TYPICAL CLUSTER SIZE FOR 2-DIM PERCOLATION PROCESSES (Revised)

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Abstract: In this paper we discuss the typical cluster size for 2-dim percolation models. We show that, for \( W_0 = \{x \in \mathbb{Z}^2 : 0 \rightarrow x\} \),

\[
-\lim_{n \to \infty} \frac{1}{n} \log_p \mathbb{P}(|W_0| = n) \sim |p - p_c|^{-\Delta} \quad \text{as} \quad p \uparrow p_c \quad \text{provided that}
\]

\[
\mathbb{E}_p(|W_0|^2)/\mathbb{E}_p(|W_0|) \equiv |p - p_c|^{-\Delta} \quad \text{as} \quad p \uparrow p_c.
\]

Furthermore, we introduce a new quantity \( f_s(p) \), which may be thought of as the singular part of the free energy, and show that \( f_s(p) \equiv |p - p_c|^{2\nu} \) provided that the correlation length \( \equiv |p - p_c|^{-\nu} \) as \( p \uparrow p_c \).

Keywords: Percolation, typical cluster size, singular part of the free energy

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