1. INTRODUCTION. There has been recent interest in linear ordinary differential equations with polynomial coefficients since algebraic functions, generating functions with combinatorial significance, and the special functions of mathematical physics satisfy such equations. Formal power series solutions of linear ordinary differential equations with polynomial coefficients - termed D-finite - are useful since, for example, their coefficients satisfy a linear recurrence relation of fixed order and consequently can be computed fast. Efficient criteria for determining whether a power series is algebraic or D-finite is needed [10].

One useful method for determining whether a power series is algebraic is the rate of growth of the coefficients, thus the recipriocal of an algebraic function is algebraic. For D-finite power series, there is also a growth restriction on the coefficients. Indeed, if $\sum_{n=0}^{\infty} c_n x^n$ is a formal power series with complex coefficients which satisfies any algebraic differential equation, then there exist two positive contants γ_1, γ_2 such that $|c_n| < \gamma_1 (n!)^{\gamma 2}$ [3,4]. This estimate is best possible since $\sum_{n=0}^{\infty} (n!)^k x^n$ satisfies a linear ordinary differential equation with polynomial coefficients; e.g.

$$\sum_{n=0}^{\infty} n! x^n \quad \text{satisfies} \quad x^2 y'' + (3x - 1) y' + y = 0.$$

Such functions are of Gevrey type and have been shown recently to have fundamental importance in the theory of linear ordinary differential equations with an irregular singular point [5,6]. Unfortunately, even though the reciprocal of a function of Gevrey type is of Gevrey type, such functions (as we shall show) are not necessarily D-finite, or not necessarily solutions of linear ordinary differential equations with analytic coefficients!

In this note we characterize the class of power series which together with their reciprocals satisfy linear ordinary differential equations.