen, 1976).
  - Earthquakes and seismic monitoring
  - Environmental remediation
  - Manufacturing of composite materials
  - Catalysis
  - Agglomeration of particles

- Qualitative understanding vs. Quantitative prediction

- Quantitative methods:
  - Monte Carlo simulations
  - Analytical methods

System Parameters

- Real systems:
  - Multiplicative noise
  - Multi-modal distributions
  - Statistical heterogeneity
  - Complex correlation structures

- Classical models:
  - Additive noise
  - Uni-modal idealized distributions, e.g., Gaussian
  - Statistical homogeneity
  - Simplified correlation structures, e.g., white noise

Additive vs. Multiplicative Noise

$$\mathcal{L}_{\{p\}} u = f$$

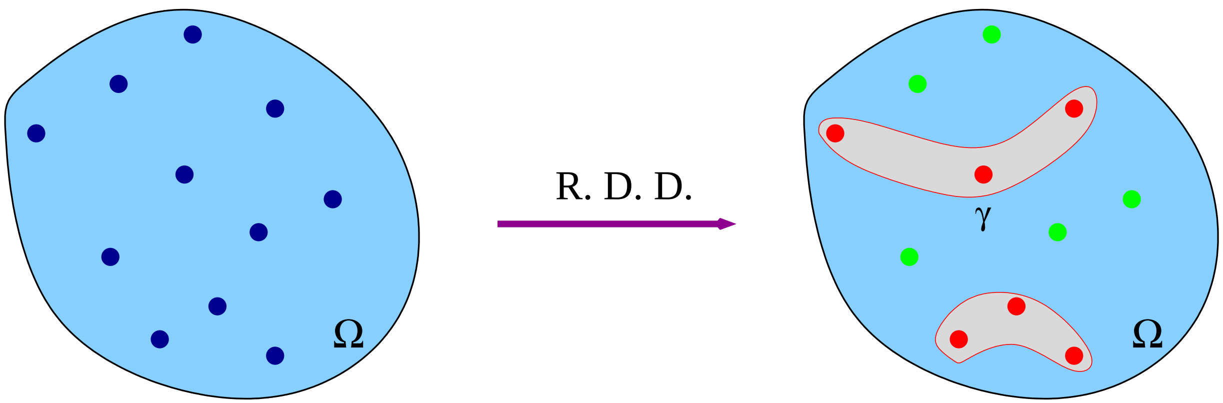
- Additive noise
  - Random driving forces
  - Deterministic parameters

$$\mathcal{L}_{\{p\}} \langle u \rangle = \langle f \rangle$$

- Multiplicative noise
  - Random parameters and driving forces
  - Closure approximations

$$\mathcal{L}_{\{\langle p \rangle\}} \langle u \rangle + \mathcal{L}^* \langle \{p\}, u \rangle = \langle f \rangle$$

Random Domain Decomposition



$p(\{\Pi\})$

$p(\{\Pi\}, \Gamma)$

Multi-modal distributions
High variances
Complex correlation structures

Uni-modal distributions
Low variances
Simple correlation structures

Strategy for Domain Decomposition

- Two-stage approach:
  - Step 1

$$\int \mathcal{L}_{\{\Pi\}} u \, p(\{\Pi\}|\gamma) \, d\{\Pi\} \quad \rightarrow \quad \langle u|\gamma\rangle$$

  - Step 2

$$\langle u\rangle = \int \langle u|\Gamma\rangle \, p(\Gamma) \, d\Gamma$$

Impact

- Natural way for incorporating statistical heterogeneity and multiplicative noise
- Robust approximations for statistics of the system parameters and states
- Incorporation of physical intuition and “expert knowledge”
- Novel definition of “effective” parameters
- Analysis of random geometries delineating random fields
- Ability to make predictions and to quantify uncertainty in realistic applications

Preliminary Results

- Random geometries and random fields (Tartakovsky and Winter, *SIAM J. Appl. Math.*, 61, 2001)

- Composite media (Winter and Tartakovsky, *Geophys. Res. Lett.*, 27, 2000; Winter et al., *Surv. Geophys.*, 2001, under review)

- Nonlinear diffusion (Tartakovsky and Guadagnini, *Phys. Rev. Lett.*, 2001, under review)

- Effective parameters (Tartakovsky et al., *Transp. Porous Media*, 2001, under review)