Remarks about equilibrium configurations of crystals

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1. Introduction

The morphology of a crystal may show several phases and these may be altered with changes in its mechanical or thermal environment. For example, some crystals may be deformed to consist of several twin related phases. Indeed, our interest is to study defect structures in materials which have mobile phase boundaries whose existence, position, and orientation are sensitive to applied loads, temperature, and electromagnetic fields.

To set these phenomena into the context of thermoelasticity theory, Ericksen ([19],[20],[21],[22]) has derived a stored energy density which exhibits invariance with respect to change of the crystallographic lattice basis of the material. Such a density is invariant with respect to an infinite discrete group as well as frame indifferent. A body governed by it is rendered highly unstable with respect to certain motions. For example, at a smooth local minimum of energy in a constant temperature heat bath, the Cauchy stress reduces to a pressure, cf. Ericksen [18]. So it seems unlikely that even setting homogeneous boundary conditions leads to a homogeneous extremal.

In this note we take up a direct method for finding and analyzing equilibrium configurations under displacement loading conditions. We favor this approach as a means of surmounting the difficulties imposed by the defect structures on the stability of smooth solutions. We are able to illustrate how an important role in the thermodynamics of the crystal is played by its subenergy. This concept, related to the traditional free energy used by many workers, was introduced by Ericksen [23] and is based in part on a method of Flory [30]. Often the energy assumed by a configuration is its subenergy and its stress is a pressure determined by it. This may be reconciled with one common thermodynamic view where