ON THE ACCURACY OF VORTEX METHODS AT LARGE TIMES

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Vortex methods simulate incompressible flow, without viscosity or at high Reynolds number, by a collection of computational elements of vorticity which are transported along computed particle paths. The velocity field can be computed from the vorticity in order to move the elements forward in time. Here we will survey the formulation and convergence theory of such methods, primarily for inviscid flow without boundaries in two or three dimensions. We also discuss a modification of the basic method intended to improve accuracy at later times and illustrate its performance with a simple test problem. It is found that the error is significantly reduced in this case. This modified method can be shown to converge, and details of the proof will be given elsewhere. Very similar ideas have been experimented with by Chris Anderson, and it is a pleasure to thank him for his helpful comments and suggestions.

For the important case of incompressible flow past boundaries at high Reynolds number, methods such as those described here can be used as one part of a general method developed by Chorin and others [12-14,31]. In this setting additional elements of vortex sheets are generated at the boundary at each time step to satisfy the no-slip condition, and a random walk simulates the effect of viscosity. Calculations illustrating the ability of this method to capture important features of the flow with good accuracy have been done by a number of workers, most recently by Ghoneim, Sethian et al. [20,21]. T. Leonard and his coworkers have applied vortex methods to a number of problems in inviscid dynamics. Leonard has given a comprehensive survey of these and related methods in [31,32].

1. Formulation and Theory in Two Dimensions.

The Euler equations in two dimensions

\[ u_t + (u \cdot v)u + \nabla p = 0 \]
\[ \nabla \cdot u = 0 \]

lead to the equation for the vorticity \( \omega = u_{2,1} - u_{1,2} \)

\[ \omega_t + (u \cdot v)\omega = 0 \] (1)