§0 Introduction

The purpose of the present paper is to investigate the hydrodynamical behavior of a scalar Ginzburg-Landau model on \( \mathbb{R} \). The dynamics of the system is described by a stochastic partial differential equation (SPDE). A nonlinear diffusion equation will be derived under appropriate space-time hydrodynamical scaling limit. The result will be formulated as a law of large numbers for the SPDE.

We start with explaining the Ginzburg-Landau model at a formal level. The configuration (or order parameter) \( S = (S(x), x \in \mathbb{R}) \) over the state space \( \mathbb{R} \) is a real-valued function on \( \mathbb{R} \). We associate an energy with each configuration \( S \):

\[
(0.1) \quad \mathcal{H}(S) = \int_{\mathbb{R}} \left( \frac{1}{2} |\nabla S(x)|^2 + U(S(x)) \right) \, dx , \quad \nabla = \frac{d}{dx} ,
\]

where the self-potential \( U \) is a real-valued function on \( \mathbb{R} \). The functional \( \mathcal{H} \) is usually called the Ginzburg-Landau-Wilson free energy. The corresponding statistical mechanical system (equilibrium state) is described by a Gibbs distribution and formally given by

\[
(0.2) \quad \mu(dS) = Z^{-1} e^{-\mathcal{H}(S)} \, dS \quad , \quad dS = \Pi \, dS(x) .
\]

More mathematically, \( \mu \) is a probability measure on the space \( C(\mathbb{R}) \) of all configurations, which is defined through the DLR equation (see Section 3) or given by taking the thermodynamical limit:

\[
(0.3) \quad \mu(dS) = \lim_{L \to \infty} Z_{L}^{-1} e^{-\int_{-L}^{L} U(S(x)) \, dx} \mu_{-L,0;L,0}(dS) ,
\]

where \( Z_L \) is a normalizing constant and \( \mu_{-L,0;L,0} \) is a probability distribution on \( C(\mathbb{R}) \) of the pinned Brownian motion satisfying \( S(-L) = S(L) = 0 \) and \( S(x) = 0 \), \( |x| > L \) for convenience.