A Lagrangian Cost Model for a Fractal Transform Compressor

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January 19, 2001

Introduction

- Image compression is a trade-off between achieving the optimal rate and the resultant quality.

- Propose a design (unimplemented) for a fractal still image compressor based on an affine video compression system.

Outline

- The “classical” fractal transform
- Rate-distortion theory and Lagrangian costing
  - Optimal codebook generation
- Quad-tree partitioning
- De-blocking technique (Fuzzy Pixels)

The Classical Fractal Transform

Fractal Transform

- Image is partitioned into (square) domain blocks
- To each domain associate:
  - a range block of twice the size
  - an additive (brightness) adjustment **q**
  - a multiplicative (contrast) adjustment **p**
  - A dihedral symmetry operation

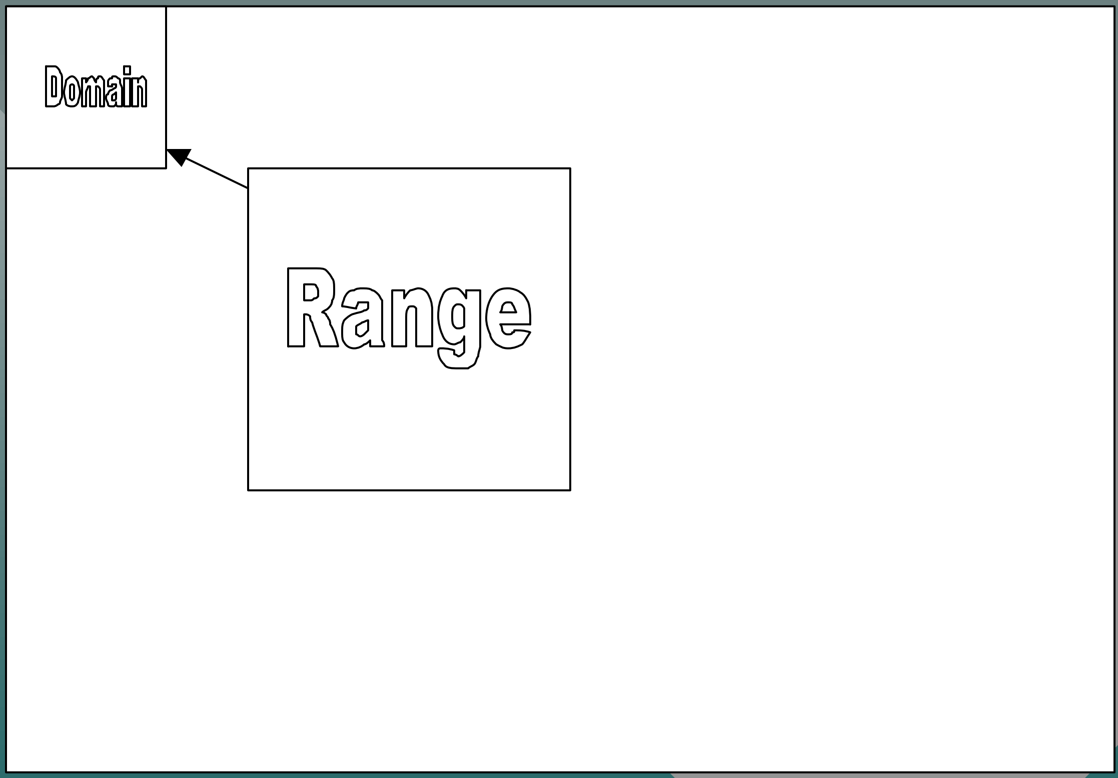
Simplifications

- Will assume:
- **p** is held to be constant (= 3/4)
  - all that is required for convergence is |**p**| < 1
  - can assume that range screen is computed once
- The identity symmetry

Image is recovered by:

- Set all pixels to a known initial state (such as neutral gray)
- Iterate:
  - Replace domain pixels by range pixels
    - downsampled spatially by 2
    - multiplied by p
    - added to q (q varies block by block)

Fractal Transform



Collage Theorem

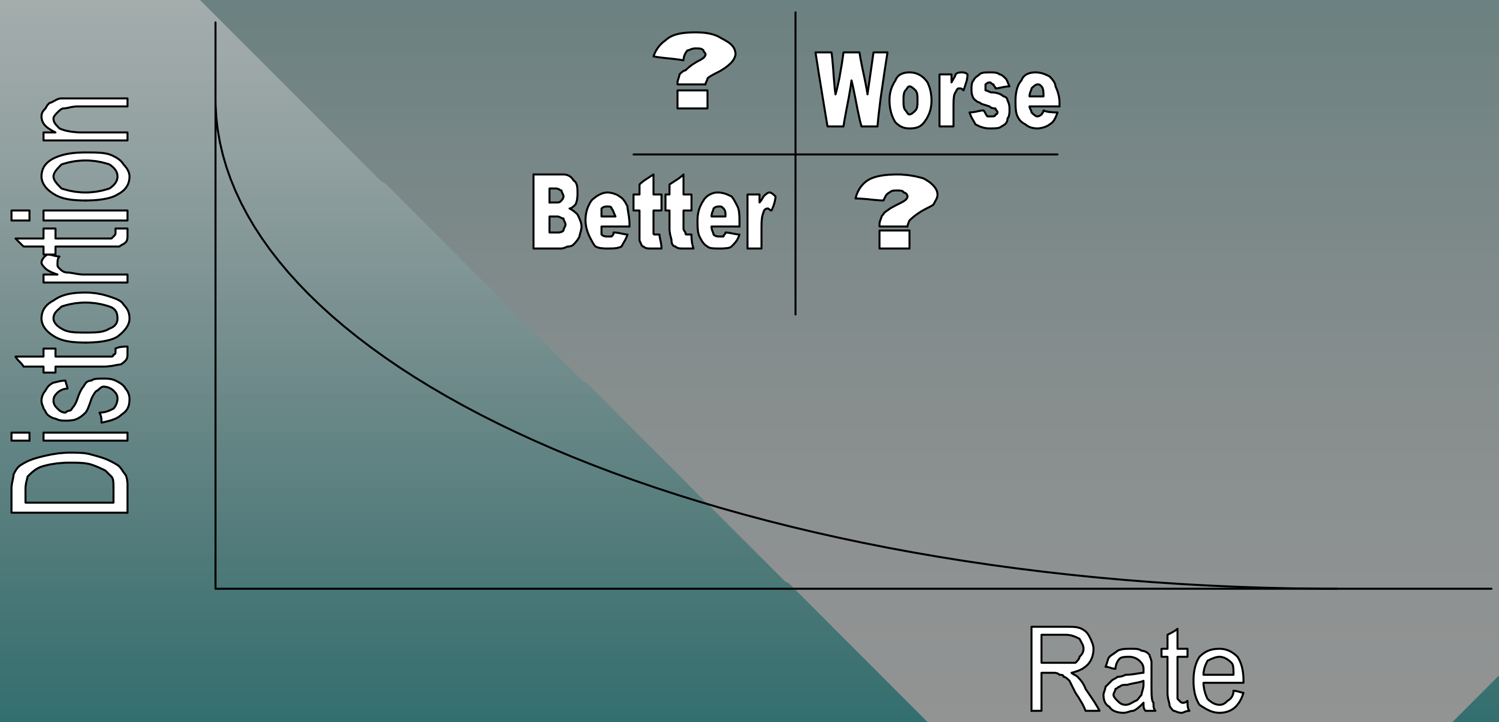
- As long as |**p**| < 1 for all blocks, the above procedure converges to a fixed point
- Distance from a transformed range block to the corresponding block in the original is the “collage error”

Dependencies

- Collage error is not the same as final decompressed error
- High correlation
- The measure under our direct control

Information Theory and Costing

R-D Curve



How do we compress

- Choose a fixed distortion metric such as SSE (sum of squared errors)
- Pick domain block and **q** to minimize this metric

Information Theory

- Everything to date has been solely geometric
- Encoding method completely independent of cost of sending description

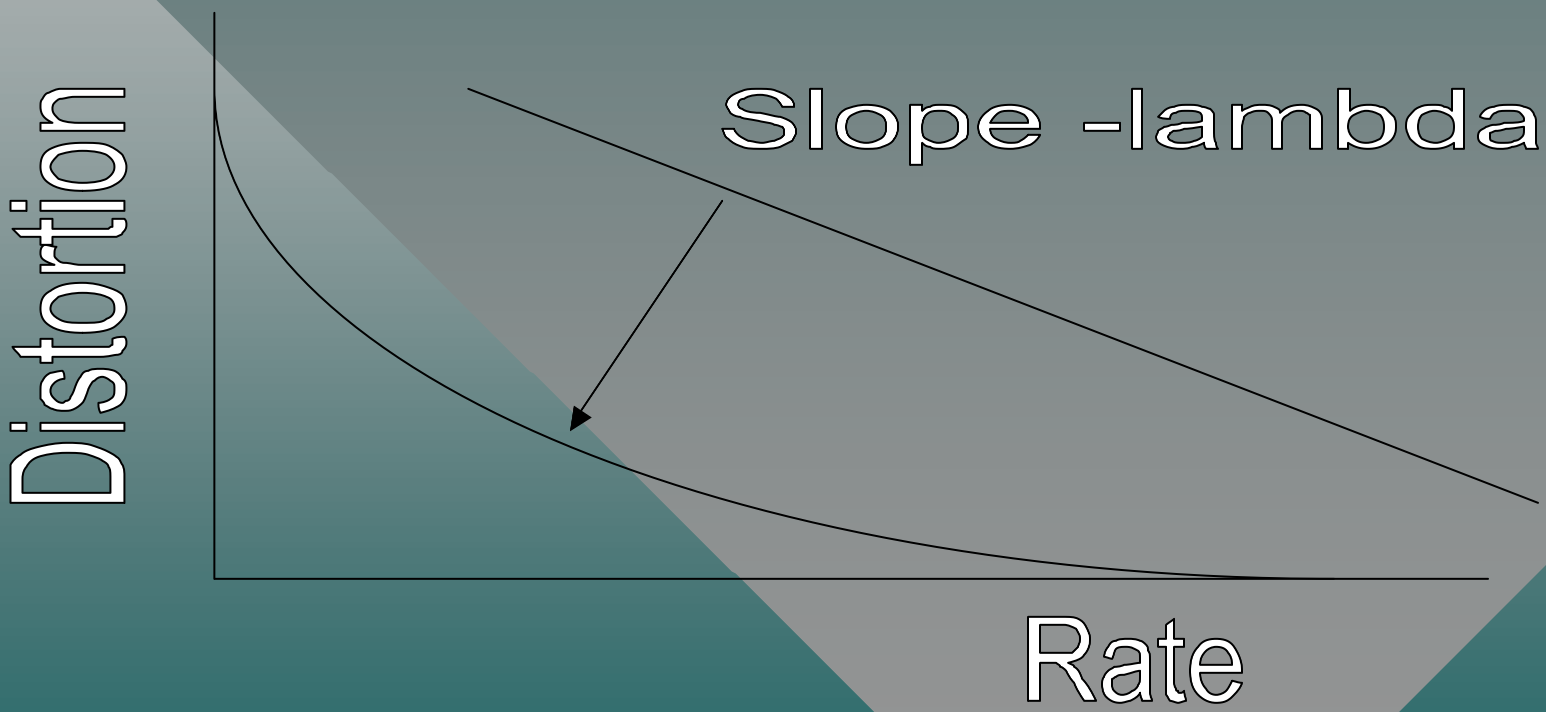
To encode:

- Choose range block from a discrete set of candidates
  - usually locally determined around block
  - Choose **q** from a fixed set of intensity displacements (usually -255 to 255)
- Classically, representation requires $-\mathbf{\log_2 f}$ bits where **f** is the probability (normalized frequency)

Lagrangian Costing

- Choose a positive $\lambda > 0$
- Pick candidate which optimizes cost: $C = D + \lambda R$
  – C = cost
  – D = distortion
  – R = rate

R-D Curve with costing



How do we establish rate?

- Circular (rate determines probability determines rate)
- Set probability distribution to uniform
- Compute optimal codes via formula
- Observe probabilities
- Iterate with new probabilities

Observations

- Problem does not depend on simplifying assumptions above
- In presence of costing, adding parameters always improves compression (while increasing complexity)

Problems

- Calculation is dependent on lambda
  - “Solution” is to use a typical lambda
- Calculations can converge to a local minimum
  - Simulated Annealing
- Use simplified variables such as x and y displacement

Can improve Performance via Deterministic Annealing

- Keep track of expected values rather than tracking a discrete histogram

Quad-tree Partitioning

Partial Quadtree

- To each block assign a decomposition taken from one of 16 possibilities
- Shape determines how many child blocks with respect to this parent

Shapes


Parent Parent Parent Parent
Parent Parent Parent Child
Parent Parent Child Parent
Parent Parent Child Child
Child Parent Parent Parent
Child Parent Parent Child
Child Parent Child Parent
Child Parent Child Child
Parent Child Parent Parent
Parent Child Parent Child
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Optimization

- Two basic approaches:
  - Top-down
  - Bottom-up
- The latter is optimal

Bottom-Up Compression

- Assign a cost-optimal code to each smallest subblock
- For each group of four quadrants decide on optimal shape (possibly merging subblocks)

Details

- If distortion is SSE (sum squared errors) the calculation can be done independent of **q** by change of variables.
- By making range address the outer loop, one can optimize shape for each choice of range block.

Hiding block boundaries

Fuzzy Pixels

- Instead of pasting range block, add it
- Range is multiplied by a 3x3 mask
- By adding copies of mask, can obtain mask for any blocksize
- Generalization of Overlapped Block Motion Compensation (OBMC) used in H.263 (video standard)

3x3 Mask (for 1x1 block)

2/25	3/25	2/25
3/25	5/25	3/25
2/25	3/25	2/25

6x6 Mask (for 4x4 block)

2/25	5/25	7/25	7/25	5/25	2/25
5/25	13/25	18/25	18/25	13/25	5/25
7/25	18/25	1	1	18/25	7/25
7/25	18/25	1	1	18/25	7/25
5/25	13/25	18/25	18/25	13/25	5/25
2/25	5/25	7/25	7/25	5/25	2/25

Advantages

- Applicable at all length scales
- Deblocking explicitly calculated in distortion measure

Conclusions

Conclusions

- Lagrangian costing cleanly makes decisions between coding methods
- Major advantage is guarantee of a point on convex hull of R-D curve
- Disadvantage is uncertainty of exact rate (can be approached iteratively)

Remarks

- Optimization all based on collage error
- Local activity measure can be handled by changing lambda locally

To be done...

- Implement a fractal compressor according to design outlined
- Evaluate versus existing fractal compressors

Main Reference

- S. Calzone, K. Chen, C.-C. Chuang, A. Divakaran, S. Dube, L. Hurd, J. Kari, G. Liang, F.-H. Lin, J. Muller, H. K. Rising III
- “Video Compression by Mean-Corrected Motion Compensation of Partial Quadtrees”
  - IEEE Trans. On Circuits and Systems for Video Technology, Feb 1997, Vol. 7, No. 1 pp 86-96