Well posedness and singular limits for nonlinear hyperbolic systems

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Recent work on hyperbolic systems of conservation laws has established the well posedness of the Cauchy problem,

$$u_t + f(u)_x = 0, u(0, x) = u_0(x),$$

in a class of functions with small total variation. Namely, entropy weak solutions are unique and depend Lipschitz continuously on the initial data, in the L^1 norm.

A major remaining open problem is whether these entropic solutions can be obtained as singular limits from various types of approximations. In particular:

1) Vanishing viscosity

$$u_t + f(u)_x = \epsilon u_{xx}$$

2) Finite difference schemes

3) Relaxations

$$U_t + AU_x = \frac{1}{\epsilon} g(U)$$

This talk will present some recent progress in this direction, and outline a general strategy for obtaining this kind of results.

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