Instability in a combustion model with free boundary in $I\!R^2$

Claude-Michel Brauner
Mathématiques Appliquées de Bordeaux, Université Bordeaux I
33405 Talence cedex, France
brauner@math.u-bordeaux.fr

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In the theory of NEF's (Near-Equidiffusional Flames), the system for temperature θ, enthalpy S, and the flame front defined by $x = s(t,y)$ read

$$\frac{\partial \theta}{\partial t} + \frac{\partial \theta}{\partial x} = \Delta\theta, \;\; x < s(t,y), \quad \theta = 1, \;\; x \geq s(t,y), \tag{1}$$

$$\frac{\partial S}{\partial t} + \frac{\partial S}{\partial x} = \Delta S - \lambda\Delta\theta, \;\; x \neq s(t,y). \tag{2}$$

At the front, θ and S are continuous, while the following jump conditions occur for the normal derivatives:

$$\left[\frac{\partial \theta}{\partial n}\right] = -\exp(S), \quad \left[\frac{\partial S}{\partial n}\right] = \lambda\left[\frac{\partial \theta}{\partial n}\right]. \tag{3}$$

We prove the instability of the planar travelling wave for $\lambda > 1$. The method is based on the elimination of the front $s(t,y)$: in fact the system (1)–(3) is equivalent to a *fully nonlinear* system

$$\mathbf{u}_t = L\mathbf{u} + F(\mathbf{u}), \quad B(\mathbf{u}) = G(\mathbf{u}), \tag{4}$$

where $\mathbf{u} = (v,w)$ and $s(t,y) = [v](t,y)$. The main difficulty is due to the spectrum of the linearized operator, which contains an interval $[0,\omega_c]$, so that we cannot construct backward solutions. We use an argument about instability of dynamical systems to prove pointwise instability of the front.