1. Introduction.

In 1956, Kelly [5] introduced a plan for portfolio management based on the criterion of maximizing, at each stage, the expected value of the logarithm of the portfolio value. This "Kelly criterion" was further studied by Breiman [3] in 1961, who established certain asymptotic optimality properties of it. More recently, Bell and Cover [2] constructed a natural model in which the Kelly criterion yields optimal performance in a single period.

The above results are all for discrete-time models. We consider here a continuous-time version in which the stochastic process \( \{X_t, t \geq 0\} \), where \( X_t \) represents the fortune of an investor at time \( t \), has continuous sample paths. If the object is to reach a fixed goal in minimum expected time, there are problems for discrete-time models of overshooting the goal when the Kelly criterion is followed, and thus it is not optimal in general. However, there are no overshoot problems for our continuous models and an infinitesimal form of the Kelly criterion is proved optimal in section 4 below.

The continuous-time portfolio selection problem is formulated in the next section. Its solution uses results on continuous-time stochastic control from Pesenti and Sudderth [7]. These results are explained in section 3 and are then applied to solve the portfolio selection problem in section 4.

2. Portfolio selection.

Consider the problem of managing a portfolio consisting of stocks, bonds and cash to minimize the expected time to reach a given total worth. More precisely, consider a gambler, whose fortune at time \( t \) is denoted by \( X_t \), who can choose gambles (make investments) from a set of gambles depending in a linear way on his current fortune. Suppose that trading costs are negligible, and that the trader can buy or sell securities at the same price. In addition, suppose that the use of margin and short sales is not allowed.