A UNIFORMLY ACCURATE FINITE ELEMENT METHOD
FOR THE MINDLIN-REISSNER PLATE*

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Abstract. We present and analyze a simple finite element method for the Mindlin-Reissner plate model in the primitive variables. Our method uses nonconforming linear finite elements for the transverse displacement and conforming linear finite elements enriched by bubbles for the rotation, with the computation of the element stiffness matrix modified by the inclusion of a simple elementwise averaging. We prove that the method converges with optimal order uniformly with respect to thickness.

Key words. Mindlin, Reissner, plate, finite element, nonconforming

AMS(MOS) subject classifications. 65N30, 73K10, 73K25

1. Introduction. The Mindlin-Reissner model describes the deformation of a plate subject to a transverse loading in terms of the transverse displacement of the midplane and the rotation of fibers normal to the midplane. This model, as well as its generalization to shells, is frequently used for plates and shells of small to moderate thickness. We present and analyze here a simple finite element method for the Mindlin-Reissner plate model. Our method uses linear finite elements for the transverse displacement and the rotation (the finite element space for the rotations are in fact slightly enriched by interior degrees of freedom) with the element stiffness matrix altered through the use of a simple elementwise average in the computation of the shear energy. We prove that the approximate values of the displacement and the rotation, together with their first derivatives, all converge at an optimal rate uniformly with respect to thickness. As far as we know, this is the only method for the Mindlin-Reissner problem in the primitive variables for which uniform optimal convergence results have been established.

Although the Mindlin-Reissner model is simple in appearance, its discretization is not straightforward. Most seemingly reasonable choices of finite element spaces lead to an approximate solution which is far more sensitive to the plate thickness than the true solution, and which grossly underestimates the displacement of thin plates. The root of this difficulty, referred to as locking of the numerical solution, is well-understood. As the plate thickness tends to zero, the Mindlin-Reissner model enforces the Kirchhoff constraint that the rotation of the normal fibers equal the gradient of the transverse displacement. On the continuous level this simply means that the solution of the Mindlin-Reissner model

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