

# Waveform relaxation methods for stochastic differential equations

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## Abstract

An operator equation  $X = \mathbb{T}X + G$  in a Banach space  $\mathcal{E}$  of  $\mathcal{F}_t$ -adapted random elements describing an initial- or boundary value problem of a system of stochastic differential equations (SDEs) is considered. Our basic assumption is that the underlying system consists of weakly coupled subsystems. The proof of the convergence of corresponding waveform relaxation methods depends on the property that the spectral radius of an associated matrix is less than one. The entries of this matrix depend on the Lipschitz-constants of a decomposition of  $\mathbb{T}$ . In proving an existence result for the operator equation we show how the entries of the matrix depend on the right hand side of the stochastic differential equations. We derive conditions for the convergence under “classical” vector-valued Lipschitz-continuity of an appropriate splitting of the system of stochastic ODEs. A generalization of these key results under one-sided Lipschitz continuous and anticoercive drift coefficients of SDEs is also presented. Finally, we consider a system of SDEs with different time scales (singularly perturbed SDEs) as an illustrative example.

*Key words:* Waveform Relaxation Methods; Stochastic Differential Equations; Stochastic-Numerical Methods; Iteration Methods; Large Scale Systems.

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