0. INTRODUCTION

This paper is devoted to the investigation of global properties of solutions of the following semilinear hyperbolic problem

\[
\begin{align*}
    &u_{tt} - u_{xx} + u^3 = 0 \quad \text{for } t \in \mathbb{R}, \quad 0 < x < 1 \\
    &u(t, 0) = u(t, 1) = 0 \quad \text{for } t \in \mathbb{R}.
\end{align*}
\]

(0.1)

It is well known (\cite{13}, Theorem VI.1.2.3) that for any \( g \in C^1(\mathbb{R}) \) such that

\[
\forall \, u \in \mathbb{R}, \quad g(u)u \geq 0
\]

(0.2)

and any \( (u_0, v_0) \) in \( H^1_0(0, 1) \times L^2(0, 1) \), there exists a unique function \( u: \mathbb{R} \times [0, 1] \rightarrow \mathbb{R} \) such that

\[
\begin{align*}
    &u \in C(\mathbb{R}, H^1_0(0, 1)) \cap C^1(\mathbb{R}, L^2(0, 1)) \\
    &u_t \in C^1(\mathbb{R}, H^{-1}(0, 1)) \\
    &u_{tt} = u_{xx} - g(u) \text{ in } C(\mathbb{R}, H^{-1}(0, 1)) \\
    &u(0) = u_0 \quad \text{and} \quad u_t(0) = v_0.
\end{align*}
\]

(0.3) (0.4) (0.5) (0.6)

When \( g(u) = u^3 \) this solves the initial value problem associated to (0.1).

When \( g \) is odd, i.e.,

\[
\forall \, u \in \mathbb{R}, \quad g(-u) = -g(u)
\]

(0.7)

it is convenient, following \cite{6}, to introduce the function \( \tilde{u} \in C(\mathbb{R}^2) \) defined by the conditions

\[
\begin{align*}
    &\tilde{u}(t, x) = -\tilde{u}(t, -x) \quad \forall \, (t, x) \in \mathbb{R}^2 \\
    &\tilde{u}(t, x+2) = \tilde{u}(t, x) \quad \forall \, (t, x) \in \mathbb{R}^2 \\
    &\tilde{u}(t, x) = u(t, x) \quad \forall \, (t, x) \in \mathbb{R} \times [0, 1]
\end{align*}
\]

(0.8)