MULTIPLE INTEGRALS WITH RESPECT TO
L-MIXING PROCESSES

By

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Abstract

We prove a moment inequality for multiple integrals of the Volterra-type with respect to zero-mean $L$-mixing processes. The result is the extension of an earlier result for single integrals. This investigation was motivated by a problem in the theory of recursive identification for linear systems.
1. Introduction

An important notion in system identification is the notion of \( L \)-mixing processes introduced in Gerencsér (1989). To define the class of \( L \)-mixing processes we first introduce the following definition, in which the \( k \)-dimensional Euclidean space will be denoted by \( \mathbb{R}^k \).

**Definition 1.1.** Let \((x_t)\) be an \( \mathbb{R}^k \)-valued stochastic process. We say that \((x_t)\) is \( M \)-bounded if for all \( 1 \leq q < \infty \)

\[
M_q(x) = \sup_{t \geq 0} \mathbb{E}^{1/q}|x_t|^q < \infty.
\]

Let us consider a family of monotone increasing \( \sigma \)-algebras \((\mathcal{F}_t), t \geq 0\) and a family of monotone decreasing \( \sigma \)-algebras \((\mathcal{F}_t^+), t \geq 0\), such that \( \mathcal{F}_t, \mathcal{F}_t^+ \) are independent for all \( t \) and \( \mathcal{F}_t = \sigma(\cup_{\epsilon>0} \mathcal{F}_{t+\epsilon}) \) for all \( t \).

**Definition 1.2.** We say that a stochastic process \((x_t)\) is \( L \)-mixing with respect to \((\mathcal{F}_t, \mathcal{F}_t^+)\) if it is \((\mathcal{F}_t)\)-progressively measurable, \( M \)-bounded and if we set for \( q \geq 1, \quad \tau > 0 \)

\[
\gamma_q(\tau, x) = \gamma_q(\tau) = \sup_{t \geq \tau} \mathbb{E}^{1/q}|x_t - \mathbb{E}(x_t|\mathcal{F}_{t-\tau})|^q
\]

then we have

\[
\Gamma_q = \Gamma_q(x) = \int_0^\infty \gamma_q(\tau) d\tau < \infty.
\]

A fundamental inequality of the theory of \( L \)-mixing processes is the following:

**Theorem A.** (Theorem 1.1 in Gerencsér (1989)) Let \((u_t), t \geq 0\) be an \( L \)-mixing process with \( \mathbb{E} u_t = 0 \) for all \( t \). Let \((f_t)\) be a function in \( L_2[0, T] \). Then we have for all \( 1 \leq m < \infty \)

\[
\mathbb{E}^{1/2m} |\int_0^T f_t u_t dt|^{2m} \leq C_m \left( \int_0^T f_t^2 dt \right)^{1/2} M_{2m}^{1/2}(u) \Gamma_{2m}^{1/2}(u)
\]

where \( C_m = (4m - 2)^{1/2} \).
1. Introduction

The proof of this theorem is partly based on the following lemma:

**Lemma B.** (Lemma 2.3 in Gerencsér (1989)) Let \((x_t)\) be a zero-mean \(L\)-mixing process with respect to \((\mathcal{F}_t, \mathcal{F}_t^+)\) and let for some \(0 < s < t\) \(\eta\) be an \(\mathcal{F}_s\)-measurable random variable. Then

\[
|E x_t \eta| \leq 2 \gamma_p (t-s, x) E^{1/q} |\eta|^q
\]

for every \(1 < p, q < \infty\) such that \(1/p + 1/q = 1\). The proposition remains true for \(p = \infty\) and \(q = 1\).

An important problem is whether the deterministic function \(f_t\) can be replaced by an adapted stochastic process. It is easy to see that this is not possible without any qualification. Set e.g. \(u_t = f_t\), then the integral in Theorem A grows as \(T\) rather than \(T^{1/2}\). However, we may expect that if the process \((f_t)\) has a lot of "information-content" in the past then the theorem may remain true. A possible assumption is that \(f_t\) can be written as

\[
f_t = \int_0^t g_s v_s ds
\]

where \(g_s\) is a deterministic locally square integrable function and \((v_s)\) is a zero-mean \(L\)-mixing process. Thus the integral in Theorem A can be represented as a double integral. Estimation of double integrals is important in the theory of recursive identification, E.g. double integrals of the form

\[
I_{T_0} = \int_{T_0}^{T} \frac{1}{t} u_t \int_{T_0}^{t} \frac{1}{s} v_s ds dt
\]

has to be estimated, where \((u_t), (v_s)\) are zero-mean \(L\)-mixing processes (c.f. Gerencsér (1990)) to get a characterization of the parameter estimation error process. The theorem below gives a general answer to the problem raised above. Also, the theorem has a potential for being used in the analysis of nonlinear stochastic systems.

**Theorem** Let \((u_{i,t}), t \geq 0, i = 1, ..., N\) be zero-mean \(L\)-mixing processes with respect to a pair of families for \(\sigma\)-algebras \((\mathcal{F}_t, \mathcal{F}_t^+)\) and let \((f_{i,t}), t \geq 0, i = 1, ..., N\) be locally square-
integrable functions. Then for the multiple integral

\[ x_{N,T} = \int_0^T \int_0^{t_1} \cdots \int_0^{t_2} (f_{1,t_1} u_{1,t_1}) \cdots (f_{N,t_N} u_{N,t_N}) dt_1 \cdots dt_N. \quad (1.1) \]

we have for any \( m \geq 1 \)

\[ E^{1/2m} x_{N,T}^{2m} \leq \varphi_{N,2m,T} \quad (1.2) \]

where \( \varphi_{N,2m,T} \) are defined recursively as follows: for \( N = 0 \) we set \( \varphi_{0,2m,T} = 1 \), for \( N = 1 \) we have

\[ \varphi_{1,2m,T} = 4m^{1/2} M_{2m}^{1/2}(u_1) \Gamma_{2m}^{1/2}(u_1) \left( \int_0^T f_{1,t_1}^2 dt_1 \right)^{1/2} \]

and for \( N > 1 \) we first define

\[ h_s = \int_s^T 2m_1N(t-r,u_N)|f_{N,t}|dt \]

and then set

\[ \varphi_{N,2m,T} = 4m M_{2mN}^{1/2}(u_N) \left( \int_0^T |f_{N,s}| \cdot h_s \cdot \varphi_{N-1,2mN/(N-1),s}^2 ds \right)^{1/2} + 8m M_{2mN}(u_N) \int_0^T |f_{N-1,s}| \cdot h_s \cdot \varphi_{N-2,2mN/(N-2),s} ds. \]

**Remark** If the functions \( |f_{i,t}| \) are monotone nonincreasing then we have

\[ h_s \leq \int_s^t \gamma_{2mN}(t-r,u_N)|f_{N,s}|ds \leq \Gamma_{2mN}(u_N)|f_{N,s}| \]

and a simpler upper bound can be obtained using the recursion

\[ \varphi_{N,2m,T} 4m^{1/2} M_{2mN}^{1/2}(u_N) \Gamma_{2mN}^{1/2}(u_N) \cdot \left( \int_0^T f_{N,s}^2 \varphi_{N-1,2mN/(N-1),s}^2 ds \right)^{1/2} + 8m \cdot M_{2mN}(u_{N-1}) \Gamma_{2mN}(u_N) \int_0^T |f_{N-1,s}| \cdot f_{N,s} \cdot \varphi_{N-2,2mN/(N-2),s} ds. \]

To illustrate the usefulness of the theorem we shall apply it to estimate the moments of the double integral \( I_{T_0} \) above as a function of \( T_0 \). For \( N = 1 \) we have

\[ \varphi_{1,2m,T} = O \left( \int_{T_0}^{1/2} ds \right)^{1/2} = O(T_0^{-1/2}). \]
2. The proof.

For $N = 2$ we note that $h_s = O(s^{-1})$. Hence

$$\varphi_{2,2m,T} = O\left(\int_{T_0}^{T} \frac{1}{s^2} T_0^{-1} ds\right)^{1/2} + O\left(\int_{T_0}^{T} \frac{1}{s^2} ds\right) = O(T_0^{-1}).$$

Thus we finally get

$$I_{T_0} = \int_{T_0}^{T} \frac{1}{t} u_t \int_{T_0}^{t} \frac{1}{s} v_s ds dt = O_M(T_0^{-1}).$$

**Remark** Note that if $u_t d_t$ and $v_s d_s$ are replaced by Gaussian white-noise say $d u_t$ and $d v_s$ we have a similar result, i.e.

$$\int_{T_0}^{T} \frac{1}{t} d u_t \int_{T_0}^{t} \frac{1}{s} d v_s = O_M(T_0^{-1}),$$

which can easily be seen using standard moment inequalities for stochastic integrals.

2. The proof.

We have for $m \geq 1$

$$\frac{d}{d T} |x_{N,T}|^{2m} = 2m |x_{N,T}|^{2m-1} (\text{sgn} x_{N,T}) f_{N,T} u_{N,T} \cdot x_{N-1,T}. \tag{2.1}$$

Setting

$$y_{N,T} = 2m |x_{N,T}|^{2m-1} (\text{sgn} x_{N,T}) \cdot x_{N-1,T}$$

we can write (2.1) also in the form

$$|x_{N,T}|^{2m} = \int_{0}^{T} f_{N,t} u_{N,T} \cdot y_{N,t} dt. \tag{2.2}$$

To get a similar integral representation for $y_{N,T}$ we note that

$$\frac{d}{d T} |x_{N,T}|^{2m-1} \text{sgn} x_{N,T} = (2m - 1) |x_{N,T}|^{2m-2} \cdot f_{N,T} u_{N,T} \cdot x_{N-1,T}$$

hence

$$\frac{d}{d T} y_{N,T}^{2m-1} (2m - 1) |x_{N,T}|^{2m-2} \cdot f_{N,T} u_{N,T} \cdot x_{N-1,T} \cdot x_{N-1,T}$$

$$+ 2m |x_{N,T}|^{2m-1} (\text{sgn} x_{N,T}) \cdot f_{N-1,T} u_{n-1,T} \cdot x_{N-2,T}. $$

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Setting
\[ z_{N,T} = 2m(2m - 1)|x_{N,T}|^{2m-2} \cdot (x_{N-1,T})^2 \]
and
\[ w_{N,T} = 2m|x_{N,T}|^{2m-1}(\text{sgn}x_{N,T}) \cdot x_{N-2,T} \]
we can write
\[ y_{N,T} = \int_0^T (f_{N,t}u_{N,t} \cdot z_{N,t} + f_{N-1,t}u_{N-1,t} \cdot w_{N,t})dt. \]
Substituting this expression into (2.2) we get
\[ |x_{N,T}|^{2m} = \int_0^T f_{N,t}u_{N,t} \int_0^t (f_{N,s}u_{N,s} \cdot z_{N,s} + f_{N-1,s}u_{N-1,s} \cdot w_{N,s})dsdt \quad (2.3) \]
Take expectations of both sides and apply the "improved Hölder's inequality" given in Lemma B. We have in general
\[ |E u_{N,t} \cdot u_{N,s} \cdot z_{N,s}| \leq 2 \gamma p_1(t - s, u_N) \cdot M_{q_1}(u_N) \cdot E^{1/r_1}|z_{N,s}|^{r_1} \]
with any \( p_1, q_1, r_1 > 0 \) such that \( 1/p_1 + 1/q_1 + 1/r_1 = 1 \). Furthermore
\[ E^{1/r_1}|z_{N,s}|^{r_1} \leq 2m(2m - 1)E^{1/r_1'}|x_{N,s}|^{(2m-2)r_1'}E^{1/r_1''}|x_{N-1,s}|^{2r_1''} \]
where \( r_1', r_1'' > 0 \) are such that \( 1/r_1' + 1/r_1'' = 1/r_1 \). Now let us set
\[ r_1' = 2m/(2m - 2) \quad \text{i.e.} \quad 1/r_1' = 1 - 1/m. \quad (2.4) \]
Then using inductive hypothesis we get
\[ |E u_{N,t} \cdot u_{N,s} \cdot z_{N,s}| \leq 4m(2m - 1) \cdot \gamma p_1(t - s, u_N) \cdot M_{q_1}(u_N) \cdot F_{N,s}^{1-1/m} \cdot \varphi_{N-1,2r_1''}^2 \quad (2.4) \]
for any \( p_1, q_1, r_1'' > 0 \) such that \( 1/p_1 + 1/q_1 + 1/r_1'' = 1 - 1/r_1' = 1/m \). Note that \( 2r_1'' > 2 \) therefore the use of the inductive hypothesis is justified. Similarly
\[ |E u_{N,t} \cdot u_{N-1,s} \cdot w_{N,s}| \leq 2 \gamma p_2(t - s, u_N) \cdot M_{q_2}(u_{N-1}) \cdot E^{1/r_2}|w_{N,s}|^{r_2} \]
with any \( p_2, q_2, r_2 > 0 \) such that \( 1/p_2 + 1/q_2 + 1/r_2 = 1 \). Furthermore
\[
E^{1/r_2}|w_{N,s}|r_2'' \leq 2m \cdot E^{1/r_2'}|x_{N,s}|^{(2m-1)r_2'}E|x_{n-2,s}|^{r_2''}
\]
with any \( r_2', r_2'' > 0 \) such that \( 1/r_2' + 1/r_2'' = 1/r_2 \). Choosing
\[
r_2' = 2m/(2m - 1) \quad \text{i.e.} \quad 1/r_2' = 1 - 1/2m
\]
and using the inductive hypothesis we get
\[
|E_{u_{N,t}} \cdot u_{N-1,s} \cdot w_{N,s}| \leq 4m \cdot \gamma_{p_2}(t - s, u_N) \cdot M_{q_2}(u_{N-1}) \cdot F_{N,s}^{1-1/2m} \cdot C_{N-2,r_2''} \varphi_{N-2,r_2'''}s
\]
with any \( p_2, q_2, r_2'' > 0 \) such that \( 1/p_2 + 1/q_2 + 1/r_2'' = 1 - 1/r_2' = 1/2m \). Note again that \( r_2'' > 2 \) hence the use of the inductive hypothesis is justified. Substituting into (2.3) we get
\[
E|x_{N,T}|^{2m} \leq \int_0^T \int_0^t f_{N,t} f_{N,s} \cdot 4m(2m - 1) \cdot \gamma_{p_1}(t - s, u_N) \cdot M_{q_1}(u_N) \cdot F_{N,s}^{1-1/m} \cdot \varphi_{N-1,2r_1''}^2 s dt + \int_0^T \int_0^t f_{N,t} f_{N-1,s} \cdot 4m \cdot \gamma_{p_2}(t - s, u_N) \cdot M_{q_2}(u_{N-1}) \cdot F_{N,s}^{1-1/2m} \cdot \varphi_{N-2,r_2'''} s dt.
\]
Interchanging the order of integration and introducing the notations
\[
h_{1,s} = \int_s^T f_{N,t} \cdot \gamma_{p_1}(t - s, u_N) dt \quad \text{and} \quad h_{2,s} = \int_s^T f_{N,t} \cdot \gamma_{p_2}(t - s, u_N) dt
\]
we get
\[
E|x_{N,T}|^{2n} \leq \int_0^T f_{N,s} h_{1,s} \cdot 4m(2m - 1) \cdot M_{q_1}(u_N) \cdot F_{N,s}^{1-1/m} \cdot \varphi_{N-1,2r_1''}^2 s ds + \int_0^T f_{N-1,s} h_{2,s} \cdot 4m \cdot M_{q_2}(u_{N-1}) \cdot F_{N,s}^{1-1/2m} \cdot \varphi_{N-2,r_2'''} s ds
\]
and here
\[
1/p_1 + 1/q_1 + 1/r_1'' = 1/m
\]
and
\[
1/p_2 + 1/q_2 + 1/r_2'' = 1/2m.
\]
Setting $\psi_s = E^{1/2m}|x_{N,s}|^{2m}$ we get the inequality

$$\psi_T^{2m} \leq \int_0^T (a_s \psi_s^{2m-2} + b_s \psi_s^{2m-1})ds$$  \hspace{1cm} (2.10)

with

$$a_s = C_a^2 f_N h_{1,s}^2 \phi_{n-1,s}^2 \text{ and } b_s = C_b^2 f_{N-1,s} h_{2,s} \phi_{N-2,s}$$

where

$$C_a^2 = 4m(2m-1)M_{q_1}(u_N) \text{ and } C_b^2 = 4mM_{q_2}(u_{N-1})$$

We need the following lemma below:

**Lemma 2.1.** Let $(\psi_t)$, $t \geq 0$ be a nonnegative monotone increasing measurable function satisfying

$$\psi_T^{2m} \leq \int_0^T (a_s \psi_s^{2m-2} + b_s \psi_s^{2m-1})ds$$  \hspace{1cm} (2.11)

where $(a_s)$, $(b_s)$ are nonnegative locally integrable functions and $n \geq 1$. Then we have

$$\psi_T \leq \sqrt{2} \left( \int_0^T a_s ds \right)^{1/2} + \frac{\sqrt{2} + 1}{2} \left( \int_0^T b_s ds \right).$$

**Proof:** Increase the right hand side of (2.11) replacing $\psi_s$ by $\psi_T$ and then divide both sides by $\psi_T^{2m-2}$. (If $\psi_T = 0$ then the proposition is trivial). Then we get

$$\psi_T^2 \leq \left( \int_0^T a_s ds \right) + \left( \int_0^T b_s ds \right) \psi_T = A + B\psi_T.$$

from which we get

$$\psi_T \leq \frac{1}{2} (B + (B^2 + 4A)^{1/2}).$$  \hspace{1cm} (2.12)

Now $B^2 + 4A \leq 2\max(B^2, 4A)$, hence $(B^2 + 4A)^{1/2} \leq 2^{1/2}\max(B, 2A^{1/2}) \leq 2^{1/2}(B + 2A^{1/2})$, and substituting into (2.12) we get the proposition of the lemma.

Applying Lemma (2.1) to (2.10) and using $(\sqrt{2} + 1)/2 \leq 2$ we get

$$E^{1/2m}|x_{N,t}|^{2m} \leq 4mM_{q_1}^{1/2}(u_N) \left( \int_0^T f_N h_{1,s}^2 \phi_{N-1,2r_{1,s}}^{2m-1} ds \right)^{1/2}$$

$$+ 8mM_{q_2}(u_{N-1}) \left( \int_0^T f_{N-1,s} h_{2,s} \phi_{N-2,2r_{2,s}} ds \right).$$  \hspace{1cm} (2.13)
Let us now choose $p_1 = q_1 = p_2 = q_2 = 2nN$. Then we get from (2.8) that $r_1'' = mN/(N - 1)$ and from (2.10) that $r_2'' = 2nN/(N - 2)$, and the proposition of the theorem follows.

3. REFERENCES


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