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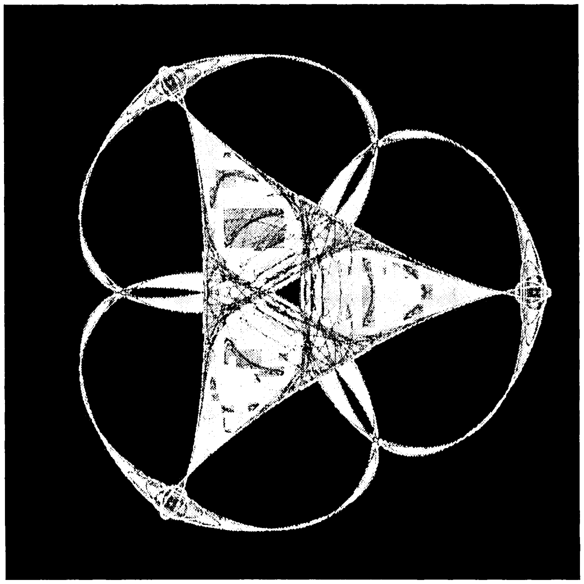

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FORMATION OF FINITE MULTIPLICATIVE GROUP FOR n- LATERAL MULTIPLE HYPERGEOMETEIC FUNCTIONS

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Some properties of special functions from the point of view of group theory, or more specifically, from the theory of group representations have been discussed in research monographs. Here an attempt has been made to generate finite multiplication group for multiple hypergeometric functions involving two and three variables.

Key Words :- Lauricella function / multiplicative group / multiple hypergeometric function / identity element.

1 :- INTRODUCTION

The fruitful nature of the theory of single hypergeometeic functions has led to generalizations, giving rise to multiple hypergeometric functions compiled by Exton[3]. Equivalent systems of partial differential equations associated with triple hypergeometric functions have been again constructed by Exton[4,5]. A new function has been generated by authors in their earlier studies[1] to meet the requirements of statisticians, whose product in the Lauricella functon pattern has led to the generation of n-lateral multiple hypergeometric function. This general function[2], whose particular cases include almost all known multiple hypergeometric functions, till date, is being represented in a numeral form for the formation of a finite multiplicative group.

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2:- GENERAL FORMULATION

Let us express the B- function, defined as

$$B\,{}^{p\ \ r}_{q\ \ s}\left[\begin{matrix}\alpha_p & \gamma_r \\ ; & ; \\ \beta_q & \delta_r\end{matrix} ; \quad x_1, x_2----, x_n\right]$$

$$= \sum_{m_1, m_2---m_n=0}^{\infty} \left(\frac{(\alpha_p)_n \quad (\gamma_r)_{-n}}{(\beta_q)_n \quad (\delta_s)_{-n}}\right) \frac{x_1^{m_1} x_2^{m_2}---x_n^{m_n}}{m_1!\ m_2!---m_n!} \qquad \text{.....................(2.1)}$$

Where $n = m_1 + m_2 + \ldots m_n$

in the following form

$$\sum_{m_1,m_2,----m_n=0}^{\infty} \left(\frac{(\alpha^1{}_p)_{a_1\Sigma m_1 + m_1 \Sigma a_2}(\alpha^2{}_p)_{a_1 \Sigma m_2 + m_2 \Sigma a_3} \cdots\cdots (\alpha^n{}_p)_{a_n m_n}(\gamma_r)_{-\Sigma m_1}}{(\beta_q)_{\Sigma m_1} \qquad (\delta_s)_{-\Sigma m_1}}\right)$$

$$\cdot \frac{x_1^{m_1}\ x_2^{m_2}----x_n^{m_n}}{m_1!\ m_2!------m_n!} \qquad \text{........(2.2)}$$

When $\sum_{i=0}^{n} a_i = 1$ and a_i may be equal to 1 or 0.

Supposing $(a_1, a_2,----a_n) = (1,0,0,-------0)$ the function given by (2.2) gets converted as

$$B^0 = \sum_{m_1,m_2,---m_n=0}^{\infty} \left(\frac{(\alpha^1{}_p)_{m_1+m_2+---m_n}(\gamma_r)_{-(m_1+m_2+---m_n)}}{(\beta_q)_{m_1+m_2+-----m_n}\ (\delta_s)_{-(m_1+m_2+----m_n)}}\right) \frac{x_1^{m_1} \quad x_2^{m_2}\ --------x_n^{m_n}}{m_1!\ \ m_2!\ ---------\ m_n!}$$

$$\text{...............(2.3)}$$

Where a^1_p shall be obviously a_p and therefore shall be single hypergeometric function in n - variables.

In case if we take

$$(a_1, a_2, ----a_n) = (0,1,0,-------0),$$

We observe that the B - function is transmogrified into a Lauricella type function, in which the second parameter is repeated (n -1) times, having the value

$$\sum_{m_1,m_2,---m_n=0}^{\infty} \left(\frac{(a^1_p)_{m_1} (a^2_p)_{m_2+---m_n} \quad (\gamma_r)_{-(m_1+m_2+----m_n)}}{(\beta_q)_{m_1+m_2+------m_n} \quad (\delta_s)_{-(m_1+m_2+----m_n)}} \right) \cdot \frac{x_1^{m_1} x_2^{m_2} -------x_n^{m_n}}{m_1!\ m_2!----------m_n!} \qquad \text{....................(2.4)}$$

Which represents trilateral hypergeometric function in n - variables.

This process of generalization, when prolonged further, can generate trilateral, quadrilateral, pentalateral and so on upto n- lateral hypergeometric functions in n-variables for corresponding values of (a_1,a_2,a_3----a_n) equal to (1,0,0,0,-----0) (0,1,0,0---0)(0,0,1,0,-----0), (0,0,0,1,0,----0), (0,0,0,0,1,---0) and (0,0,0,-----1).

3:- FORMATION OF MULTIPLICATIVE GROUP

For the B -function expressed by (2.2) we shall have

$$B^{1\,1}_{1\,1}\left[\begin{matrix} a & & c & & \\ & ; & & ; & x,y \\ b & & d & & \end{matrix}\right] = \sum_{m,n=0}^{\infty} \left(\frac{(a)_{m+n}\ (C)_{-(m+n)}}{(b)_{m+n}\ (d)_{-(m+n)}} \right) \frac{x^m}{m!} \frac{y^n}{n!} \qquad \text{................(3.1)}$$

from which, according to the following explanation

{In $\begin{bmatrix} a \\ b \end{bmatrix}$

if upper parameter is changed, the notation shall be 1;
if both parameter are changed, the notation shall be 2;
if only lower parameter is changed, the notation shall be 3
and 0 shall correspond for the normal function.}

we can produce

$$B^{10} = (a_1, a_2) = \begin{bmatrix} (a_2,a_1) & (c_1,c_2) \\ (b_1,b_2) & (d_1,d_2) \end{bmatrix} = \begin{bmatrix} (1,0) & (1,0) \\ (1,0) & (1,0) \end{bmatrix}$$

$$= \sum_{m,n=0}^{\infty} \left(\frac{(\alpha)_m (\alpha^1)_n \; (\gamma)_{-(m+n)}}{(\beta)_{m+n} \; (\delta)_{-(m+n)}} \right) \frac{x^m \; y^n}{m! \; n!} \quad \text{................(3.2)}$$

$$B^{20} = (a_1,a_2) \; (b_1,b_2) = \begin{bmatrix} (a_2,a_1) & (c_1,c_2) \\ (b_2,b_1) & (d_1,d_2) \end{bmatrix}$$

$$= \sum_{m,n=0}^{\infty} \left(\frac{(\alpha)_m \; (\alpha^1)_n \; (\gamma)_{-(m+n)}}{(\beta)_m \; (\beta^1)_n \; (\delta)_{-(m+n)}} \right) \frac{x^m \; y^n}{m! \; n!} \quad \text{..............(3.3)}$$

$$B^{30} = (b_1,b_2) = \begin{bmatrix} (a_1,a_2) & (c_1,c_2) \\ (b_2,b_1) & (d_1,d_2) \end{bmatrix} = \sum_{m,n=0}^{\infty} \left(\frac{(\alpha)_{m+n} \; (\gamma)_{-(m+n)}}{(\beta)_m \; (\beta^1)_n \; (\delta)_{-(m+n)}} \right) \frac{x^m \; y^n}{m! \; n!} \quad \text{....(3.4)}$$

$$B^{01} = (c_1,c_2) = \begin{bmatrix} (a_1,a_2) & (c_2,c_1) \\ (b_1,b_2) & (d_1,d_2) \end{bmatrix}$$

$$= \sum_{m,n=0}^{\infty} \left(\frac{(\alpha)_{m+n} \; (\gamma)_{-m} \; (\gamma^1)_{-n}}{(\beta)_{m+n} \; (\delta)_{-(m+n)}} \right) \frac{x^m \; y^n}{m! \; n!} \quad \text{..........(3.5)}$$

$$B^{23} = (a_1,a_2) \; (b_1,b_2) \; (d_1,d_2) = \sum_{m,n=0}^{\infty} \left(\frac{(\alpha)_m \; (\alpha^1)_n \; (\gamma)_{-(m+n)}}{(\beta)_m \; (\beta^1)_n \; (\delta)_{-m} \; (\delta^1)_{-n}} \right) \frac{x^m \; y^n}{m! \; n!}$$

........................(3.6)

Which in total shall be fifteen.

They shall form the elements of the finite multiplicative group $(S,\cdot)$, for which

$$S= \{ B^{10}=(a_1,a_2), \quad B^{20}=(a_1,a_2)(b_1,b_2), \quad B^{30}=(b_1,b_2),$$

$$B^{01}=(c_1,c_2), \quad B^{02}=(c_1,c_2)(d_1,d_2), \quad B^{03}=(d_1,d_2),$$

$$B^{11}=(a_1,a_2)(c_1,c_2), \quad B^{12}=(a_1,a_2)(c_1,c_2)(d_1,d_2),$$

$$B^{13}=(a_1,a_2)(d_1,d_2), \quad B^{21}=(a_1,a_2)(b_1,b_2)(c_1,c_2),$$

$$B^{22}=(a_1,a_2)(b_1,b_2)(c_1,c_2)(d_1,d_2), \quad B^{23}=(a_1,a_2)(b_1,b_2)(d_1,d_2),$$

$$B^{31}=(b_1,b_2)(c_1,c_2), \quad B^{32}=(b_1,b_2)(c_1,c_2)(d_1,d_2), \quad B^{33}=(b_1,b_2)(d_1,d_2)\} \quad ..(3.7)$$

The order of the group shall be

$2^{p+q+r+s}-1$ and B^{22} shall be identity multiplication element.

Similarly for three variables, we can generate, in order to generate a group

$$\overset{1\ 1}{\underset{1\ 1}{B}}\left[\begin{matrix} a \\ b\end{matrix} ; \begin{matrix} c \\ d\end{matrix} ; x,y,z\right] = \sum_{m,n,p=0}^{\infty} \left(\frac{(a)_{m+n+p} \quad (c)_{-(m+n+p)}}{(b)_{m+n+p} \quad (d)_{-(m+n+p)}} \right) \frac{x^m \ y^n \ z^p}{m! \ n! \ p!}$$

......................(3.8)

So that the total element of the multiplicative group shall be 80.

Groups for functions involving more then three variables can be proliferated by the generalization of results obtained in this section.

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