ON THE EXISTENCE OF THE ENTROPIC SOLUTIONS
FOR THE TRANSONIC FLOW PROBLEM

By

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ABSTRACT. Full Potential Equation is studied using the time dependent formulation. The original entropic solution of FPE is obtained as an asymptotic solution of a quasilinear parabolic equation.

INTRODUCTION

Results contained in the presented paper are the further development of the paper by P.Klouček, J.Nečas [1]. The spirit of the analysis for the pseudo-time-dependent approach is taken from the paper of A.Friedman, J.Nečas [2].

The "nearest" mathematical model, bringing the problem of transonic flow within the framework of well-established theory, is the so-called Unsteady Transonic Flow Problem . The analysis of the existence and eventually the unicity of the resulting nonlinear second order hyperbolic equation is still an open problem.

Most of the numerical methods for the calculating of transonic flow can be rewritten in a form, from which a successive iteration could be derived, using an artificial time coordinate (see e.g. Jameson [3],[4], Hafez,South,Murman [5]). Typically, one ends up with

$$\alpha_1 u_{xt} + \alpha_2 u_{yt} + \alpha_3 u_t = - \text{div}(\rho(|\nabla u|^2) \nabla u),$$

where $u$ is the velocity potential. Implementing the code onto the parallel systems, one has to avoid the sequential updating and this requirement calls for introducing the term $u_{tt}$ (see Jameson, Keller [6]). Further comments on the time-dependent approach can be found in Zanetti et al. [7], on the pseudo-unsteady methods in Veuilhot, Vivian [51]. All the indications show that the time-dependent or pseudo-time-dependent approaches to the Transonic Flow Problems have the advantages from the point of view of the physical approach, mathematical model and implementation. Though it is not clear whereas there is uniqueness or not in the original stationary problem (Steinhoff, Jameson [9]), we can use these models to capture the steady-state.

Here, we deal with a similar situation. We start from the assumption of adiabatic, isentropic, nonviscous, irrational steady flow, we introduce an artificial time coordinate and we add the damping

$$u_{tt} - \Delta u_t,$$

into the original equation. In such a way we solve the quasilinear parabolic equation. We expect to obtain a solution which converges to the entropic solution of the original stationary problem, if time goes to infinity.

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1. Formulation of the problem

Prior to the further development of the theory we briefly mention the formulation of the problem and results from [1].

We consider the steady, inviscid, irrotational, adiabatic and isentropic flow of an ideal gas in two or three dimensions. This situation is covered by the following system of equations:

\(\nabla \cdot (\rho \cdot \vec{v}) = 0,\) \hspace{1cm} \text{conservation of mass}, \hspace{1cm} (1.1)

\(\vec{v} \cdot (\nabla \vec{v}) = -\frac{1}{\rho} \nabla p,\) \hspace{1cm} \text{conservation of momentum}, \hspace{1cm} (1.2)

\(\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^\kappa,\) \hspace{1cm} \text{adiabatic, isentropic, ideal gas flow}, \hspace{1cm} (1.3)

\(\nabla \times \vec{v} = 0\) \hspace{1cm} \text{and irrotationality.} \hspace{1cm} (1.4)

Here \(\vec{v}\) is the velocity vector, \(\rho\) the density, \(p\) the pressure and \(\kappa\) the heat conductivity \((\kappa = 1.4\ \text{for air})\) and \(p_0, \rho_0\) are the stagnation values of the density and pressure.

We will consider the system to be in a bounded domain \(\Omega \subset \mathbb{R}^n, \ n = 2, 3\). We assume, that the boundary \(\partial \Omega\) is smooth enough and thus there exists a function \(u \in C^2(\Omega)\) such that \(\nabla u = \vec{v}\) which is called velocity potential. From (1.2)-(1.4) we can obtain the density as the function of the velocity potential

\(\rho = \rho_0 \left(1 - \frac{\kappa - 1}{2a_0^2} |\nabla u|^2\right)^{\frac{1}{\kappa - 1}},\) \hspace{1cm} (1.5)

where \(a\) is the local speed of sound and \(a_0\) corresponds to the stagnation values of \(\rho\) and \(p\). The density \(\rho(s)\) is defined for \((s \equiv |\nabla u|^2)\)

\(0 \leq s \leq \frac{2a_0^2}{\kappa - 1}.\) \hspace{1cm} (1.6)

Here, we will consider the density to be extended smoothly to all of \(\mathbb{R}^+,\) such that the following conditions hold

\(0 < \rho_1 \leq \rho(s) \leq \rho_2 < \infty,\) \hspace{1cm} (1.7)

\(\rho'(s) \leq 0,\) \hspace{1cm} (1.8)

\(-\rho'(s)s \leq C_E < \infty.\) \hspace{1cm} (1.9)

Equation (1.1) with the density (1.5), in two dimensions can also be written as

\((a^2 - u_x^2)u_{xx} - 2u_xu_yu_{xy} + (a^2 - u_y^2)u_{yy} = 0,\) \hspace{1cm} (1.10)

(the indexes \(x\) and \(y\) denote the derivatives with respect to \(x\) - and \(y\) - direction), has clearly the elliptic/hyperbolic character. The equation is elliptic (and the flow subsonic) in a region \(\Omega_1 \subset \Omega,\) if

\(|\nabla u|^2 < \frac{2a_0^2}{\kappa + 1},\) \hspace{1cm} \text{in } \Omega_1,\) \hspace{1cm} (1.11)
and hyperbolic (and the flow transonic) in a region $\Omega_2 \subset \Omega$, if

\begin{equation}
|\nabla u|^2 > \frac{2a_0^2}{\kappa + 1}, \quad \text{in } \Omega_2.
\end{equation}

To keep in mind the physical origin of the problem, we usually have to restrict ourselves to the class of the solutions for which there exists a certain constant, say $C_{ph}$, that

\begin{equation}
|\nabla u|^2 \leq C_{ph} < \infty.
\end{equation}

This condition ensures that the norm of the solution can be only in that part of $\mathcal{R}^+$, for which the density is naturally defined. We do not apply this condition in the subsequent theory but it plays a very important role in numerical experiments.

2. Method of Stabilization

Instead of the steady state velocity potential $u = u(x)$, we will consider a time dependent function

\begin{equation}
u = u(x, t), \quad x \in \Omega, \quad t \in [0, T].\end{equation}

We perturbate the equation (1.12) to obtain its regularized parabolic counterpart

\begin{equation}
\dddot{u} - \Delta \dot{u} - \text{div}(\rho(|\nabla u|^2)\nabla u) = 0.
\end{equation}

For this equation, we formulate the initial-boundary-value problem.

Let $\Omega$ be a bounded domain in $\mathcal{R}^n$, $n = 2, 3$, with $C^2$ boundary $\partial \Omega$; for any $T > 0$ we set

\begin{align*}
\Omega_T &= \{(x, T) \mid x \in \Omega\} \quad \text{and} \\
Q_T &= \{(x, t) \mid x \in \Omega, t \in [0, T]\}.
\end{align*}

Let us consider the equation (2.2) in $Q_T$, with the initial conditions

\begin{align*}
u(x, 0) &= u_0(x), \quad \text{for } x \in \Omega, \\
\dot{u}(x, 0) &= u_1(x), \quad \text{for } x \in \Omega,
\end{align*}

and the boundary condition

\begin{equation}
\frac{\partial \dot{u}}{\partial n}(x, t) + \rho(|\nabla u(x, t)|^2)\frac{\partial u}{\partial n}(x, t) = g(x), \quad \text{for } x \in \partial \Omega, \quad t \in [0, T].
\end{equation}

Here $\frac{\partial}{\partial n}$ denotes the differentiation with respect to the outer normal of $\partial \Omega$. We assume that

\begin{align*}
\|g(x)\| \leq \text{const.} < \infty, \quad \text{for } x \in \partial \Omega \quad \text{and} \\
\int_{\partial \Omega} g(x) \, dS &= 0.
\end{align*}

The last requirement is natural; it corresponds to the conservation of mass.
3. Weak formulation of the problem

Definition 3.1. A function \( \dot{u} \) is called a weak solution to (2.2)-(2.5) if

\[
\begin{align*}
\dot{u} &\in L^\infty([0, T], W^{1,2}(\Omega)), \\
\bar{u} &\in L^2(Q_T)
\end{align*}
\]

and (2.2)-(2.5) are satisfied in the following sense

\[
\int_0^T \int_{Q_T} (\bar{u} \cdot \varphi + \nabla \dot{u} \cdot \nabla \varphi + \rho(|\nabla u|^2) \nabla u \cdot \nabla \varphi) \, dx \, dt = \int_0^T \int_{\partial \Omega} g \varphi \, dS \, dt,
\]

for any \( \varphi \in L^2([0, T], W^{1,2}(\Omega)) \), and any \( T > 0 \).

Remark 3.2. We assume that

\[
u_0 \in W^{1,2}(\Omega), \, u_1 \in W^{1,2}(\Omega).
\]

We use the following notation. \( L^p([0, T], X) \) is a space of time dependent, \( X \)-valued functions, defined as follows:

\[
L^p([0, T], X) = \left\{ x: x(t) \in X \text{ a.e. on } [0, T], x \text{ is a measurable function of } t, \int_0^T \| x(t) \|_X^p \, dt < \infty \right\}.
\]

In the case \( p = \infty \), we replace the last condition in the above definition by

\[
\sup_{t \in [0, T]} \| x(t) \|_X < \infty.
\]

We identify \( L^p(Q_T) \) with \( L^p([0, T], L^p(\Omega)) \) and write \( u \in W^{k,p}_{\text{loc}}(\Omega) \), if \( u \in W^{k,p}(\Omega') \) for any compact subdomain \( \Omega' \) of \( \Omega \). By \( D(\Omega) \) we denote the set of all functions from \( C^\infty(\overline{\Omega}) \) with a compact support in \( \Omega \) and by \( D^+(\Omega) \), the functions from \( D(\Omega) \) which are positive.

We observe that from (3.1) that

\[
\dot{u} \in C([0, T], L^2(\Omega));
\]

thus \( u(., 0) \) and \( \dot{u}(., 0) \) are well defined.

4. Global existence and unicity of the Strong solution

The following theorems and lemmas can be found with detail proofs in [1].

Theorem 4.1. Assume that the conditions (1.7)-(1.9) for the density, the bounds (2.6), (2.7) for the boundary function \( g \) and the regularity requirements (3.3) for the initial values, are satisfied. Then, there exists a uniquely determined function \( \dot{u} \) which is the weak solution of (2.2)-(2.5), for any \( T > 0 \).
Theorem 4.2. Let $\Omega$ be a bounded domain with a Lipschitz boundary and let the assumptions for the density (1.7)-(1.9) and (2.6),(2.7) for the boundary function $g$, the regularity requirements (3.3) for the initial values, be valid. Let $\tilde{u} \in L^\infty([0,T],W^{1,2}(\Omega))$ be the weak solution asserted by Theorem 4.1. Further, let $\sigma$ be a smooth positive function, zero on the boundary of $\Omega$. Then

$$
(4.1) \quad \iint_{Q_T} \left( \frac{\partial^2 \tilde{u}}{\partial x_i \partial x_j} - \sigma \right)^2 \, dx \, dt \leq C \left( \| \tilde{u} \|^2_{L^\infty([0,T],W^{1,2}(\Omega))} + \| \tilde{u} \|^2_{L^2(Q_T)} \right),
$$

for $1 \leq i, j \leq n$.

Under somewhat stronger assumptions (compare with those of Theorem 4.2, we can guarantee the global existence of the strong solution.

Lemma 4.3. Let all the assumptions of Theorem 4.2 be satisfied and, moreover, let

$$
(4.2) \quad \int_{\Omega} u(x,0) \, dx = \int_{\Omega} \tilde{u}(x,0) \, dx = 0.
$$

Then there exists a constant $C$ which does not depend on $T$, such that

$$
(4.3) \quad \int_{\Omega_T} |\tilde{u}|^2 \, dx + \iint_{Q_T} |\nabla \tilde{u}|^2 \, dx \, dt + \int_{\Omega_T} |\nabla u|^2 \, dx \leq C < \infty.
$$

Lemma 4.4. Under the assumptions of the preceding Lemma 4.3, there exists a constant $C$ which does not depend on $T$, such that

$$
(4.4) \quad \iint_{Q_T} |\tilde{u}|^2 \, dx \, dt + \int_{\Omega_T} |\nabla \tilde{u}|^2 \, dx \leq C < \infty.
$$

Theorem 4.5. Let $\Omega$ be a bounded domain with a Lipschitz boundary and let the assumptions for the density (1.7)-(1.9) and (2.6),(2.7) for the boundary function $g$, be valid. Let, moreover, (4.2) and (3.3) for the initial solutions, be true. Then there exists the unique global strong solution of the problem (2.2)-(2.5).

5. Additional regularity of the weak solution

Using the lemmas of the preceding paragraph, we can derive additional regularity results.

Theorem 5.1. Let $\Omega$ be a bounded domain with a Lipschitz boundary and let the assumptions for the density (1.7)-(1.9) and (2.6),(2.7) for the boundary function $g$, be valid. Let (4.2) for the initial solutions, be true. Let assume, moreover, that

$$
 u_0 \in W^{2,2}(\Omega), \quad u_1 \in W^{2,2}(\Omega).
$$

Then there exists a constant $C$ which does not depend on $T$, such that

$$
(5.1) \quad \int_{\Omega_T} |\tilde{u}|^2 \, dx + \iint_{Q_T} |\nabla \tilde{u}|^2 \, dx \, dt \leq C.
$$
Proof. We approximate the initial values \( u_0 \) and \( u_1 \) by the smooth initial values \( u_0^\lambda, u_1^\lambda \), as \( \lambda \to 0 \), in \( C^2(\Omega) \). By a fixed point argument we can prove (since the density \( \rho \) is a smooth positive bounded function) that the problem (2.2)-(2.5) with the initial data \( u_0^\lambda, u_1^\lambda \) has a classical solution in \( Q_{T_*} \), if \( T_* \) is small enough. The strong solution, asserted by Theorem 4.5, has to coincide with the classical solution for any \( t < T_* \) and, as \( \lambda \to 0 \),

\[
(5.2) \quad u^\lambda \to u \quad \text{in} \quad C^1([0,T_*], W^{1,2}(\Omega)).
\]

We, now, work with finite differences in time. Let us denote

\[
(5.3) \quad \tilde{\partial}_t \tilde{u}(x,t) \equiv \frac{1}{\Delta t} (\tilde{u}(x,t + \Delta t) - \tilde{u}(x,t)).
\]

In the interval \([0, T - \Delta t]\) which is again written as \([0,T]\), we have the existence of the weak solution \( \tilde{\partial}_t u^\lambda \), for the initial values \( u_0^\lambda, u_1^\lambda \). Because the function \( g \) from the definition of the boundary condition (2.5) is constant in time, we have

\[
(5.4) \quad \iint_{Q_T} \tilde{\partial}_t \tilde{u}^\lambda \cdot \varphi \, dx \, dt + \iint_{Q_T} \nabla \tilde{\partial}_t \tilde{u}^\lambda \cdot \nabla \varphi \, dx \, dt + \iint_{Q_T} \tilde{\partial}_t (\rho(|\nabla u^\lambda|^2) \nabla u^\lambda) \cdot \nabla \varphi \, dx \, dt = 0,
\]

where \( \tilde{\partial}_t \rho(|\nabla u|^2) \nabla u = \frac{1}{\Delta t} \left( (\rho(|\nabla u|^2) \nabla u)|_{t+\Delta t} - (\rho(|\nabla u|^2) \nabla u)|_t \right) \). We introduce

\[
(5.5) \quad w(\tau) = \tau u^\lambda(t + \Delta t) + (1 - \tau) u^\lambda(t)
\]

\[
(5.6) \quad \tilde{\xi}(\tau) = \rho(|\nabla w(\tau)|^2) \nabla w(\tau)
\]

In view of the above notation it holds

\[
\tilde{\partial}_t (\rho(|\nabla u^\lambda|^2) \nabla u^\lambda) = \frac{1}{\Delta t} \int_0^1 \frac{d}{d\tau} \tilde{\xi}(\tau) \, d\tau =
\]

\[
(5.7) \quad \int_0^1 \left( (\rho(|\nabla w|^2) \tilde{\partial}_t \nabla u^\lambda + 2 \rho'(|\nabla w|^2) (\nabla w \cdot \tilde{\partial}_t u^\lambda) \nabla w \right) \, d\tau.
\]

Thus, we can rewrite (5.4), substituting \( \varphi = \tilde{\partial}_t \tilde{u}^\lambda \), as

\[
(5.8) \quad \frac{1}{2} \int_{\Omega_T} |\tilde{\partial}_t \tilde{u}^\lambda|^2 \, dx + \iint_{Q_T} |\nabla \tilde{\partial}_t \tilde{u}^\lambda|^2 \, dx \, dt = \frac{1}{2} \int_{\Omega_0} |\tilde{\partial}_t \tilde{u}^\lambda|^2 \, dx -
\]

\[
\iint_{Q_T} \int_0^1 \left( \rho(|\nabla w|^2) \left( \nabla \tilde{\partial}_t u^\lambda \cdot \nabla \tilde{\partial}_t \tilde{u}^\lambda \right) + 2 \rho'(|\nabla w|^2) \left( \nabla w \cdot \nabla \tilde{\partial}_t u^\lambda \right) \left( \nabla w \cdot \nabla \tilde{\partial}_t \tilde{u}^\lambda \right) \right) \, d\tau \, dx \, dt.
\]

This yields

\[
(5.9) \quad \frac{1}{2} \int_{\Omega_T} |\tilde{\partial}_t \tilde{u}^\lambda|^2 \, dx + (1 - C \cdot \varepsilon) \iint_{Q_T} |\nabla \tilde{\partial}_t \tilde{u}^\lambda|^2 \, dx \, dt \leq
\]

\[
\frac{1}{2} \int_{\Omega_0} |\tilde{\partial}_t \tilde{u}^\lambda|^2 \, dx + C \int_{Q_T} |\nabla \tilde{\partial}_t u^\lambda|^2 \, dx \, dt.
\]
As $\Delta t \to 0_+$, the right-hand side of (5.9) converges to

$$
\frac{1}{2} \int_{\Omega_0} |\tilde{u}^\lambda|^2 \, dx + C \int_Q |\nabla \tilde{u}^\lambda|^2 \, dx \, dt.
$$

In view of Lemma 4.3, the second term in (5.10) is bounded independently of $T$ and therefore independently of $\lambda$. Since, in view of the assumptions for $u_0$, $u_1$, we can choose the approximations $u_0^\lambda$, $u_1^\lambda$ such, that the first term in (5.10) is bounded independently of $\lambda$, it thus follows from (5.10), that

$$
\int_{\Omega_T} |\tilde{\partial}_t \tilde{u}^\lambda|^2 \, dx + \int_Q |\nabla \tilde{\partial}_t \tilde{u}^\lambda|^2 \, dx \, dt \leq C
$$

and $C$ does not depend on $\lambda$ and $T$. That is

$$
\int_{\Omega_T} |\tilde{u}^\lambda|^2 \, dx \, dt + \int_Q |\nabla \tilde{u}^\lambda|^2 \, dx \, dt \leq C.
$$

Letting $\lambda \to 0_+$ and using (5.2) we get the assertion of this Theorem. $\Box$

**Theorem 5.2.** Let $\Omega$ be a bounded domain with a Lipschitz boundary and let the assumptions for the density (1.7)-(1.9) and (2.6),(2.7) for the boundary function $g$, be valid. Let (4.2) for the initial solutions be true. Let as assume that $u_0 \in W^{2,2}(\Omega)$, $u_1 \in W^{2,2}(\Omega)$, and that the weak solution $\tilde{u}$ satisfies

$$
||\nabla \tilde{u}||_{L^2(\Omega)}^2 \leq \text{const.} < \infty.
$$

Then there exists a constant $C$, which does not depend on $T$, such that

$$
\int_Q |\tilde{u}|^2 \, dx \, dt + \int_{\Omega_T} |\nabla \tilde{u}|^2 \, dx \leq C.
$$

**Proof.** If we use the weak formulation for the finite differences $\tilde{\partial}_t \tilde{u}$, introduced in (5.4) and substituting $\varphi = \tilde{\partial}_t \tilde{u}$, we end up with

$$
\int_{Q_T} |\tilde{\partial}_t \tilde{u}|^2 \, dx \, dt + \int_{Q_T} \nabla \tilde{\partial}_t \tilde{u} \cdot \nabla \tilde{\partial}_t \tilde{u} \, dx \, dt + \int_{Q_T} \tilde{\partial}_t (\rho(|\nabla u|^2) \nabla u) \nabla \tilde{\partial}_t \tilde{u} \, dx \, dt = 0.
$$

It holds

$$
\int_{Q_T} \nabla \tilde{\partial}_t \tilde{u} \cdot \nabla \tilde{\partial}_t \tilde{u} \, dx \, dt = \frac{1}{2} \int_{\Omega_T} |\nabla \tilde{\partial}_t \tilde{u}|^2 \, dx - \frac{1}{2} \int_{\Omega_0} |\nabla \tilde{\partial}_t \tilde{u}|^2 \, dx.
$$

We can write

$$
\int_{Q_T} \tilde{\partial}_t (\rho(|\nabla u|^2) \nabla u) \nabla \tilde{\partial}_t \tilde{u} \, dx \, dt =
$$

$$
\int_{Q_T} \frac{d}{dt} (\tilde{\partial}_t (\rho(|\nabla u|^2) \nabla u) \nabla \tilde{\partial}_t \tilde{u}) \, dx \, dt - \int_{Q_T} \frac{d}{dt} (\tilde{\partial}_t (\rho(|\nabla u|^2) \nabla u)) \nabla \tilde{\partial}_t \tilde{u} \, dx \, dt.
$$
Now, using $\bar{\xi}(\tau)$ defined in (5.6), we have
\[
\iint_{Q_T} \frac{d}{dt} \left( \partial_t \rho(|\nabla u|^2) \nabla u \right) \cdot \nabla \partial_t \dot{u} \, dx \, dt = \frac{1}{\Delta t} \iint_{Q_T} \frac{d}{d\tau} \iint_{0}^{1} \rho(\nabla w) \cdot |\nabla \partial_t \dot{u}|^2 \, d\tau \, dx \, dt + \\
\iint_{Q_T} \rho'(|\nabla w|^2) \frac{\partial w}{\partial x_j} \cdot \frac{\partial \dot{w}}{\partial x_i} \, d\tau \frac{\partial}{\partial x_j} \partial_t u \cdot \frac{\partial}{\partial x_i} \partial_t \dot{u} \, dx \, dt + \\
\iint_{Q_T} \rho'(|\nabla w|^2) \frac{\partial w}{\partial x_j} \cdot \frac{\partial w}{\partial x_i} \, d\tau \frac{\partial}{\partial x_j} \partial_t u \cdot \frac{\partial}{\partial x_i} \partial_t \dot{u} \, dx \, dt + \\
+ \iint_{Q_T} \rho'(|\nabla w|^2) \frac{\partial w}{\partial x_j} \cdot \frac{\partial \dot{w}}{\partial x_i} \, d\tau \frac{\partial}{\partial x_j} \partial_t u \cdot \frac{\partial}{\partial x_i} \partial_t \dot{u} \, dx \, dt + \\
\iint_{Q_T} \rho''(|\nabla w|^2) \frac{\partial w}{\partial x_k} \cdot \frac{\partial \dot{w}}{\partial x_j} \frac{\partial w}{\partial x_i} \frac{\partial \dot{w}}{\partial x_i} d\tau \frac{\partial}{\partial x_j} \partial_t u \cdot \frac{\partial}{\partial x_i} \partial_t \dot{u} \, dx \, dt.
\]

(5.17)

Using (5.15)-(5.17) and the inequality
\[
||\nabla \partial_t u||_{L^2(\Omega)} \leq ||\nabla \dot{u}||_{L^2(\Omega)},
\]
we get from the equation (5.17)
\[
\left| \iint_{Q_T} \frac{d}{dt} \left( \partial_t \rho(|\nabla u|^2) \nabla u \right) \cdot \nabla \partial_t \dot{u} \, dx \, dt \right| \leq \\
C \left( 1 + \iint_{Q_T} |\nabla \partial_t \dot{u}| \, dx \, dt \right).
\]

(5.18)

Summing up (5.15),(5.17) and (5.18) we obtain
\[
\iint_{Q_T} \partial_t |\partial_t \dot{u}|^2 \, dx \, dt + \frac{1}{2} \int_{\Omega_T} |\nabla \partial_t \dot{u}|^2 \, dx \leq C \left( 1 + \iint_{Q_T} |\nabla \partial_t \dot{u}|^2 \, dx \, dt \right) + \\
\frac{1}{2} \int_{\Omega_0} |\nabla \partial_t \dot{u}|^2 \, dx + \frac{1}{2} \int_{\Omega_0} \partial_t \left( \rho(|\nabla u|^2) \nabla u \right) \nabla \partial_t \dot{u} \, dx - \\
\frac{1}{2} \int_{\Omega_0} \partial_t \left( \rho(|\nabla u|^2) \nabla u \right) \nabla \partial_t \dot{u} \, dx \, dt.
\]

(5.19)
Also because

\[
\int_{\Omega_T} \partial_t (\rho(|\nabla u|^2) \nabla u) \nabla \partial_t \hat{u} \, dx =
\]

(5.20)

\[
\int_{\Omega_T} \int_0^1 (\rho(|\nabla w|^2) + 2\rho'(|\nabla w|^2) \frac{\partial w}{\partial x_i} \cdot \frac{\partial w}{\partial x_j}) \, d\tau \frac{\partial}{\partial x_j} \partial_t \hat{u} \cdot \frac{\partial}{\partial x_i} \partial_t \hat{u} \, dx \leq
\]

\[
C + \varepsilon \int_{\Omega_T} |\nabla \partial_t \hat{u}|^2 \, dx,
\]

we get from (5.19)

\[
\int_{Q_T} |\partial_t \hat{u}|^2 \, dx + \int_{\Omega_T} |\nabla \partial_t \hat{u}|^2 \, dx \leq
\]

(5.21)

\[
C \left( 1 + \int_{Q_T} |\nabla \partial_t \hat{u}|^2 \, dx \right) +
\]

\[
\frac{1}{2} \int_{\Omega_0} |\nabla \partial_t \hat{u}|^2 \, dx + C \left( \int_{\Omega_0} |\nabla \partial_t u|^2 \, dx + \int_{\Omega_0} |\nabla \partial_t \hat{u}|^2 \, dx \right).
\]

Now, similarly as in the above theorem, we assume that \( u \equiv u^\lambda \) is considered as the weak solution of the initial-boundary value problem with the initial data \( u_0^\lambda, u_1^\lambda \in C^2(\Omega) \), as \( \lambda \to 0_+ \).

Considering again the short time interval \([0, T_*] \), we have (5.2) from the proof of the preceding theorem.

In fact, as \( \Delta t \to 0_+ \), the first bracket on the right-hand side of (5.21) converges to

(5.22)

\[
C \left( 1 + \int_{Q_T} |\nabla \hat{u}^\lambda|^2 \, dx \right).
\]

We can choose \( u_0^\lambda \) and \( u_1^\lambda \) in such a way that, in view of Theorem 5.1 and Lemmas 4.3, 4.4, the other integrals in (5.21) are bounded independently of \( T \) and \( \lambda \).

As \( \Delta t \to 0_+ \), the last three integrals in (5.21) converge to

(5.23)

\[
C \left( \int_{\Omega_0} |\nabla \hat{u}^\lambda|^2 \, dx + \int_{\Omega_0} |\nabla \hat{u}^\lambda|^2 \, dx \right).
\]

Similarly as in the preceding proof, (5.22), (5.23) can be again bounded independently of \( \lambda \) and the assertion follows. □

6. **Generic Solution of the Transonic Flow Problems**

The following Definition and Theorem can be found in [1].
Definition 6.1. We say that a sequence \( \{u_m\}_{m=1}^{\infty} \) is a generic solution of the Transonic Flow Problem if there exist mappings \( F_m \in (W^{1,2}({\Omega}))^* \), such that

\[
\int_{\Omega} \rho(|\nabla u_m|^2) \nabla u_m \cdot \nabla \varphi \, dx = \int_{\partial \Omega} g \cdot \varphi \, dS + \langle F_m, \varphi \rangle,
\]

for all \( \varphi \in W^{1,2}(\Omega) \), and

\[
\|F_m\|_{(W^{1,2}({\Omega}))^*} \longrightarrow 0, \quad \text{as} \quad m \longrightarrow \infty.
\]

Here, \( \langle \cdot, \cdot \rangle \) denotes duality pairing.

Theorem 6.2. Let \( \Omega \) be a bounded domain with a Lipschitz boundary and let the assumptions for the density (1.7)-(1.9) and (2.6),(2.7) for the boundary function \( g \), be valid. Let (3.3), (4.2) for the initial solutions be true. Then there exists a sequence \( \{t_m\}_{m=1}^{\infty} \), such that \( \{u(\cdot, t_m)\}_{m=1}^{\infty} \) forms a generic solution of the Transonic Flow Problem.

An immediate consequence of this theorem is the fact that the stable limit of \( u(t) \), as \( t \longrightarrow \infty \), is the solution of the Subsonic Flow, which is characterized by the inequality

\[
0 \leq |\nabla u|^2 \leq \frac{2a_0^2}{\kappa + 1}.
\]

This constrain implies (\( s \equiv |\nabla u|^2 \))

\[
\rho(s) + 2\rho'(s)s \geq \rho_3 > 0, \quad \text{for each} \quad s \in \left[0, \frac{2a_0^2}{\kappa + 1}\right].
\]

Theorem 6.3. Let \( \Omega \) be a bounded domain with a Lipschitz boundary and let the assumptions for the density (1.7)-(1.9) and (2.6),(2.7) for the boundary function \( g \), be valid. Let (3.3), (4.2) for the initial solutions be true. Let, moreover, (6.10) be true. Then there exists a function \( \bar{u} \in W^{1,2}(\Omega) \) such that

\[
u(t) \longrightarrow \bar{u} \quad \text{strongly in} \quad W^{1,2}(\Omega) \quad \text{as} \quad t \longrightarrow \infty
\]

and \( \bar{u} \) is the solution of the Subsonic Flow:

\[
\int_{\Omega} \rho(|\nabla \bar{u}|^2) \nabla \bar{u} \cdot \nabla \varphi \, dx = \int_{\partial \Omega} g \varphi \, dS \quad \forall \varphi \in W^{1,2}(\Omega).
\]

Proof. Using the estimates already obtained, we have

\[
u(t) \longrightarrow \bar{u} \quad \text{weakly in} \quad L^2(Q_\infty), \quad \text{as} \quad t \longrightarrow \infty \quad \text{and} \quad \nabla \nu(t) \longrightarrow \nabla \bar{u} \quad \text{weakly-star in} \quad L^\infty([0, \infty], L^2(\Omega)), \quad \text{as} \quad t \longrightarrow \infty.
\]

From Definition 3.1 of the weak solution with the test function \( \varphi = \nu(t) - \bar{u} \), we get

\[
\rho_3 \int_{\Omega} |\nabla(\nu(t) - \bar{u})|^2 \, dx \leq 
\]

\[
\int_{\Omega} \bar{u}(\bar{u} - \nu(t)) \, dx + \int_{\Omega} \nabla \bar{u} \cdot \nabla(\bar{u} - \nu(t)) \, dx - \int_{\Omega} \rho(|\nabla \bar{u}|^2) \nabla \bar{u} \cdot \nabla(\nu(t) - \bar{u}) \, dx.
\]
Because of \( u(t) \to \bar{u} \) a.e. in \( \Omega \) and \( \bar{u} \in L^1([0, \infty), L^2(\Omega)) \), the right-hand side of (6.12) converges to zero, as \( t \to \infty \). Thus

\[
(6.6) \quad \nabla u(t) \to \nabla \bar{u} \quad \text{a.e. in } \Omega.
\]

Because of the continuity of the density function \( \rho \), we have

\[
\rho(|\nabla u(t)|^2) \to \rho(|\nabla \bar{u}|^2), \quad \text{a.e. in } \Omega \text{ for } t \to \infty.
\]

We know (Theorem 6.2) that there exists a sequence \( \{t_m\} \) such that \( \{u(t_m)\} \) forms the generic solution, thus the proof is complete. \( \Box \)

7. Existence of the entropic solution of the Transonic Flow Problems

We have already pointed out (in (1.11) and (1.12)) that, with respect to the value of the velocity, the domain of computation can be divided into two parts: \( \Omega_1 \) and \( \Omega_2 \). The boundary between these two subdomains - \( \Gamma_{\text{SHOCK}} \) - contains possible shocks with jumps in \( \vec{v}, \rho \) and \( p \), or equivalently: \( \vec{v}, \rho \) and \( p \) are not continuously differentiable across the ”shock/sonic line”.

It can be shown using the standard arguments that the following Rankine-Hugoniot conditions hold across the shock:

\[
(7.1) \quad \nabla u_- \cdot \vec{i} = \nabla u_+ \cdot \vec{i},
\]

\[
(7.2) \quad \rho(|\nabla u_-|^2)\nabla u_- \cdot \vec{n} = \rho(|\nabla u_+|^2)\nabla u_+ \cdot \vec{n},
\]

where +, - denotes the quantities in front of the shock and behind the shock, respectively, \( \vec{i}, \vec{n} \) are the tangential and normal vectors to the shock.

While these conditions are automatically satisfied by the weak solution, there is a so-called Entropy condition, which has to be satisfied by the resulting solution to be a physical one. It can be formulated (cf. Ladam-Lifschitz [10]) as follows:

\[
(7.3) \quad \rho(|\nabla u_-|^2) \leq \rho(|\nabla u_+|^2). \quad \text{on } \Gamma_{\text{SHOCK}}.
\]

This can be expressed as ”a compression shocks are the only acceptable ones”. In view of (7.1)-(7.3), this requirement can be also formulated as

\[
(7.4) \quad \nabla u_- \cdot \vec{n} > \nabla u_+ \cdot \vec{n} \quad \text{on } \Gamma_{\text{SHOCK}}.
\]

This condition is usually used in the weaker form (see Bristeau et. al [11]):

**Definition 7.1.** We say that \( u \in W^{1,2}(\Omega) \) satisfies the entropy condition if

\[
(7.5) \quad \forall \varphi \in D^+(\Omega) : \quad -\int_{\Omega} \nabla u \cdot \nabla \varphi \, dx \leq K \int_{\Omega} \varphi \, dx,
\]

where \( K \) is a positive fixed constant.

**Remark.** (i) If \( u \) satisfies the entropy condition (7.5), then it can be shown that \( u \) satisfies also (7.4), provided that \( u|_{\Omega_i} \in C^2(\Omega_i) \cap C^1(\bar{\Omega}_i), \quad i = 1, 2 \) and that the boundary \( \Gamma_{\text{SHOCK}} \) is smooth enough.

(ii) There is also another possibility to formulate the entropy condition:

\[
(7.6) \quad \forall \varphi \in D^+(\Omega) : \quad -\int_{\Omega} \rho'(|\nabla u|^2)|\nabla u|^2 \nabla u \cdot \nabla \varphi \, dx \leq K \int_{\Omega} \varphi \, dx,
\]
**Theorem 7.2.** Let us assume that there exists a constant \( C > 0 \) s.t. \( |\nabla u(t)|^2 \leq C \), for any \( t > 0 \) and assume that all the assumptions which guarantee the global existence of the strong solution are valid (assumptions of Theorem 4.5). Let \( u_0 \equiv u(\cdot, 0) \in C^2(\Omega) \) and, moreover,

\[
(7.7) \quad \int_0^T \left( \rho'(|\nabla u|^2) \frac{\partial u}{\partial x_i} \frac{\partial u}{\partial x_j} \frac{\partial^2 u}{\partial x_i \partial x_j} \right)^- \, dt \rightarrow f(x) \quad \text{weakly in } L^2_{loc}(\Omega) \quad \text{as } T \rightarrow \infty.
\]

Then there exists a function \( \bar{u} \in W^{1,2}(\Omega) \) such that

\[
u(t) \rightarrow \bar{u}, \quad \text{strongly in } W^{1,2}(\Omega)
\]

and \( \bar{u} \) is the weak solution of transonic flow problem which is entropic in the sense

\[
\Delta \bar{u} \leq M_0 + f,
\]

where \( M_0 \) is a constant such that

\[
(7.8) \quad -\int_\Omega \nabla u_0 \cdot \nabla \varphi \, dx \leq M_0 \int_\Omega \varphi \, dx, \quad \forall \varphi \in \mathcal{D}^+(\Omega).
\]

**Remark.** In virtue of Theorem 4.2, we have \( \dot{u} \in L^2([0, \infty]; W^{2,2}_{loc}(\Omega)) \), thus

\[
u \in L^2([0, T]; W^{2,2}_{loc}(\Omega)), \quad \text{for any } T > 0.
\]

**Proof.** For the sake of simplicity we will assume that \( \dot{u}(0) = 0 \). We already know that

\[
(7.9) \quad \int_{\Omega_T} \dot{u} \varphi \, dx + \int_{\Omega_T} \nabla \dot{u} \cdot \nabla \varphi \, dx + \int_{\Omega_T} \rho(|\nabla u|^2) \nabla u \cdot \nabla \varphi \, dx = 0, \quad \forall \varphi \in \mathcal{D}^+(\Omega).
\]

Integrating the above equation over the time interval \([0, T] \), we get

\[
(7.10) \quad \int_{\Omega_T} \dot{u} \varphi \, dx + \int_{\Omega_T} \nabla u \cdot \nabla \varphi \, dx + \int_{Q_T} \rho(|\nabla u|^2) \nabla u \cdot \nabla \varphi \, dx \, dt = \int_{\Omega_0} \nabla u \cdot \nabla \varphi \, dx, \quad \forall \varphi \in \mathcal{D}^+(\Omega).
\]

This yields the inequality \( \forall \varphi \in \mathcal{D}^+(\Omega) \)

\[
(7.11) \quad -\int_{\Omega_T} \nabla u \cdot \nabla \varphi \, dx \leq -\int_{\Omega_0} \nabla u \cdot \nabla \varphi \, dx + \int_{\Omega_T} \dot{u} \varphi \, dx + \int_{Q_T} \left( \frac{\partial}{\partial x_i} \left( \rho(|\nabla u|^2) \nabla u \right) \right)^- \cdot \varphi \, dx \, dt.
\]

Now, in view of Lemma 4.3 there exists a sequence \( \{t_m\}_{m=1}^{\infty} \), s.t.

\[
(7.12) \quad u(t_m) \rightarrow \bar{u} \quad \text{weakly in } W^{1,2}(\Omega), \quad \text{as } t_m \rightarrow \infty.
\]

Because of the bound of the gradient of \( u \), we get, by the application of the result of Mayers [12],

\[
(7.13) \quad \|u(t_m)\|_{W^{2,r}(\Omega)} \leq C \quad \text{for some } p > 2,
\]
and because a bounded, closed, convex set \( \{u(t_m) | t_m \geq 0\} \) is weakly closed, also
\[
\|\overline{u}\|_{W^{2,p}(\Omega)} \leq C
\]

Let us define \( G(T) \in (W^{1,2}_0(\Omega'))^* \) as
\[
< G(T), \varphi > \equiv - \int_{\Omega_0} \nabla u \cdot \nabla \varphi \, dx + \int_{\Omega_T} \dot{u} \varphi \, dx + 2 \int_{Q_T} \left( \frac{\partial}{\partial x_i} (\rho(|\nabla u|^2) \nabla u) \right) \cdot \varphi \, dx \, dt + \int_{\Omega_T} \nabla u \cdot \nabla \varphi \, dx,
\]
where \( \Omega' \equiv \Omega_h = \{x \in \Omega | \text{dist}(x, \partial \Omega) \geq h\} \). Now, we claim that there exists a sequence \( \{T_m\}_{m=1}^\infty \)

\[
\int_{\Omega_{T_m}} \hat{u} \varphi \, dx \longrightarrow 0, \quad T_m \longrightarrow \infty, \quad \forall \varphi \in D^+(\Omega).
\]

To show this, we estimate
\[
\int_{\Omega_{T_m}} \hat{u} \varphi \, dx \leq \|\varphi\|_{L^2(\Omega)} \cdot \|\hat{u}(T_m)\|_{L^2(\Omega)} \leq \|\varphi\|_{L^2(\Omega)} \cdot \|\nabla \hat{u}(T_m)\|_{L^2(\Omega)}.
\]

Using the result of Theorem 6.2 we get a subsequence of \( \{T_m\}_{m=1}^\infty \), s.t.
\[
|\nabla \hat{u}(T_{m_k})| \longrightarrow 0, \quad \text{as } T_{m_k} \longrightarrow \infty.
\]

Thus, if we define \( G \in \left(W^{1,2}_0(\Omega')\right)^* \)

\[
< G, \varphi > \equiv - \int_{\Omega_0} \nabla u \cdot \nabla \varphi \, dx + \int_{\Omega} f \cdot \varphi \, dx + \int_{\Omega} \nabla \overline{u} \cdot \nabla \varphi \, dx,
\]

we see that
\[
G(T) \longrightarrow G \quad \text{weakly in } (W^{1,2}_0(\Omega'))^* \quad \text{and}
\]
\[
< G(T), \varphi > \geq 0, \quad \forall \varphi \in D^+(\Omega).
\]

Using the stronger version of the Murat's theorem [13] (see Nečas [14]) we obtain
\[
G(T) \longrightarrow G \quad \text{strongly in } (W^{1,p}_0(\Omega'))^*.
\]

Let us denote \( w(t) = (u(t) - \overline{u})\sigma_h^2 \), with \( \sigma_h \) defined in (5.1)-(5.3). Then \( w(t) \in W^{1,p}_0(\Omega') \).

Moreover, we have
\[
|\nabla w(T)|^2 = < G(T), w(T) > - < G, w(T) > - \int_{\Omega_T} \hat{u} w \, dx + \int_{\Omega_T} f w \, dx - \int_{Q_T} \left( \frac{\partial}{\partial x_i} (\rho(|\nabla u|^2) \frac{\partial u}{\partial x_i}) \right) \cdot w \, dx \, dt.
\]

In view of the compact embedding of \( W^{1,2}(\Omega) \) into \( L^2(\Omega) \), (7.20) and (7.7), the right-hand side of (7.21) converges to zero. The continuity of the density function \( \rho \) and (5.1) give
\[
\rho(|\nabla u(T)|^2) \longrightarrow \rho(|\nabla \overline{u}|^2) \quad \text{a.e. in } \Omega \text{ as } T \longrightarrow \infty.
\]

We already know that there exists a sequence \( \{T_k\} \) so that \( \{u(T_k)\} \) is generic, thus the proof follows.  \( \Box \)
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