WELL-POSEDNESS AND REGULARITY RESULTS FOR A DYNAMIC VON KÁRMÁN PLATE

By

M.E. Bradley

IMA Preprint Series # 1122
March 1993
WELL-POSEDNESS AND REGULARITY RESULTS
FOR A DYNAMIC VON KÁRMAÑ PLATE

M.E. BRADLEY*

Abstract. We consider the problem of well-posedness and regularity of solutions for a dynamic von Kármán plate which is clamped along one portion of the boundary and which experiences boundary damping through “free edge” conditions on the remainder of the boundary. We prove the existence of unique strong solutions for this system.

Key words. von Kármán plate, strong regularity, well-posedness

AMS(MOS) subject classifications. 35B65, 47N20, 73K10

1. Introduction. In this paper, we consider the well-posedness of the von Kármán system given by

\[ \begin{align*}
    w_{tt} - \gamma^2 \Delta w_{tt} + \Delta^2 w &= [w, F(w)] \quad \text{in} \quad Q = \Omega \times (0, T) \\
    w(0, \cdot) = w_0; w_t(0, \cdot) = w_1 &\quad \text{in} \quad \Omega \\
    w = \frac{\partial}{\partial \nu} w = 0 &\quad \text{on} \quad \Sigma_0 = \Gamma_0 \times (0, T) \\
    \Delta w + (1 - \mu) B_1 w &= -\frac{\partial}{\partial \nu} w_t \quad \text{on} \quad \Sigma_1 = \Gamma_1 \times (0, T) \\
    \frac{\partial}{\partial \nu} \Delta w + (1 - \mu) B_2 w - \gamma^2 \frac{\partial}{\partial \nu} w_{tt} &= w_t - \frac{\partial^2}{\partial \tau^2} w_t \quad \text{on} \quad \Sigma_1,
\end{align*} \]

(1.1)

where we assume \( \Omega \subset \mathbb{R}^2 \), with sufficiently smooth boundary \( \Gamma = \Gamma_0 \cup \Gamma_1 \). Here, \( 0 < \mu < \frac{1}{2} \) represents Poisson’s ratio and the boundary operators \( B_1 \) and \( B_2 \) are given by

\[ \begin{align*}
    B_1 w &= 2n_1 n_2 w_{xy} - n_1^2 w_{yy} - n_2^2 w_{xx} \\
    B_2 w &= \frac{\partial}{\partial \tau} \left[ (n_1^2 - n_2^2) w_{xy} + n_1 n_2 (w_{yy} - w_{xx}) \right] \\
\end{align*} \]

(1.1)(b)

Also, \( F(w) \) satisfies the system of equations

\[ \begin{align*}
    \Delta^2 F &= -[w, w] \\
    F = \frac{\partial}{\partial \nu} F &= 0 \quad \text{on} \quad \Sigma = \Gamma \times (0, \infty)
\end{align*} \]

(1.2)

where

\[ [\phi, \psi] = \frac{\partial^2 \phi}{\partial x^2} \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \phi}{\partial y^2} \frac{\partial^2 \psi}{\partial x^2} - 2 \frac{\partial^2 \phi}{\partial x \partial y} \frac{\partial^2 \psi}{\partial x \partial y}. \]

The well-posedness and regularity of such a system is both a delicate and interesting problem. Such results are important in solving the problem of stabilization for system (1.1). Usual PDE techniques require the existence and uniqueness of “smooth”

---

* Department of Mathematics, University of Louisville, Louisville, KY 40292.
solutions to justify computations used in determining the stability and controllability of dynamical models. The stabilization of thin plates (and particularly the von Kármán system) is of current interest in the literature (see ([L-1], [L-2], [B-L], [P-T], [L-L])). The von Kármán nonlinearity poses many difficulties in obtaining the well-posedness and regularity results we seek. Difficulties also arise from the higher order boundary conditions on \( \Sigma \). To handle these difficulties we adapt abstract results proven in [F-L] to our more difficult boundary conditions.

This paper will proceed as follows. In Section 2 we state the main results of our paper. After this we state the appropriate abstract results from [F-L] which will be useful in the proofs of our results. In Section 3 we prove the results stated in Section 2.

2. Statement of Results. Before stating the results we intend to prove, we define the meaning of “weak solutions” through a variational equality. Let

\[
H^2_{\Gamma_0}(\Omega) = \left\{ w \in H^2(\Omega) : w = \frac{\partial w}{\partial \nu} = 0 \text{ on } \Gamma_0 \right\}
\]

with norm

\[
\|w\|^2_{H^2_{\Gamma_0}(\Omega)} = \int_\Omega ((\Delta w)^2 + 2(1 - \mu)(w_{xy}^2 - w_{xx}w_{yy}))d\Omega
\]

and let

\[
H^1_{\Gamma_0}(\Omega) = \left\{ w \in H^1(\Omega) : w = 0 \text{ on } \Gamma_0 \right\}
\]

with norm

\[
\|w\|^2_{H^1_{\Gamma_0}(\Omega)} = \int_\Omega (w^2 + \gamma^2|\nabla w|^2)d\Omega.
\]

We define the solution space \( \mathcal{H} = H^2_{\Gamma_0}(\Omega) \times H^1_{\Gamma_0}(\Omega) \).

**Definition 2.1.** A function pair \((w, w_\tau) \in C((0, T); \mathcal{H})\) is said to be a weak solution to system (1.1) if \((w(\cdot, 0), w_\tau(\cdot, 0)) = (w_0, w_1)\) and \(w\) satisfies the variational equation

\[
\frac{d}{dt} \left( (I - \gamma^2\Delta)w_\tau, \varphi \right) + (\Delta^2 w, \varphi) - ([w, F], \varphi)\forall \varphi \in H^2_{\Gamma_0}(\Omega),
\]

where here and throughout the paper \((\cdot, \cdot)\) denotes either the \(L^2\)-inner product or the duality pairing between \(H^2_{\Gamma_0}(\Omega)\) and \([H^2_{\Gamma_0}(\Omega)]',\) as is appropriate by context. We note that (2.1) holds in \(H^{-1}[0, T]\).

**Theorem 2.1.** Given initial data \((w_0, w_1) \in \mathcal{H},\) there exists a unique weak solution to system (1.1), \((w, w_\tau) \in C([0, T], \mathcal{H})\) for any \(T > 0\).
THEOREM 2.2. (Regularity): Assume in addition to Theorem 2.1 that the initial data satisfy

\begin{align*}
( i ) & \quad w_0 \in H^3(\Omega); \quad w_1 \in H^2_{\Gamma_0}(\Omega); \\
( ii ) & \quad \begin{cases}
\Delta w_0 + (1 - \mu) B_1 w_0 = -\frac{\partial}{\partial v} w_1 \\
\frac{\partial^2 w_0}{\partial v^2} + (1 - \mu) B_2 w_0 = w_1 - \frac{\partial^2}{\partial v^2} w_1
\end{cases} \quad \text{on } \Gamma_1.
\end{align*}

Then the unique solution to (1.1) has the regularity

\begin{align*}
( i ) & \quad (w, w_t) \in C((0, T); (H^3(\Omega) \cap H^2_{\Gamma_0}(\Omega)) \times H^2_{\Gamma_0}(\Omega)); \\
( ii ) & \quad w_{tt} \in C((0, T); H^1_{\Gamma_0}(\Omega)) \\
( iii ) & \quad \text{equation (2.1) is satisfied for every } t \in [0, T).
\end{align*}

THEOREM 2.3. (Strong Regularity): In addition to Theorems 2.1 and 2.2 we assume that

\begin{align*}
( i ) & \quad w_0 \in H^4(\Omega); \quad w_1 \in H^3(\Omega) \cap H^2_{\Gamma_0}(\Omega), \\
( ii ) & \quad \begin{cases}
\Delta w_0 + (1 - \mu) B_1 w_1 = -\frac{\partial w_{tt}(0)}{\partial v} \\
\frac{\partial^2 w_0}{\partial v^2} + (1 - \mu) B_2 w_1 = w_{tt}(0) - \frac{\partial^2 w_{tt}(0)}{\partial v^2}
\end{cases} \quad \text{on } \Gamma_1.
\end{align*}

where \( w_{tt}(0) \) is derived from the equation (1.1). Then the unique solution guaranteed by Theorem 2.1 has the following regularity properties:

\begin{align*}
( i ) & \quad (w, w_t) \in C((0, T); (H^4(\Omega) \cap H^2_{\Gamma_0}(\Omega)) \times (H^3(\Omega) \cap H^2_{\Gamma_0}(\Omega))); \\
( ii ) & \quad w_{tt} \in C((0, T); H^1_{\Gamma_0}(\Omega)) \\
( iii ) & \quad w_{ttt} \in C((0, T); H^1_{\Gamma_0}(\Omega)).
\end{align*}

Moreover, equation (1.1) holds in the \( L^2 \)-sense for each \( t \in [0, T] \).

The proofs of Theorems 2.1–2.3 will be based primarily on the work of Favini and Lasiecka [F-L]. That paper deals with abstract problems of the form

\begin{equation}
\begin{cases}
M w_{tt}(t) + A w(t) + A G^* A w(t) + A G f(w)(t) = F(w)(t) \\
w(t = 0) = w_0; \quad w_t(t = 0) = w_1,
\end{cases}
\end{equation}

which will be described in detail shortly. Our intention in this paper is to recast system (1.1) in the abstract framework of (2.4). We will then show that the results of [F-L] may be applied directly to or may be adapted for our system. For the purpose of self-containment, we now state the necessary background and results from [F-L] which will be useful in this present context.
Let $\mathcal{A}$ be a closed, positive self-adjoint operator on a Hilbert space $H$ with $\mathcal{D}(\mathcal{A}) \subset H$. Let $V$ be another (appropriately chosen) Hilbert space such that
\[
\mathcal{D}(\mathcal{A}^{1/2}) \subset V \subset H \subset V' \subset [\mathcal{D}(\mathcal{A}^{1/2})]'.
\]
We assume that $M : V \to V'$ is both bounded and boundedly invertible so that the restriction $\tilde{M} \equiv M|_H$ with domain $\mathcal{D}(\tilde{M}) = \{u \in V : Mu \in H\}$ gives that $V = \mathcal{D}(\tilde{M})^{1/2})$.

The operator $G$ is defined on another Hilbert space, $U$, whose inner product is denoted by $\langle \cdot, \cdot \rangle$. It is assumed that $G : U \to H$ is a bounded linear operator such that $G^*\mathcal{A} \in \mathcal{L}(\mathcal{D}(\mathcal{A}^{1/2}) ; H)$.

Finally, the nonlinear term $\mathcal{F} : \mathcal{D}(\mathcal{A}^{1/2}) \to V'$ is assumed to be Frechét differentiable with derivative, denoted $D\mathcal{F}$, satisfying
\[
\|D\mathcal{F}(u)h\|_{V'} \leq C(\|u\|_{\mathcal{D}(\mathcal{A}^{1/2})}) \|h\|_{\mathcal{D}(\mathcal{A}^{1/2})}.
\]
We note that for our purposes, $f \equiv 0$.

We now state the results from [F-L] which form the framework for Theorem 2.1–2.3.

**Theorem 2.4.** (F-L Theorem 2.1): For each initial data $(w_0, w_1) \in \mathcal{D}(\mathcal{A}^{1/2}) \times V$, there exists $T_0 > 0$ such that there exists a unique weak solution $(w(t), w_t(t))$ to (2.4).

**Theorem 2.5.** (F-L Theorem 2.4): In addition to the hypotheses of Theorem 2.4 we assume that for all $\tilde{w} = (w, w_1) \in C(0, T_0; \mathcal{D}(\mathcal{A}^{1/2}) \times V)$ and such that $G^*\mathcal{A}w_t \in L^2(0, T_0; U)$ the following inequality holds for all $t \in [0, T_0)$:

\[
\int_0^t (\mathcal{F}(w(\tau)), w_t(\tau)) d\tau \leq C_1 \int_0^t (\|w(\tau)\|_{\mathcal{D}(\mathcal{A}^{1/2})}^2 + \|w_t(\tau)\|_V^2) d\tau + C_2(\|(w_0, w_1)\|_{\mathcal{D}(\mathcal{A}^{1/2}) \times V}) \equiv C_0.
\]

Then the weak solution $(w(t), w_t(t))$ is global for any $T > 0$.

**Theorem 2.6.** (F-L Theorem 2.2): Assume that the initial data $(w_0, w_1)$ satisfy
\[
(2.6) \quad (i) \quad w_1 \in \mathcal{D}(\mathcal{A}^{1/2}) \\
(ii) \quad \mathcal{A}(w_0 + \beta GG^*\mathcal{A}w_1) \in V'.
\]

Moreover, assume that
\[
(2.7) \quad \|\mathcal{A}^{-1/2}D\mathcal{F}(w)h\|_H \leq C(\|w\|_{\mathcal{D}(\mathcal{A}^{1/2})}) \|h\|_V.
\]

Then the solution satisfies (2.4) in the sense of the $[\mathcal{D}(\mathcal{A}^{1/2})]'$ topology for each $T \in (0, T)$, $(w(0), w_t(0)) = (w_0, w_1)$ and has the following regularity:

$(w, w_t) \in C(0, T; \mathcal{D}(\mathcal{A}^{1/2}) \times \mathcal{D}(\mathcal{A}^{1/2}))$,

$(w_t) \in C(0, T; V)$.

By showing that system (1.1) can be formulated in the framework of the abstract equation (2.4) while satisfying the hypotheses of Theorems 2.5–2.7, we will have proven Theorems 2.1 and 2.2. For the additional regularity given in Theorem 2.3, we will need an additional proof which does not follow directly from results of [F-L].
3. Proofs of Theorems 2.1–2.3. Let $V = H^1_{\Gamma_0}(\Omega)$, $H = L^2(\Omega)$ and $U = (L^2(\Gamma_1))^3$. We define $\mathcal{A}$ on $H^2_{\Gamma_0}(\Omega)$ by

\[
\mathcal{A}w \equiv \Delta^2 w \text{ with domain} \quad D(\mathcal{A}) = \left\{ w \in H^4(\Omega) \cap H^2_{\Gamma_0}(\Omega) : \Delta w + (1 - \mu)B_1w = 0 \right. \\
\left. \text{and } \frac{\partial}{\partial \nu}\Delta w + (1 - \mu)B_2w = 0 \text{ on } \Gamma_1 \right\},
\]

which is well-defined, positive and self-adjoint. By the results of Grisvard [G], we see that $\mathcal{D}(\mathcal{A}^{1/2}) = H^2_{\Gamma_0}(\Omega)$. We also define the Green maps, $G_1 : L^2(\Gamma) \rightarrow H^{5/2}(\Omega)$ and $G_2 : H^S(\Gamma) \rightarrow H^{7/2+S}(\Omega)$, by

\[
G_1 h = v \iff \Delta^2 v = 0 \quad \text{in } Q \\
\begin{align*}
\Delta v &= 0 \\
\frac{\partial}{\partial \nu}v &= h \\
\frac{\partial}{\partial \nu}\Delta v + (1 - \mu)B_2v &= 0
\end{align*} \\
\begin{align*}
& \text{on } \Sigma_0 \\
& \text{on } \Sigma_1
\end{align*}
\]

(3.2)

and

\[
G_2 h = v \iff \Delta^2 v = 0 \quad \text{in } Q \\
\begin{align*}
\Delta v &= 0 \\
\frac{\partial}{\partial \nu}v &= 0 \\
\frac{\partial}{\partial \nu}\Delta v + (1 - \mu)B_2v &= h
\end{align*} \\
\begin{align*}
& \text{on } \Sigma_0 \\
& \text{on } \Sigma_1 .
\end{align*}
\]

(3.3)

A straightforward computation shows that for $w \in \mathcal{D}(\mathcal{A})$,

\[
G^*_1 \mathcal{A}w = \frac{\partial w}{\partial \nu} |_{\Gamma_1} \\
G^*_2 \mathcal{A}w = -w |_{\Gamma_1}.
\]

(3.4)

Let $\bar{u} \in [L^2(\Gamma)]^3$. Define $G\bar{u} = -G_1u_1 + G_2(-u_2 + \frac{\partial^2}{\partial \tau^2}u_3)$. Then $G : [L^2(\Gamma)]^3 \rightarrow L^2(\Omega)$ is bounded and $G^* \mathcal{A} \in \mathcal{L}(H^2_{\Gamma_0}(\Omega); [L^2(\Gamma)]^3)$.

We now introduce the operator $M : \mathcal{D}(M) \subset H^2(\Omega) \rightarrow L^2(\Omega)$,

\[
Mw = (I - \gamma^2 \Delta)w + \gamma^2 G^*_2 \frac{\partial w}{\partial \nu}.
\]

(5.1)

We observe that for $v, w \in H^1_{\Gamma_0}(\Omega)$,

\[
(Mw, v) = (w, v) + \gamma^2(\nabla w, \nabla v) \\
+ \gamma^2 \left( \frac{\partial w}{\partial \nu}, G^*_2 \mathcal{A}v \right)_{L^2(\Gamma)} - \gamma^2 \left( \frac{\partial w}{\partial \nu}, v \right)_{L^2(\Gamma)} \\
= (v, w) + \gamma^2(\nabla v, \nabla w),
\]

(3.5)

where we have interpreted the $\gamma^2 G^*_2 \frac{\partial w}{\partial \nu}$ term in the sense of duality. Using (3.5), we see that $M : H^1_{\Gamma_0}(\Omega) \rightarrow [H^1_{\Gamma_0}(\Omega)]'$ is an isomorphism (by the Lax-Milgram Theorem).
Defining $\mathcal{F}(w) = [w, F(w)]$, we can now rewrite system (1.1) in the form of (2.4).

To see that the von Kármán nonlinearity is Fréchet differentiable, we define the operator

$$
(3.6) \quad A_0w = \Delta^2w \text{ with } D(A_0) = H^4(\Omega) \cap H_0^2(\Omega).
$$

Then $F(w) = -A_0^{-1}[w, w]$ so that $\mathcal{F}(w) = -[w, A_0^{-1}[w, w]]$. By straight-forward (but somewhat lengthy) computations we see that

$$
(3.7) \quad D\mathcal{F}(w)h = [h, A_0^{-1}[w, w]] + 2[w, A_0^{-1}[w, h]].
$$

To prove that $\|D\mathcal{F}(w)h\|_{[H^1_0(\Omega)]'} \leq C(\|w\|_{H^2(\Omega)})\|h\|_{H^2_0(\Omega)}$, we use the following lemma, which is proved in [B-L].

**Lemma 3.1.** The mapping $(u, v, w) \rightarrow [u, A_0^{-1}[v, w]]$ is continuous from $[H^2(\Omega)]^3 \rightarrow H^{-2}(\Omega)$ for $0 < \varepsilon < 1/2$.

Consequently, we have

$$
\|D\mathcal{F}(w)h\|_{[H^1_0(\Omega)]'} \leq \|D\mathcal{F}(w)h\|_{H^{-2}(\Omega)} \leq C\|w\|_{H^2(\Omega)}^2\|h\|_{H^2_0(\Omega)}.
$$

**Remark.** An interesting estimate which arises in the proof of Lemma 3.1 is

$$
(3.8) \quad \|A_0^{-1}[w, v]\|_{H^{2-\varepsilon}(\Omega)} \leq C\|w\|_{H^2(\Omega)}\|v\|_{H^2(\Omega)}.
$$

This will be useful to us later in the proof. □

**Proof of Theorem 2.1:** To complete the proof, it suffices to show that (2.5) holds. Let $(w, w_t) \in C([0, T]; H^2_0(\Omega) \times H^1_0(\Omega))$. Then

\[
\int_0^t \int_\Omega [w, F(w)]w_t d\Omega dt = \int_0^t \int_\Omega [w, w_t]F(w) d\Omega dt
\]

\[
= \int_0^t \int_\Omega \frac{1}{2} \left( \frac{d}{dt} [w, w] \right) F(w) d\Omega dt
\]

\[
= -\frac{1}{2} \int_0^t \int_\Omega \frac{d}{dt} (\Delta^2 F(w)) F(w) d\Omega dt
\]

\[
= -\frac{1}{2} \int_0^t \int_\Omega \frac{d}{dt} (\Delta F(w))^2 d\Omega dt
\]

\[
\leq \frac{1}{4} \int_\Omega (\Delta F(w_0))^2 d\Omega = C\|F(w_0)\|_{H^2(\Omega)}^2
\]

\[
\leq C\|w_0\|_{H^2(\Omega)}^2,
\]

where the last inequality holds by (3.8). Hence, (2.5) holds with $C_1 \equiv 0$. □
Proof of Theorem 2.2: It suffices to verify (2.6) and (2.7) and to apply Theorem 2.6. We note that (2.6)(i) is satisfied by hypothesis (2.2)(i) in Theorem 2.2. As for (2.6)(ii), we see that in p.d.e. form this is equivalent to

\[
\Delta^2 w_0 \in [H^1_{\Gamma_0}(\Omega)]', \\
w_0 = \frac{\partial w_0}{\partial \nu} = 0 \quad \text{on } \Gamma_0, \\
\Delta w_0 + (1 - \mu) B_1 w_0 = -\frac{\partial w_0}{\partial \nu}, \\
\frac{\partial \Delta w_0}{\partial \nu} + (1 - \mu) B_2 w_0 = w_1 - \frac{\partial^2 w_1}{\partial \tau^2} \quad \text{on } \Gamma_1.
\]

But then if \( w_0 \in H^3(\Omega) \cap H^1_{\Gamma_0}(\Omega) \) and \((w_0, w_1)\) satisfy the compatibility relation (2.2)(ii), we see that (2.6) must hold also.

We now prove (2.7). We need to show that for \( w \in H^2_{\Gamma_0}(\Omega) \), \( h \in H^1_{\Gamma_0}(\Omega) \) and \( \varphi \in H^2_{\Gamma_0}(\Omega) \) that

\[
|(D\mathcal{F}(w)h, \varphi)| \leq C\left(\|w\|_{H^2_{\Gamma_0}(\Omega)}\|h\|_{H^1_{\Gamma_0}(\Omega)}\|\varphi\|_{H^2_{\Gamma_0}(\Omega)}\right).
\]

Recalling (3.6)–(3.7), we compute

\[
\begin{align*}
|([h, A_0^{-1}[w, w]], \varphi)| &= |([\varphi, F(w)], h)| \\
&\leq \|h\|_{H^1_{\Gamma_0}(\Omega)}\|\varphi, F(w)\|_{H^2_{\Gamma_0}(\Omega)}' \\
&\leq \|h\|_{H^1_{\Gamma_0}(\Omega)}\|\varphi, A_0^{-1}[w, w]\|_{H^{-\epsilon}(\Omega)} \\
&\leq C(\|w\|_{H^2_{\Gamma_0}(\Omega)}\|h\|_{H^1_{\Gamma_0}(\Omega)}\|\varphi\|_{H^2_{\Gamma_0}(\Omega)}),
\end{align*}
\]

where we have used Lemma 3.1.

We now compute

\[
\begin{align*}
|([w, A_0^{-1}[w, h]], \varphi)| &= |([w, \varphi], A_0^{-1}[w, h])| \\
&\leq \|w\|_{H^1(\Omega)}\|\varphi\|_{H^{-1-\epsilon}(\Omega)}\|A_0^{-1}[w, h]\|_{H^1(\Omega)}' \\
&\leq C\|w\|_{H^2_{\Gamma_0}(\Omega)}\|\varphi\|_{H^2(\Omega)}\|A_0^{-\frac{3}{4}+\epsilon/4}[w, h]\|_{L^2(\Omega)},
\end{align*}
\]

where we have again used the results of Grisvard [G] to give us \( \mathcal{D}(A_0^{\frac{3}{4}+\epsilon/4}) \sim H^{1+\epsilon}(\Omega) \).

We now examine the term \( \|A_0^{-3/4+\epsilon/4}[w, h]\|_{L^2(\Omega)} \). Let \( \psi \in \mathcal{D}(A_0^{\frac{3}{4}-\epsilon/4}) \) so that (again by Grisvard’s results) we have \( \psi \in H^{3-\epsilon}(\Omega) \cap H^3_0(\Omega) \). Then

\[
|([w, h], \psi)| = |([w, \psi], h)| \leq C\|w\|_{H^2(\Omega)} \left(\int_{\Omega} (\psi_{yy}^2 + \psi_{xx}^2 + \psi_{xy}^2) h^2 d\Omega\right)^{1/2}.
\]

But then since \( h \in H^1(\Omega) \subset L^q(\Omega), 1 \leq q < \infty \), and by Hölder’s inequality, we have, for example,

\[
\int_{\Omega} \psi_{yy}^2 h^2 d\Omega \leq \left(\int_{\Omega} \psi_{yy}^{2p} d\Omega\right)^{1/p} \left(\int_{\Omega} h^{2q} d\Omega\right)^{1/q} \\
= \|\psi_{yy}\|_{L^{2p}(\Omega)}^2 \|h\|_{L^{2q}(\Omega)}^2 \\
\leq C\|\psi_{yy}\|_{L^{2+\epsilon_0}(\Omega)}^2 \|h\|_{H^1(\Omega)}^2.
\]
Using the Sobolev imbeddings (see [Adams], Theorem 7.58 p. 218), this implies
\[ \| \psi \eta h \|_{L^2(\Omega)} \leq C \| \psi \|_{H^{2+\varepsilon_1}(\Omega)} \| h \|_{H^1(\Omega)}, \]
where \( \varepsilon_1 = \frac{\varepsilon_0}{2+\varepsilon_0} \). Substituting back into (3.11), we obtain
\[
(3.12) \quad |([w, \psi], h)| \leq \tilde{C} \| w \|_{H^2(\Omega)} \| \psi \|_{H^{2+\varepsilon}(\Omega)} \| h \|_{H^1(\Omega)} \\
\quad \leq C \| w \|_{H^2(\Omega)} \| \psi \|_{H^{2-\varepsilon}(\Omega)} \| h \|_{H^1(\Omega)}.
\]
Putting (3.10)–(3.12) together implies
\[
(3.13) \quad |([w, A_0^{-1}[w, h]], \varphi)| \leq C \| w \|_{H^2_0(\Omega)} \| h \|_{H^2_0(\Omega)} \| \varphi \|_{H^2_0(\Omega)}.
\]
Then taking (3.9) with (3.13) gives us the estimate in (2.7). Applying Theorem 2.6, we have the result. \( \Box \)

**Proof of Theorem 2.3**: Here we would like to use the following strong regularity result from [F-L].

**Theorem 3.2.** [F-L] Theorem 2.3 – regular revisited: In addition to the assumptions of the previous theorem (our Theorem 2.6) assume that \( F \) is twice Fréchet differentiable \( D(A^{1/2}) \rightarrow V' \). Moreover, assume
\[
(3.14) \quad \tilde{M}^{-1} \in \mathcal{L}(H; D(A^{1/2}))
\]
\[
(3.15) \quad F(w_0) \in H;
\]
and
\[
(3.16) \quad \left\{ \begin{array}{l}
(i) \quad w_0 + \beta GG^* Aw_1 \in D(\tilde{A}) \\
(ii) \quad A(w_1 + \beta GG^* A[-M^{-1}[\tilde{A}(w_0 + \beta GG^* Aw_1) - F(w_0)]]) \in V'.
\end{array} \right.
\]
Then,
\[
(3.17) \quad (w_{tt}, w_{ttt}) \in C([0, T]; D(A^{1/2}) \times V);
\]
\[
(3.18) \quad \left\{ \begin{array}{l}
\tilde{A}(w + \beta GG^* Aw_t) - F(w) \in C([0, T]; H), \\
\tilde{A}(w_{tt} + \beta GG^* Aw_{tt}) - D \mathcal{F}(w)w_t \in C([0, T]; V'),
\end{array} \right.
\]
and the equation
\[
(3.19) \quad Mw_{tt} + A(w(t) + \beta GG^* Aw_t(t)) - F(w(t)) = 0
\]
holds for all \( t \geq 0 \) on \( H \).
Unfortunately, system (1.1) fails to satisfy hypothesis (3.14), since for general \( L^2 \)-functions, \( \mathcal{M}^{-1} \) cannot recover both boundary conditions on \( \Gamma_0 \). However, to follow the proof of the theorem given in [F-L], we need only

\[
(3.20) \quad M^{-1}A(w_0 + \beta GG^*Aw_1) + M^{-1}\mathcal{F}(w_0) \in \mathcal{D}(\mathcal{A}^{1/2}),
\]

which, in terms of system (1.1) requires \( w_{tt}(0) \in \mathcal{D}(\mathcal{A}^{1/2}) \). By virtue of hypothesis on \( w_0, w \in \mathcal{D}(\mathcal{A}^{1/2}) \), it suffices that \( \mathcal{M}^{-1} : L^2(\Omega) \to H^2(\Omega) \). But this follows directly from the definition of \( \mathcal{M} \). Consequently, system (1.1) satisfies the weaker, but sufficient, hypothesis (3.20). We now show that under the hypotheses of Theorem 2.3, we may apply the modified version of Theorem 3.1 to system (1.1).

By straightforward computations one can see that the von Kármán nonlinearity is twice Frechét differentiable with

\[
(3.21) \quad D^2\mathcal{F}(w)(h, v) = [-2A_0^{-1}[w, h], v]
+ \quad [-2A_0^{-1}[v, h], w] + [h, -2A_0^{-1}[w, v]].
\]

By Lemma 3.1 we see that for \( w, h, v \in H^2_{\Gamma_0}(\Omega) \) with \( \varepsilon < 1/2 \),

\[
\|D^2\mathcal{F}(w)(h, v)\|_{H^2_{\Gamma_0}(\Omega)} \leq C\|w\|_{H^2_{\Gamma_0}(\Omega)}\|h\|_{H^2_{\Gamma_0}(\Omega)}\|v\|_{H^2_{\Gamma_0}(\Omega)}.
\]

By hypothesis (2.3)(i), we see that \( \mathcal{F}(w_0) \in L^2(\Omega) \) is trivially satisfied.

In terms of the p.d.e., (3.16)(i) is equivalent to (2.3)(i) with (2.2)(ii). We also observe that by (2.4)

\[
-\mathcal{M}^{-1}[A(w_0 + \beta GG^*Aw_1) - \mathcal{F}(w_0)] = w_{tt}(0),
\]

So that the p.d.e. equivalent of (3.16)(ii) is

\[
\Delta^2 w_1 \in [H^1_{\Gamma_0}(\Omega)]',
\]

\[
w_1 = \frac{\partial w_1}{\partial \nu} = 0 \quad \text{on} \quad \Gamma_0,
\]

\[
\Delta w_1 + (1 - \mu)B_1 w_1 = -\frac{\partial}{\partial \nu} w_{tt}(0)
\]

\[
\frac{\partial \Delta w_1}{\partial \nu} + (1 - \mu)B_2 w_1 = w_{tt} - \frac{\partial^2}{\partial \tau^2} w_{tt}(0) \quad \text{on} \quad \Gamma_1.
\]

But these are precisely satisfied by hypothesis (2.3).

Applying the results of Theorem 3.1, we obtain the regularity results of Theorem 2.3. \( \square \)
REFERENCES


<table>
<thead>
<tr>
<th>#</th>
<th>Title</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1041</td>
<td>Least squares estimation of the linear model with autoregressive errors</td>
<td>Neerchal K. Nagaraj and Wayne A. Fuller</td>
</tr>
<tr>
<td>1042</td>
<td>A characterization of continuous dependence of trajectories with respect to the input for control-affine systems</td>
<td>H.J. Sussmann &amp; W. Liu</td>
</tr>
<tr>
<td>1043</td>
<td>Protocol verification using discrete-event systems</td>
<td>Karen Rudie &amp; W. Murray Wonham</td>
</tr>
<tr>
<td>1044</td>
<td>Nucleation, kinetics and admissibility criteria for propagating phase boundaries</td>
<td>Rohan Abeyaratne &amp; James K. Knowles</td>
</tr>
<tr>
<td>1045</td>
<td>Computation of pseudo-differential operators</td>
<td>Gang Bao &amp; William W. Symes</td>
</tr>
<tr>
<td>1046</td>
<td>Nonsmooth analysis and shape optimization in flow problem</td>
<td>Srdjan Stojanovic</td>
</tr>
<tr>
<td>1047</td>
<td>Row ordering in sparse QR decomposition</td>
<td>Miroslav Tuma</td>
</tr>
<tr>
<td>1048</td>
<td>On the computation of suboptimal $H^\infty$ controllers for unstable infinite dimensional systems</td>
<td>Onur Toker &amp; Hitay Özbay</td>
</tr>
<tr>
<td>1049</td>
<td>$H^\infty$ optimal controller design for a class of distributed parameter systems</td>
<td>Hitay Özbay</td>
</tr>
<tr>
<td>1050</td>
<td>The Weierstrass condition for a special class of elastic materials</td>
<td>J.E. Dunn &amp; Roger Fosdick</td>
</tr>
<tr>
<td>1051</td>
<td>A free boundary problem arising in the modeling of internal oxidation of binary alloys</td>
<td>Bei Hu &amp; Jianhua Zhang</td>
</tr>
<tr>
<td>1052</td>
<td>Global attractors for semilinear wave equations with locally distributed nonlinear damping and critical exponent</td>
<td>Eduard Feireisl &amp; Enrique Zuazua</td>
</tr>
<tr>
<td>1053</td>
<td>Stability of equilibria for a class of time-reversible, $D_\alpha\mathcal{O}(2)$-symmetric homogeneous vector fields</td>
<td>I-Heng McComb &amp; Chjan C. Lim</td>
</tr>
<tr>
<td>1054</td>
<td>A state-space approach to a one-dimensional mathematical model for the dynamics of phase transitions in pseudoelastic materials</td>
<td>Ruben D. Spies</td>
</tr>
<tr>
<td>1055</td>
<td>Multiphase averaging for generalized flows on manifolds</td>
<td>H.S. Dumas, F. Golse, and P. Lochak</td>
</tr>
<tr>
<td>1056</td>
<td>Global solutions and quenching to a class of quasilinear parabolic equations</td>
<td>Bei Hu &amp; Hong-Ming Yin</td>
</tr>
<tr>
<td>1057</td>
<td>Projection finite element methods for semiconductor device equations</td>
<td>Zhangxin Chen</td>
</tr>
<tr>
<td>1058</td>
<td>Statistical analysis of biological monitoring data</td>
<td>Peter Guttorp</td>
</tr>
<tr>
<td>1059</td>
<td>Abnormal sub-Riemannian minimizers</td>
<td>Wensheng Liu &amp; Héctor J. Sussmann</td>
</tr>
<tr>
<td>1060</td>
<td>A combinatorial perturbation method and Arnold's whiskered Tori in vortex dynamics</td>
<td>Chjan C. Lim</td>
</tr>
<tr>
<td>1061</td>
<td>Axially symmetric jet flows arising from high speed fiber coating</td>
<td>Yong Liu</td>
</tr>
<tr>
<td>1062</td>
<td>$H_2$ and $H_\infty$ designs of multirate sampled-data systems</td>
<td>Li Qiu &amp; Tongwen Chen</td>
</tr>
<tr>
<td>1063</td>
<td>Maximum principle for state-constrained optimal control problems governed by quasilinear elliptic equations</td>
<td>Eduardo Casas &amp; Jiongmin Yong</td>
</tr>
<tr>
<td>1064</td>
<td>Optimal control for degenerate parabolic equations with logistic growth</td>
<td>Suzanne M. Lenhart &amp; Jiongmin Yong</td>
</tr>
<tr>
<td>1065</td>
<td>Optimal control of a convective-diffusive fluid problem</td>
<td>Suzanne Lenhart</td>
</tr>
<tr>
<td>1066</td>
<td>Weakly nonlinear large time behavior in scalar convection-diffusion equations</td>
<td>Enrique Zuazua</td>
</tr>
<tr>
<td>1067</td>
<td>Approximate controllability of the semilinear heat equation</td>
<td>Caroline Fabre, Jean-Pierre Puel &amp; Enrike Zuazua</td>
</tr>
<tr>
<td>1068</td>
<td>Entropy solutions for diffusion-convection equations with partial diffusivity</td>
<td>M. Escobedo, J.L. Vazquez &amp; Enrike Zuazua</td>
</tr>
<tr>
<td>1069</td>
<td>A diffusion-convection equation in several space dimensions</td>
<td>M. Escobedo, J.L. Vazquez &amp; Enrike Zuazua</td>
</tr>
<tr>
<td>1070</td>
<td>Symmetries of differential systems</td>
<td>F. Fagnani &amp; J.C. Willems</td>
</tr>
<tr>
<td>1071</td>
<td>Mixed-RDG finite element methods for the 2-D hydrodynamic model for semiconductor device simulation</td>
<td>Zhangxin Chen, Bernardo Cockburn, Joseph W. Jerome &amp; Chi-Wang Shu</td>
</tr>
<tr>
<td>1072</td>
<td>Bilinear optimal control of a Kirchhoff plate</td>
<td>M.E. Bradley &amp; Suzanne Lenhart</td>
</tr>
<tr>
<td>1073</td>
<td>A cornucopia of abnormal subriemannian minimizers. Part I: The four-dimensional case</td>
<td>Héctor J. Sussmann</td>
</tr>
<tr>
<td>1074</td>
<td>Transfer function approach to disturbance decoupling problem</td>
<td>Marek Rakowski</td>
</tr>
<tr>
<td>1075</td>
<td>Optimal control of Ginzburg-Landau equation for superconductivity</td>
<td>Yuncheng You</td>
</tr>
<tr>
<td>1076</td>
<td>Global dynamics of dissipative modified Korteweg-de Vries equations</td>
<td>Yuncheng You</td>
</tr>
<tr>
<td>1077</td>
<td>Nonuniformly attracting inertial manifolds and stabilization of beam equations with structural and Balakrishnan-Taylor damping</td>
<td>Mario Taboada &amp; Yuncheng You</td>
</tr>
<tr>
<td>1078</td>
<td>Global existence and regularity of solutions of the nonlinear string equation</td>
<td>Michael Böhm &amp; Mario Taboada</td>
</tr>
<tr>
<td>1079</td>
<td>BDM mixed methods for a nonlinear elliptic problem</td>
<td>Zhangxin Chen</td>
</tr>
<tr>
<td>1080</td>
<td>On the dynamics of a closed thermosyphon</td>
<td>J.J.L. Velázquez</td>
</tr>
<tr>
<td>1081</td>
<td>Some stability concepts and their applications in optimal control problems</td>
<td>Frédéric Bonnans &amp; Eduardo Casas</td>
</tr>
<tr>
<td>1082</td>
<td>$L^2$-estimates for parabolic equations and applications</td>
<td>Hong-Ming Yin</td>
</tr>
<tr>
<td>1083</td>
<td>Smoothing and decay properties of solutions of the Korteweg-de Vries equation on a periodic domain with point dissipation</td>
<td>David L. Russell &amp; Bing-Yu Zhang</td>
</tr>
<tr>
<td>1084</td>
<td>Fluids of differential type: Critical review and thermodynamic analysis</td>
<td>J.E. Dunn &amp; K.R. Rajagopal</td>
</tr>
<tr>
<td>1085</td>
<td>Global stabilization of the von Kármán plate with boundary</td>
<td>Mary Elizabeth Bradley &amp; Mary Ann Horn</td>
</tr>
</tbody>
</table>
Mary Ann Horn & Irena Lasiecka, Global stabilization of a dynamic von Kármán plate with nonlinear boundary feedback

Vilmos Komornik, Decay estimates for a petrováki system with a nonlinear distributed feedback

Jesse L. Barlow, Perturbation results for nearly uncoupled Markov chains with applications to iterative methods

Jong-Shenq Guo, Large time behavior of solutions of a fast diffusion equation with source

Tongwen Chen & Li Qiu, $\mathcal{H}_\infty$ design of general multirate sampled-data control systems

Satyanad Kichenassamy & Walter Littman, Blow-up surfaces for nonlinear wave equations, I

Nahum Shimkin, Asymptotically efficient adaptive strategies in repeated games, Part I: certainty equivalence strategies

Caroline Fabre, Jean-Pierre Puel & Enrique Zuazua, On the density of the range of the semigroup for semilinear heat equations

Robert F. Stengel, Laura R. Ray & Christopher I. Marrison, Probabilistic evaluation of control system robustness

H.O. Fattorini & S.S. Sritharan, Optimal chattering controls for viscous flow

Kathryn E. Lenz, Properties of certain optimal weighted sensitivity and weighted mixed sensitivity designs

Gang Bao & David C. Dobson, Second harmonic generation in nonlinear optical films

Avner Friedman & Chaocheng Huang, Diffusion in network

Xinfu Chen, Avner Friedman & Tsuyoshi Kimura, Nonstationary filtration in partially saturated porous media

Walter Littman & Baisheng Yan, Rellich type decay theorems for equations $P(D)u = f$ with $f$ having support in a cylinder

Satyanad Kichenassamy & Walter Littman, Blow-up surfaces for nonlinear wave equations, II

Nahum Shimkin, Extremal large deviations in controlled I.I.D. processes with applications to hypothesis testing

A. Narain, Interfacial shear modeling and flow predictions for internal flows of pure vapor experiencing film condensation

Andrew Teel & Laurent Praly, Global stabilizability and observability imply semi-global stabilizability by output feedback

Karen Rudie & Jan C. Willems, The computational complexity of decentralized discrete-event control problems

John A. Burns & Ruben D. Spies, A numerical study of parameter sensitivities in Landau-Ginzburg models of phase transitions in shape memory alloys

Gang Bao & William W. Symes, Time like trace regularity of the wave equation with a nonsmooth principal part

Lawrence Markus, A brief history of control

Richard A. Brualdi, Keith L. Chavey & Bryan L. Shader, Bipartite graphs and inverse sign patterns of strong sign-nonsingular matrices

A. Kersch, W. Morokoff & A. Schuster, Radiative heat transfer with quasi-monte carlo methods

Jianhua Zhang, A free boundary problem arising from swelling-controlled release processes

Walter Littman & Stephen Taylor, Local smoothing and energy decay for a semi-infinite beam pinned at several points and applications to boundary control

Srdjan Stojeanovic & Thomas Svobodny, A free boundary problem for the Stokes equation via nonsmooth analysis

Bronislaw Jakubczyk, Filtered differential algebras are complete invariants of static feedback

Boris Mordukhovich, Discrete approximations and refined Euler-Lagrange conditions for nonconvex differential inclusions

Bei Hu & Hong-Ming Yin, The profile near blowup time for solution of the heat equation with a nonlinear boundary condition

Jin Ma & Jiongmin Yong, Solvability of forward-backward SDEs and the nodal set of Hamilton-Jacobi-Bellman Equations

Chaocheng Huang & Jiongmin Yong, Coupled parabolic and hyperbolic equations modeling age-dependent epidemic dynamics with nonlinear diffusion

Jiongmin Yong, Necessary conditions for minimax control problems of second order elliptic partial differential equations

Eitan Altman & Nahum Shimkin, Worst-case and Nash routing policies in parallel queues with uncertain service allocations

Nahum Shimkin & Adam Shwartz, Asymptotically efficient adaptive strategies in repeated games, part II: Asymptotic optimality

M.E. Bradley, Well-posedness and regularity results for a dynamic Von Kármán plate

Zhangxin Chen, Finite element analysis of the 1D full drift diffusion semiconductor model

Gang Bao & David C. Dobson, Diffractive optics in nonlinear media with periodic structure

Steven Cox & Enrique Zuazua, The rate at which energy decays in a damped string

Anthony W. Leung, Optimal control for nonlinear systems of partial differential equations related to ecology

H.J. Sussmann, A continuation method for nonholonomic path-finding problems