1. Introduction

In this article we wish to consider questions of almost sure structural stability for stochastic linear delay equations where diffusion coefficients do not depend on the past history of the state. Our main objective is to prove a "stable manifold theorem" for stochastic (linear) delay equations with ordinary (non-delayed) diffusion coefficients. The "stable manifold theorem" is well-known for stochastic (non-linear) o.d.e.'s (Arnold & Wihstutz [2], Arnold, Kliemann & Oeljeklaus [3], Carverhill [7], [10], [11], Baxendale [4], [5]). Although (moment) stability issues for stochastic hereditary equations were discussed by some authors (Kushner [25], Hizel & Trutzer [28], Mohammed [30], [31], Mohammed, Scheutzov & Weiszaècker [32]), so far little is known regarding almost sure structural stability for stochastic hereditary equations e.g. the one-dimensional pathological equation [29] (§3, pp. 144-149).

2. Existence of the Stochastic Semi-flow

Our approach draws heavily on an infinite-dimensional version of the Oseledec multiplicative ergodic theorem developed by Ruelle [35] within a Hilbert space context. Certain extensions of Ruelle's results to Banach spaces are given by Mane [26] and Thiûullen [36]. We will however use Ruelle's results only.

Let us now formulate the class of stochastic linear delay equations to be considered. Take \( r \geq 0 \) and \( J := [-r, 0) \). Denote by \( \mathbb{R}^n \) n-dimensional Euclidean space with the Euclidean norm \( | \cdot | \). Following Delfour & Mitter [13], [14], we shall let \( M_2 := \mathbb{R}^n \times L^2(J, \mathbb{R}^n) \) stand for the separable real Hilbert space of all