

Köln, den 16.05.2001

Scientific report: Visit from August 2000 until March 2001 at the “Institute for Mathematics and its Applications (IMA)”**I. Description of the project:**

- a) Movement is a fundamental process for almost all biological organisms, ranging from the single cell level to the population level, and two major classes of models are widely used to describe movement. In space-jump processes movement is a sequence of position jumps at random time intervals, while in velocity-jump processes movement consists of straight-line motion punctuated by random changes in velocity at random times. Space jump processes include the familiar lattice walks, and the simplest nearest-neighbour version in one space dimension leads to the master equation

$$\frac{dp_i}{dt} = \mathcal{T}_{i-1}p_{i-1} - 2\mathcal{T}_i p_i + \mathcal{T}_{i+1}p_{i+1} \quad (1)$$

for the probability density p_i . When the transition rates \mathcal{T}_i are constant across the lattice this leads in the usual way to the diffusion equation for the density $p(x, t)$. However if \mathcal{T} depends on a control substance w resp. on the density p itself the equation becomes a negative cross-diffusion resp. forward-backward parabolic problem. Thus one immediately has to think about the well-posedness of the problem and the question raises whether the consideration of the continuous PDE instead of the discrete formulation is legitimate or not. In a joint project H.G. Othmer, K.J. Painter and I studied the discrete and the continuous problems for a concrete transition rate \mathcal{T} which depends on the density p itself. We compared the results for the discrete with the results for the continuous problem and were able to show that in our concrete case it is reasonable to study the limiting continuous instead of the discrete problem.

- b) During my visit at the IMA I also participated in several workshops organized during the IMA annual program 2000 – 2001 “Mathematics in Multimedia”. To participate in the workshops gave me the opportunity to get a good insight in this interesting research field. The extremely interesting workshops on image processing encouraged me to start with some problems related to the wellknown Perona-Malik model. In this connection S. Esedoglu and I started to discuss reasonable approximations of the Perona-Malik model. S. Esedoglu and I plan to continue our discussions about this topic.

II. Collaborators:

- a) Hans G. Othmer, *School of Mathematics, University of Minnesota, Vincent Hall 207a, 206 Church Street S.E., Minneapolis, MN 55455, USA*
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- b) Selim Esedoglu, *Institute for Mathematics and its Applications (IMA), 400 Lind Hall, 207 Church Street S.E., Minneapolis, MN 55455-0436, USA*

III. Publications:

- i. “*Localization in Lattice and Continuum Models of Reinforced Random Walks*”, joint work with H.G. Othmer and K.J. Painter (in preparation)
- ii. “*Aggregation under Local Reinforcement: From Lattice to Continuum*”, joint work with H.G. Othmer and K.J. Painter (in preparation)